Métodos Matemáticos 1
Tarea 2
Espacios Vectoriales: Vectores e índices
Marzo 2004

1. Demuestre

$$
\begin{aligned}
\delta^j_j &= 3 \\
\varepsilon_{jkm}\varepsilon^{ilm} &= \delta^i_j\delta^l_k - \delta^i_k\delta^l_j = \delta^i_j\delta^l_k - \delta^l_j\delta^i_k \\
\varepsilon_{jmn}\varepsilon^{imn} &= 2\delta^i_j, \\
\varepsilon_{ijk}\varepsilon^{ijk} &= 6.
\end{aligned}
$$

2. Muestre que en general

vector	$\cdot$	vector	$=$	escalar
vector	$\cdot$	pseudovector	$=$	pseudoescalar
pseudovector	$\cdot$	pseudovector	$=$	escalar
vector	$\times$	vector	$=$	pseudovector
vector	$\times$	pseudovector	$=$	vector
pseudovector	$\times$	pseudovector	$=$	pseudovector

3. Datos los vectores $\vec{a}, \vec{b}, \vec{c}$ y $\vec{d}$ denotando el producto mixto $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}$ por $\left[\vec{a}, \vec{b}, \vec{c}\right]$ entonces, demuestre las siguientes igualdades vectoriales

   (a) $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = (\vec{c} \cdot \vec{a})\,\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c}$

   (b) $\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right) = (\vec{a} \cdot \vec{c})\left(\vec{b} \cdot \vec{d}\right) - \left(\vec{a} \cdot \vec{d}\right)\left(\vec{b} \cdot \vec{c}\right)$

   (c) $\left(\vec{a} \times \vec{b}\right) \cdot \left\{\left(\vec{b} \times \vec{c}\right) \times (\vec{c} \times \vec{a})\right\} = \left[\vec{a}, \vec{b}, \vec{c}\right]^2$

   (d) $\left\{\vec{d} \times \left(\vec{a} \times \vec{b}\right)\right\} \cdot (\vec{a} \times \vec{c}) = \left[\vec{a}, \vec{b}, \vec{c}\right]\left(\vec{a} \cdot \vec{d}\right)$

   (e) $\left(\vec{a} \times \vec{b}\right) \times \vec{c} - \vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \cdot \vec{b}\right)\vec{c} - \left(\vec{b} \cdot \vec{c}\right)\vec{a}$

   (f) $\times \left\{\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right\} = \|\vec{a}\|^2 \left(\vec{b} \times \vec{a}\right)$

4. Si denotamos el vector $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$ y $\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{\mathbf{k}}$ de tal modo que

$$\vec{\nabla} \times \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_z}{\partial y}\right)\hat{\mathbf{k}}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

demuestre

(a) $\vec{\nabla}\left(\vec{a}\cdot\vec{b}\right) = \left(\vec{a}\cdot\vec{\nabla}\right)\vec{b} + \left(\vec{b}\cdot\vec{\nabla}\right)\vec{a} + \vec{a}\times\left(\vec{\nabla}\times\vec{b}\right) + \vec{b}\times\left(\vec{\nabla}\times\vec{a}\right)$

(b) $\vec{\nabla}\cdot\left(\vec{a}\times\vec{b}\right) = \vec{b}\cdot\left(\vec{\nabla}\times\vec{a}\right) - \vec{a}\cdot\left(\vec{\nabla}\times\vec{b}\right)$

(c) $\vec{\nabla}\times\left(\vec{a}\times\vec{b}\right) = \vec{a}\left(\vec{\nabla}\cdot\vec{b}\right) - \vec{b}\left(\vec{\nabla}\cdot\vec{a}\right) + \left(\vec{b}\cdot\vec{\nabla}\right)\vec{a} - \left(\vec{a}\cdot\vec{\nabla}\right)\vec{b}$