

Métodos Matemáticos 1
Tarea 2
Espacios Vectoriales: Vectores e índices
Marzo 2004

1. Demuestre

$$\begin{aligned}\delta_j^j &= 3 \\ \varepsilon_{jkm}\varepsilon^{ilm} &= \delta_j^i\delta_k^l - \delta_k^i\delta_j^l = \delta_j^i\delta_k^l - \delta_j^l\delta_k^i \\ \varepsilon_{jmn}\varepsilon^{imn} &= 2\delta_j^i, \\ \varepsilon_{ijk}\varepsilon^{ijk} &= 6.\end{aligned}$$

2. Muestre que en general

vector	·	vector	=	escalar
vector	·	pseudovector	=	pseudoescalar
pseudovector	·	pseudovector	=	escalar
vector	×	vector	=	pseudovector
vector	×	pseudovector	=	vector
pseudovector	×	pseudovector	=	pseudovector

3. Datos los vectores $\vec{a}, \vec{b}, \vec{c}$ y \vec{d} denotando el producto mixto $(\vec{a} \times \vec{b}) \cdot \vec{c}$ por $[\vec{a}, \vec{b}, \vec{c}]$ entonces, demuestre las siguientes igualdades vectoriales

- (a) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{c} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
- (b) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$
- (c) $(\vec{a} \times \vec{b}) \cdot \{(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\} = [\vec{a}, \vec{b}, \vec{c}]^2$
- (d) $\{\vec{d} \times (\vec{a} \times \vec{b})\} \cdot (\vec{a} \times \vec{c}) = [\vec{a}, \vec{b}, \vec{c}](\vec{a} \cdot \vec{d})$
- (e) $(\vec{a} \times \vec{b}) \times \vec{c} - \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{b})\vec{c} - (\vec{b} \cdot \vec{c})\vec{a}$
- (f) $\times \left\{ \vec{a} \times (\vec{a} \times \vec{b}) \right\} = \|\vec{a}\|^2 (\vec{b} \times \vec{a})$

4. Si denotamos el vector $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ y $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$ de tal modo que

$$\vec{\nabla} \times \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{i} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{j} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \hat{k}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

demuestre

- (a) $\vec{\nabla} (\vec{a} \cdot \vec{b}) = (\vec{a} \cdot \vec{\nabla}) \vec{b} + (\vec{b} \cdot \vec{\nabla}) \vec{a} + \vec{a} \times (\vec{\nabla} \times \vec{b}) + \vec{b} \times (\vec{\nabla} \times \vec{a})$
- (b) $\vec{\nabla} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{\nabla} \times \vec{a}) - \vec{a} \cdot (\vec{\nabla} \times \vec{b})$
- (c) $\vec{\nabla} \times (\vec{a} \times \vec{b}) = \vec{a} (\vec{\nabla} \cdot \vec{b}) - \vec{b} (\vec{\nabla} \cdot \vec{a}) + (\vec{b} \cdot \vec{\nabla}) \vec{a} - (\vec{a} \cdot \vec{\nabla}) \vec{b}$