General Relativistic Radiation Hydrodynamics and Galaxy Rotation Curves without Dark Matter ?

F. Aguirre, L A. Núñez,

Universidad de Los Andes, Mérida Venezuela

P. S. Apostolopoulos, J. Carot y

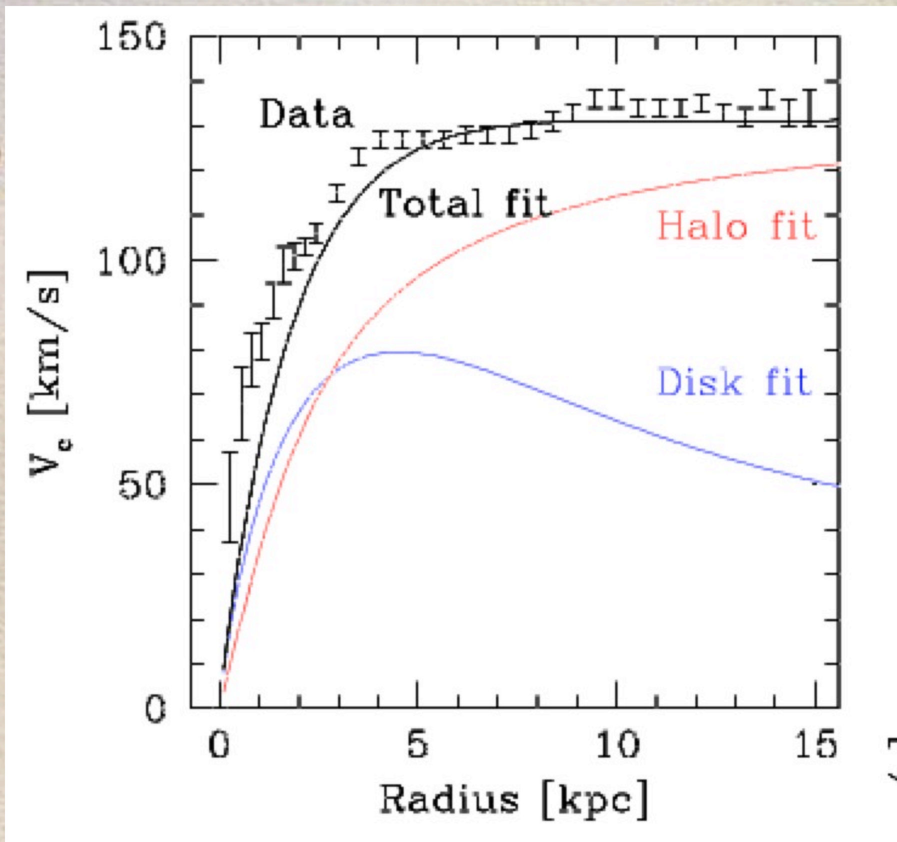
Universitat de les Illes Ballears, Mallorca España

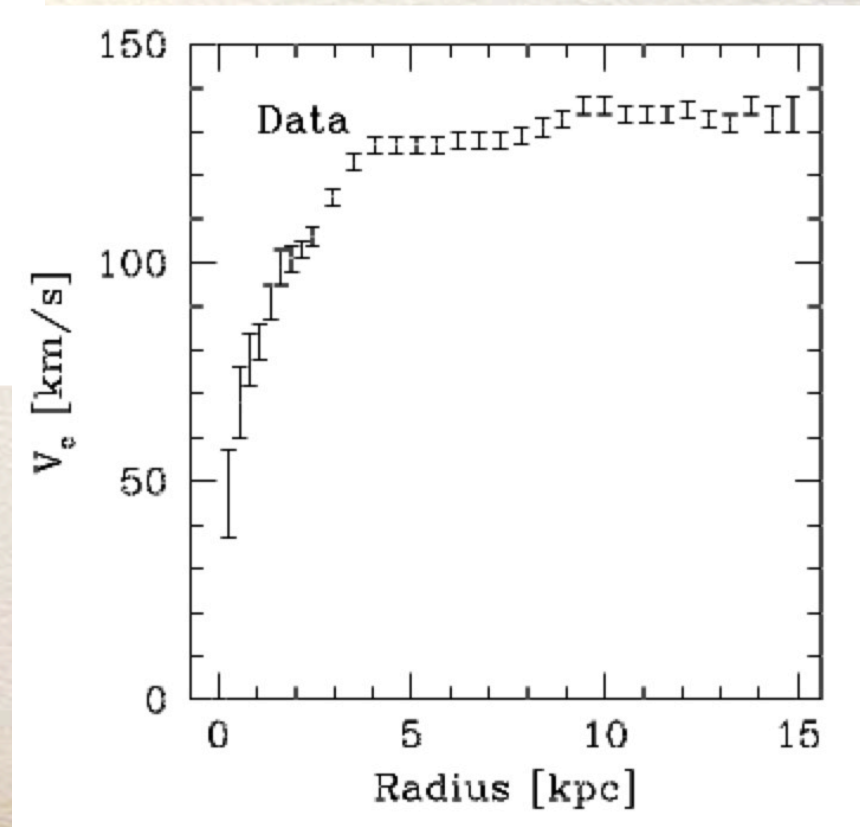
A. Muñoz.

Universidad del Zulia, Maracaibo Venezuela.

Dark Matter or GR for Galaxy Modeling: to be or not to be

- Cooperstock & Tieu (2005) astro-ph/0507619 stationary, geodesic, axial symmetric, relativistic model with no dark matter
- Korzynski (2005) astro-ph/0508377: Cooperstock-Tieu disk is singular
- Vogt and Letelier (2005) astro-ph/0510750: Cooperstock-Tieu singular disk has exotic matter
- Garfinkle (2005) gr-qc/051108 Even GR models need dark matter.


150
100
50
0
0
5
10
15
Data
Total fit
Halo fit
Disk fit
V_c [km/s]
Radius [kpc]


150
100
50
0
0
5
10
15
Data
V_c [km/s]
Radius [kpc]

Exploring the need for Dark Matter with….

Axisymmetric, Slowly rotating configuration. $\tilde{\alpha} \sim \mathcal{D} \sim \mathcal{G} \sim \omega_\phi \ll 1$

$$ds^2 = e^{2\beta}\left\{\frac{V}{r}\mathrm{d}u^2 + 2\mathrm{d}u\mathrm{d}r\right\} - (r^2 + \tilde{\alpha}^2\cos^2\theta)\mathrm{d}\theta^2 + 2\tilde{\alpha}e^{2\beta}\sin^2\theta\left\{1 - \frac{V}{r}\right\}\mathrm{d}u\mathrm{d}\phi$$
$$-2e^{2\beta}\tilde{\alpha}\sin^2\theta\mathrm{d}r\mathrm{d}\phi - \sin^2\theta\left\{r^2 + \tilde{\alpha}^2 + 2\tilde{\alpha}^2\sin^2\theta\frac{V}{r}\right\}\mathrm{d}\phi^2.$$

where $V = e^{2\beta}\left(r - \frac{2\tilde{m}(u,r,\theta)r^2}{r^2 + \tilde{\alpha}^2\cos^2\theta}\right)$

Energy Momentum Tensor

$$T_{\mu\nu} = \left(\rho + P + \frac{1}{2}(\rho_R - \mathcal{P})\right)u_\mu u_\nu - \left(P + \frac{1}{2}(\rho_R - \mathcal{P})\right)g_{\mu\nu}$$
$$+\left(P + \mathcal{P} - \left(P + \frac{1}{2}(\rho_R - \mathcal{P})\right)\right)\chi_\mu\chi_\nu + \mathcal{F}_{(\mu}u_{\nu)} + 2\mathcal{F}_{(\mu}\mathcal{D}_{\nu)} + 2\mathcal{G}u_{(\mu}\mathcal{D}_{\nu)},$$

with $D_\mu = \mathcal{O}(\omega_z^2)\delta_\mu^u + \mathcal{O}(\omega_z^2)\delta_\mu^r + \left[\mathcal{D}r\sin\theta + \mathcal{O}(\omega_z^2)\right]\delta_\mu^\phi, \quad \mathcal{F}_\mu = -\mathcal{F}\chi_\mu$

Radiation Hydrodynamics scenario

$$\mathrm{d}\mathcal{E} = \mathbf{I}(r,t;\vec{n},\nu)\mathrm{d}S\ \cos\varphi\ \mathrm{d}\Theta\ \mathrm{d}\upsilon\ \mathrm{d}t,$$

$$\rho_R = \frac{1}{2}\int_0^\infty d\nu \int_1^{-1} d\mu\ \mathbf{I}(r,t;\vec{n},\nu), \qquad \mathcal{F} = \frac{1}{2}\int_0^\infty d\nu \int_1^{-1} d\mu\ \mu\,\mathbf{I}(r,t;\vec{n},\nu)$$

$$\mathcal{P} = \frac{1}{2}\int_0^\infty d\nu \int_1^{-1} d\mu\ \mu^2\mathbf{I}(r,t;\vec{n},\nu)$$

with $f = \frac{\mathcal{F}}{\rho_R}$ and $\chi = \frac{\mathcal{P}}{\rho_R}$

$$\left.\begin{array}{l} \mathcal{F} = \mathcal{P} = \rho_R \Rightarrow f = 1 \quad \chi = 1 \\ \mathcal{P} = \frac{1}{3}\rho_R \Rightarrow f \longrightarrow 0; \quad \chi = \frac{1}{3} \end{array}\right\} \Rightarrow \quad 0 \le f \le 1 \quad \text{and} \quad \frac{1}{3} \le \chi(f) \le 1$$

$$\|f\| \le 1, \quad f^2 \le \chi \le 1 \quad \text{and} \quad -\frac{1-\chi}{1+f} \le \frac{\mathrm{d}\chi}{\mathrm{d}f} \le \frac{1-\chi}{1-f}$$

Pons, Ibañez, Miralles (2000) *Mon. Not. R. Astr. Soc*, **317**, 550

Radiation Hydrodynamics scenario

Closure	$\chi(f)$	$\left.\frac{d\chi}{df}\right\rvert_{f=1}$	$\left.\frac{d\chi}{df}\right\rvert_{f=0}$
Lorentz-Eddington (LE)	$\frac{5}{3}-\frac{2}{3}\sqrt{4-3f^2}$	2	0
Bowers-Wilson	$\frac{1}{3}\left(1-f+3f^2\right)$	$\frac{5}{3}$	$-\frac{1}{3}$
Janka (Monte Carlo) (MC)	$\frac{1}{3}\left(1+\frac{1}{2}f^{1.31}+\frac{3}{2}f^{4.13}\right)$	2.28	0
Maximum Packing (MP)	$\frac{1}{3}\left(1-2f+4f^2\right)$	2	$-\frac{2}{3}$
Minerbo (Mi)	$\chi(f)=1-2\frac{f}{\kappa}$ where $f=\coth\kappa-\frac{1}{\kappa}$	2	0
Levermore-Pomraning	$\chi(f)=f\coth\beta$ where $f=\coth\beta-\frac{1}{\beta}$	1	0

Junction Conditions

$$\tilde{m}_{1a}(1+\dot{a}) \approx \alpha \Longrightarrow \frac{a^2 4\pi}{1+\omega_{xa}^2}\left(1-\omega_{xa}\frac{2\,\tilde{m}_a}{a}\right)[\rho_a+\rho_{Ra}-\mathcal{F}_a] \approx \alpha$$

$$\beta_{1a}(1+\dot{a}) \approx \alpha \Longrightarrow \frac{2\pi a\left(1-\omega_{xa}\frac{2\tilde{m}_a}{a}\right)}{\left(1-\frac{2\,\tilde{m}_a}{a}\right)(1+\omega_{xa})}[\rho_a+\rho_{Ra}-\mathcal{F}_a] \approx \alpha$$

And it effects

Closure	$f_{r=a}$	e
Lorentz-Eddington	$\frac{3}{7} \leq f_{LE}\|_{r=a} \leq 1$	$\Lambda \leq e_{LE} \leq \Lambda\left(1+\frac{4}{3}\frac{\mathcal{F}_a}{\rho_a}\right)$
Bowers-Wilson	$\frac{1}{3} \leq f_{BW}\|_{r=a} \leq 1$	$\Lambda \leq e_{BW} \leq \Lambda\left(1+2\frac{\mathcal{F}_a}{\rho_a}\right)$
Janka (Monte Carlo)	$0.39 \leq f_{MC}\|_{r=a} \leq 1$	$\Lambda \leq e_{MC} \leq \Lambda\left(1+1.545\frac{\mathcal{F}_a}{\rho_a}\right)$
Maximum Packing	$\frac{1}{4} \leq f_{MP}\|_{r=a} \leq 1$	$\Lambda \leq e_{MP} \leq \Lambda\left(1+3\frac{\mathcal{F}_a}{\rho_a}\right)$
Minerbo	$0.40 \leq f_{M}\|_{r=a} \leq 1$	$\Lambda \leq e_{M} \leq \Lambda\left(1+1.488\frac{\mathcal{F}_a}{\rho_a}\right)$
Levermore-Pomraning	$f_{LP}\|_{r=a} = 1$	$e_{LP} = \Lambda$

Aguirre, Núñez T. Soldovieri *Variable Eddington Factor and Radiating Slowly Rotating Bodies in General Relativity* gr-qc/0503085 to appear CQG 2006

Seminumeric Approach

defining two auxiliary variables in terms of the Eddington and the Flux factor

$$\widetilde{P} = \frac{P + \mathcal{P} - \mathcal{F} - \omega_x \left(\rho + \rho_R - \mathcal{F}\right)}{1 + \omega_x} \equiv \frac{P - \omega_x \rho + \frac{1}{f}(\chi - f - \omega_x(1 - f))\mathcal{F}}{1 + \omega_x}$$

$$\widetilde{\rho} = \frac{\rho + \rho_R - \mathcal{F} - \omega_x (P + \mathcal{P} - \mathcal{F})}{1 + \omega_x} \equiv \frac{\rho - \omega_x P + \frac{1}{f}(1 - f - \omega_x(\chi - f))\mathcal{F}}{1 + \omega_x}$$

metric elements can be formally integrated from field equations

$$\beta\left(u, r\right) = \int_a^r 2\pi \bar{r} \frac{\widetilde{\rho} + \widetilde{P}}{\left(1 - \frac{2\widetilde{m}}{\bar{r}}\right)} d\bar{r} \qquad \text{and} \qquad \widetilde{m}\left(u, r\right) = \int_0^r 4\pi\ \bar{r}^2\ \widetilde{\rho} d\bar{r}$$

$$G_{\mu\nu}\left(\beta\left(\tilde{\rho}(u,r), \tilde{P}(u,r)\right), \tilde{m}\left(\tilde{\rho}(u,r), \tilde{P}(u,r)\right), \text{and derivatives}\right) = T_{\mu\nu}\left(\rho, \rho_R, P, \mathcal{P}, \mathcal{F}, \omega_r, \omega_\phi, \mathcal{D}\right)$$

Seminumeric Approach

$$\tilde{\rho}(u,r) = \rho_{st}(r)\rho_d(u) \qquad \tilde{P}(u,r) = P_{st}(r)P_d(u)$$

$$\rho_{st}(r = a(u))\rho_d(u) = \tilde{\rho}(A, F, \Omega, \text{and derivatives}) \qquad P_{st}(r = a(u))P_d(u) = \tilde{P}(A, F, \Omega, \text{and derivatives})$$

Field equations at the surface

$$G_{\mu\nu}\left(\beta\left(\tilde{\rho}(u,r), \tilde{P}(u,r)\right), \tilde{m}\left(\tilde{\rho}(u,r), \tilde{P}(u,r)\right), \text{and derivatives}\right) = T_{\mu\nu}\left(\rho, \rho_R, P, \mathcal{P}, \mathcal{F}, \omega_r, \omega_\phi, \mathcal{D}\right)$$

Surface equations

$$\dot{A} = F\,(\Omega - 1) \qquad \frac{\dot{F}}{F} = \frac{2L + (1-F)(\Omega - 1)}{A}$$

$$0 = \frac{\dot{\Omega}}{\Omega} + \frac{\dot{F}}{F} + \frac{(\tilde{\rho}_a)_{,0}}{\tilde{\rho}_a} + \frac{F\Omega^2\tilde{R}}{\tilde{\rho}_a} - \frac{2F\Omega}{A\tilde{\rho}_a}\left(P_a + \frac{\chi(f)\mathcal{F}_a}{2f}(\chi(f) - 1)\right) + (\Omega - 1)\left(\frac{F\Omega\tilde{\rho}_{1a}}{\tilde{\rho}_a} - \frac{4\pi A(1-3\Omega)\tilde{\rho}_a}{\Omega} - \frac{3+F}{2A}\right)$$

Metric elements

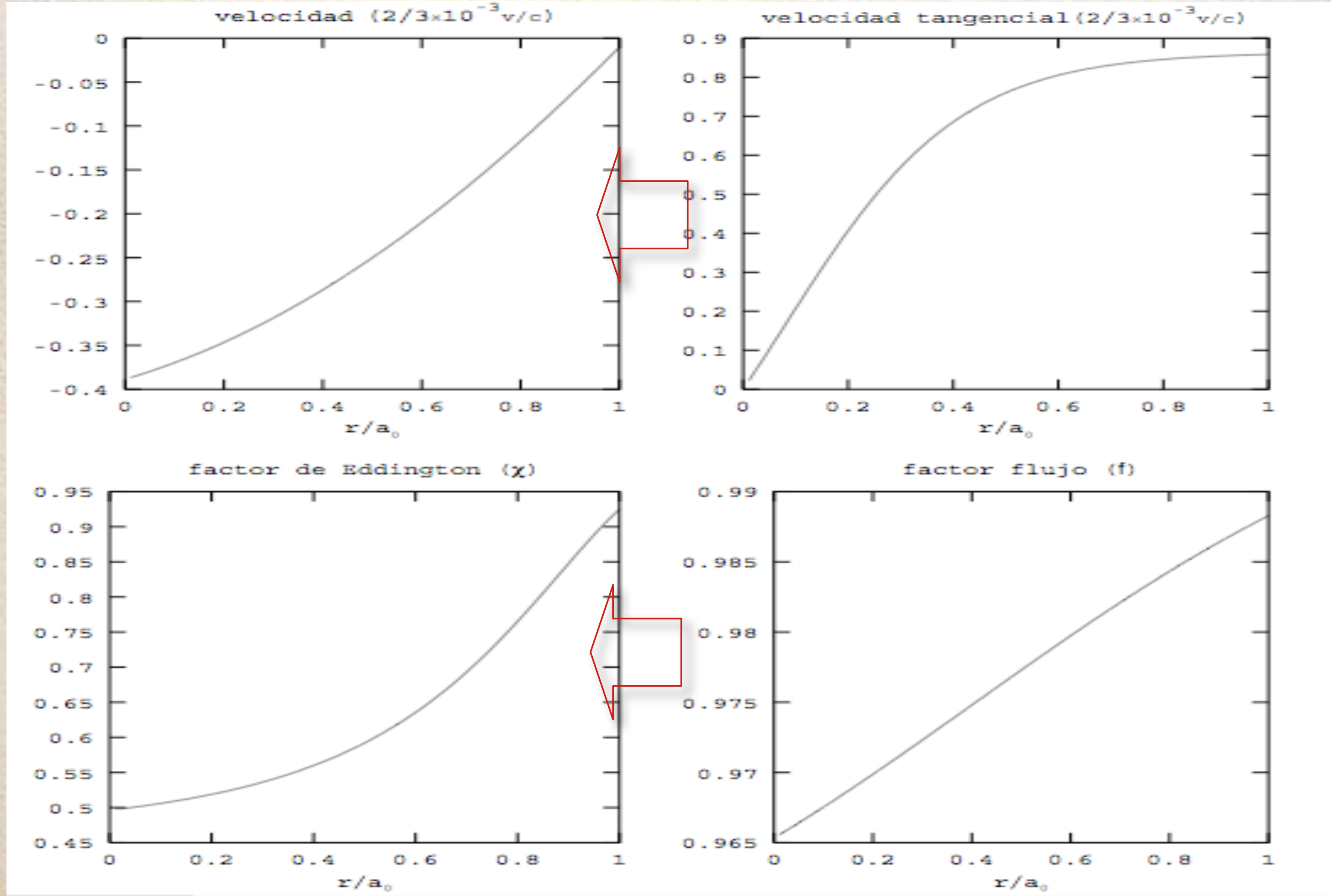
$$\beta\,(u,r) = \int_a^r 2\pi\bar{r}\frac{\tilde{\rho} + \tilde{P}}{\left(1 - \frac{2\tilde{m}}{\bar{r}}\right)}d\bar{r} \qquad \text{and} \qquad \tilde{m}\,(u,r) = \int_0^r 4\pi\;\bar{r}^2\;\tilde{\rho}d\bar{r}$$

Physical Variables **with** $f = \dfrac{\mathcal{F}}{\rho_R}$ and $\chi = \dfrac{\mathcal{P}}{\rho_R}$

$$G_{\mu\nu}\left(\beta\left(\tilde{\rho}(u,r), \tilde{P}(u,r)\right), \tilde{m}\left(\tilde{\rho}(u,r), \tilde{P}(u,r)\right), \text{and derivatives}\right) = T_{\mu\nu}\left(\rho, \rho_R, P, \mathcal{P}, \mathcal{F}, \omega_r, \omega_\phi, \mathcal{D}\right)$$

A Semi-nummeric aproach

$$\tilde{\rho} = \frac{3h(u)}{r^2} = \frac{1}{8\pi r^2}(1-F) \quad \text{and} \quad \tilde{P} = \frac{h(u)}{r^2}\frac{(1-9d(u)r)}{(1-d(u)r)} \qquad \textbf{with} \qquad d(u) = \frac{1}{3}\frac{4\Omega-1}{A(4\Omega-3)}.$$



Dark matter *ma non tropo* but….

The model should be improved

$$e = \frac{1}{r_s}\tilde{\alpha} - \frac{1}{2}\frac{1}{r_s^3}\tilde{\alpha}^3 + \cdots$$

We have found a rotating configuration where radiation variables are determinant to give this particuar velocity profile but …. Tt is low eccentricity and contracting configuration.

It seams that at least for this globular configurations matching up to it second order should be explored