Cardinals topologies and strict topologies

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Abstract

The cardinal topologies Ψ_α^X are introduced in the space of bounded continuous functions on a completely regular Hausdorff space X. If X is a discrete space it is shown that $\mid X \mid \leq \aleph_\alpha$ if and only if the unit ball $B_1(X)$ in $C_b(X)$ is Ψ_α^X-compact, and also, if and only if Ψ_α^X coincides with the topology of pointwise convergence. Also we prove that if X is discrete then β_0 and the Ψ_α^X's can be compared always. We present a characterization of real and Ulam measurable cardinals in terms of the compactness of the unit ball with respect to some known strict topologies.

1 Introduction

Wheeler in [11] characterized a discrete space X as the one for which the unit ball $B_1(X)$ in $C_b(X)$ is β_0-compact, where β_0 is the strict topology introduced by Buck in [1]. Since on a discrete space the only significant property is its cardinality, then it seems natural to ask whether there are topologies on $C_b(X)$ which characterizes the cardinality of X via the compactness of the unit ball. We introduce a family of topologies Ψ_α^X on $C_b(X)$ (that we call *cardinal topologies*) and give a definite answer to that question. We will show that the cardinal topologies we define are always comparable with the strict topology β_0.

There are some characterization of Real and Ulam measurable cardinals in terms of properties of measure spaces (see [3], [5], [6] and [7]). We will

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show that similar results can be proved looking at the compactness of the unit ball in $C_b(X)$ with respect to the strict topologies β_p and β_σ.

2 Preliminaries and notation

Let X be a completely regular Hausdorff space. $B_1(X)$ will denote the closed unit ball, i.e. the set $\{f \in C_b(X) : \|f\| \leq 1\}$, $\mid X \mid$ will denote the cardinality of X. For each $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$, let T_Y be the linear map $T_Y : C_b(X) \to C_b(Y)$ defined by $T_T(f) = f \mid Y$, i.e. the restriction of f to Y.

If (E, τ) is a Hausdorff locally convex topological vector space and E' is its topological dual then $\sigma(E, E')$ and $\tau(E, E')$ denotes the weak and Mackey topologies of the duality $< E, E' >$, respectively (see [8]). As it is customary, any locally convex topology β on E such that $\sigma(E, E') \leq \beta \leq \tau(E, E')$ is said to be consistent with the duality and in this case the dual of (E, β) is E'.

If X is a completely regular Hausdorff space then the topology β_0 is the finest locally convex topology on $C_b(X)$ which coincides on the norm-bounded sets with the compact open topology. The dual of $(C_b(X), \beta_0)$ is the space $M_t(X)$ of tight measures on X (see [12]). If X is locally compact, β_0 coincide with the strict topology of Buck [1], that is to say, β_0 is determined by the seminorms $\| \,.\, \|_h$

$$\| f \|_h = Sup\{\mid f(x)h(x) \mid : x \in X\}$$

where h is a bounded real valued function defined on X, such that $\{x :\mid h(x) \mid \geq \varepsilon\}$ is relatively compact for every $\varepsilon > 0$, i.e. h is a bounded continuous function vanishing at infinity.

When $C_b(X)$ is given the supremum norm $\| \,.\, \|$, we know (by the Alexandroff representation theorem) that its dual is given by the space $M(X)$ of all finite, finitely additive Baire measures on X (see e.g. [12]). βX denotes the Stone-Cech compactification of X. For every set $K \subseteq \beta X - X$ the spaces $C_b(X)$ and $C_b(\beta X - K)$ are isomorphic. Then the topology β_0 on $C_b(\beta X - K)$ induces a topology β_K on $C_b(X)$ which makes this two spaces homeomorphic. If we consider on $C_b(X)$ the inductive topology induced by $(C_b(X), \beta_K)$ and the identity maps when K runs on a family of subsets of $\beta X - X$, the topology obtained is often called a strict topology.

The strict topologies we are going to use in this paper are the following:

(1) If $\mathcal{K} = \{K \subseteq \beta X - X : K$ is compact $\}$ the strict topology obtained is denoted by β_τ and the dual of $(C_b(X), \beta_\tau)$ is known to be the space $M_\tau(X)$ of all Baire τ-additive measure over X (see [5]).

(2) If $\mathcal{K} = \{Z \subseteq \beta X - X : Z$ is a zero set $\}$ we get the topology β_σ which gives as dual the space $M_\sigma(X)$ of all Baire σ-additive measure over X (see [5]).

(3) If $\mathcal{K} = \{D \subseteq \beta X - X : D$ is a distinguished set $\}$ the strict topology we obtain is β_p and the corresponding dual space is $M_p(X)$ of all Baire perfect measure on X (see [5]).

(4) Finally, if $\mathcal{K} = \{C \subseteq \beta X - X :$ There is partition of unity $(f_\alpha)_{\alpha \in A}$ for X such that $f_\alpha \mid C = 0$ for all $\alpha \in A\}$ we obtain the topology β_μ which deals as dual the space $M_\mu(X)$ of all μ-additive Baire measure over X (see [5]).

We will be using β_z as a generic symbol for the various strict topologies used in this paper and $M_z(X)$ will denote its corresponding dual space. Let us recall that on a discrete space X we have that $M_t(X) = M_\tau(X) = M_\mu(X)$. If τ and τ^* are topologies on some space, $\tau \leq \tau^*$ will denote that τ^* is finer than τ. All topologies on $C_b(X)$ use in this paper will be finer than the pointwise topology, which will be denoted by t_p. In fact we have that $t_p \leq \beta_0 \leq \beta_z$. Our set theoretic notation is standard as in [4].

3 Cardinal topologies and main result

Now we introduce the cardinal topologies on the space $C_b(X)$.

Definition 3.1 *The topology Ψ_0^X on $C_b(X)$ is defined as the projective topology induced by the spaces $(C_b(Y), \| . \|)$ and the restriction maps T_Y where $Y \subseteq X$ and $\mid Y \mid < \mid X \mid$, i.e. the smallest topology for which the maps T_Y's are continuous.*

By transfinite induction we define the topologies Ψ_α^X on $C_b(X)$ for every ordinal α. If $\alpha = \beta + 1$, then Ψ_α^X is the projective topology induced by the spaces $(C_b(Y), \Psi_\beta^Y)$ and the restriction maps T_Y, where $Y \subseteq X$ and $\mid Y \mid < \mid X \mid$. Finally, if α is a limit ordinal, then Ψ_α^X is the projective topology induced by $(C_b(Y), \bigcap_{\beta < \alpha} \Psi_\beta^Y)$ and the restriction maps T_Y, where $Y \subseteq X$ and $\mid Y \mid < \mid X \mid$.

The main theorem is the following:

Theorem 3.2 *The following statements are equivalent:*

(i) $\mid X \mid \leq \aleph_\alpha$

(ii) $\Psi_\alpha^X = t_p$

(iii) $B_1(X)$ *is* Ψ_α^X*-compact.*

Before we give the proof we will show some lemmas. The following result is well known and it is the prototype of our result.

Lemma 3.3 *X is finite if and only if $B_1(X)$ is $\| \,.\, \|$-compact.* □

Now we will show some basic facts about the topologies Ψ_α^X.

Lemma 3.4 *(i) For every α, $\Psi_{\alpha+1}^X \leq \Psi_\alpha^X$. Moreover, if α and β are ordinals with $\alpha < \beta$ then $\Psi_\beta^X \leq \Psi_\alpha^X$.*

(ii) For every ordinal α, $t_p \leq \Psi_\alpha^X$, where t_p denotes the topology of pointwise convergence.

(iii) $(C_b(X), \Psi_\alpha^X)$ is a locally convex topological vector space.

Proof: (i) We want to show that $id : (C_b(X), \Psi_\alpha^X) \to (C_b(X), \Psi_{\alpha+1}^X)$ is continuous. So, let $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$ and let us show that $T_Y \, o \, id$ is continuous. It sufficies to show that for every set $Z \subseteq Y$ with $\mid Z \mid < \mid Y \mid$ the map

$$T_Z \, o \, T_Y \, o \, id : (C_b(X), \Psi_\alpha^X) \to (C_b(Z), \bigcap_{\beta<\alpha} \Psi_\beta^Z)$$

is continuous. There are two cases to be considered: either α is a successor ordinal or a limit ordinal. In both cases $T_Z \, o \, T_Y \, o \, id$ is continuous by the definition of Ψ_α^X.

(ii) and (iii) follow easily by induction on α. □

In what follows X is taken to be a discrete space.

Lemma 3.5 *$\Psi_\alpha^X = t_p$ if and only if $\bigcap_{\beta<\alpha} \Psi_\beta^Y = t_p$, for every $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$.*

Proof: Suppose $\Psi_\alpha^X = t_p$ and let $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$. There are two cases to consider: (a) If $\alpha = \gamma + 1$, then we claim that $\Psi_\gamma^Y = t_p$. In fact, by lemma 3.4 (ii) we know that $t_p \leq \Psi_\gamma^Y$. So let (f_η) be a net in $C_b(Y)$ that converges pointwise to zero (by lemma 3.4(iii) we need only to consider such nets). Then let $g_\eta : X \to \Re$ be defined as follows: if $x \in X - Y$ put

$g_\eta(x) = 0$, otherwise put $g_\eta(x) = f_\eta(x)$. Clearly $g_\eta \to 0$ pointwise, so $g_\eta \to 0$ in Ψ_α^X. Hence $T_Y(g_\eta) = f_\eta \to 0$ in Ψ_γ^Y. Thus $\Psi_\gamma^Y \leq t_p$.

When α is a limit ordinal, a similar argument shows that, if (f_η) is a net in $C_b(Y)$ that converges pointwise to zero then $f_\eta \to 0$ in the topology $\bigcap_{\beta<\alpha} \Psi_\beta^Y$.

Conversely, if $\bigcap_{\beta<\alpha} \Psi_\beta^Y = t_p$ for every $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$, then since $T_Y : (C_b(X), t_p) \to (C_b(Y), t_p)$ is continuous it follows that $\Psi_\alpha^X = t_p$. $\square$

Now we come to define a family of subsets of $C_b(X)$ which will be of particular importance when proving the main theorem and also when we try to compare the cardinals topologies Ψ_α^X with the strict topology β_0.

Definition 3.6 *Let $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$ and $\varepsilon > 0$ we define $N(X, Y, \varepsilon)$ as the set*

$$N(X, Y, \varepsilon) = \{f \in C_b(X) : Sup\{\mid f(x) \mid : x \in Y\} < \varepsilon\}$$

The next lemma says that convergence in Ψ_α^X implies uniform convergence over subsets of X of cardinality smaller than certain cardinal less than $\mid X \mid$.

Lemma 3.7 *If $\mid X \mid = \aleph_\lambda$ and $Z \subseteq X$ with $\mid Z \mid < \mid X \mid$ and $\mid Z \mid = \aleph_\beta$ and $\beta + \alpha < \lambda$, then for every $\varepsilon > 0$, $N(X, Z, \varepsilon) \in \Psi_\alpha^X$.*

Proof: The proof goes by induction on α.

(i) For $\alpha = 0$. Let $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$ and $\varepsilon > 0$, then $N(X, Y, \varepsilon) = T_Y^{-1}(Ba_\varepsilon(Y))$, where $Ba_\varepsilon(Y))$ is the open ball of radius ε. Since $T_Y : (C_b(X), \Psi_0^X) \to (C_b(Y), \|\,.\,\|)$ is continuous, then we have $N(X, Y, \varepsilon) \in \Psi_0^X$.

(ii) Suppose $\alpha = \delta + 1$. Let $Z \subseteq X$ such that $\mid Z \mid = \aleph_\beta$ and $\beta + \delta + 1 < \lambda$. Take Y such that $Z \subseteq Y \subseteq X$ and $\mid Y \mid = \aleph_{\beta+\delta+1}$. Then $\mid Z \mid < \mid Y \mid$ and $N(X, Z, \varepsilon) = T_Y^{-1}(N(Y, Z, \varepsilon))$. So, it sufficies to show that $N(Y, Z, \varepsilon) \in \Psi_\delta^Y$. But this follows from the inductive hypothesis, because $\mid Y \mid = \aleph_{\beta+\delta+1}$, $\mid Z \mid = \aleph_\beta$ and $\beta + \delta < \beta + \delta + 1$.

(iii) A similar argument works for the case α a limit ordinal. $\square$

The following corollary will be used in the proof of the main theorem.

Corollary 3.8 *Let $\mid X \mid = \aleph_\lambda$ with $\lambda > 0$ and $\alpha < \lambda$ then for every countable subset $Z \subseteq X$ we have $N(X, Z, \varepsilon) \in \Psi_\alpha^X$.*

Proof: Use the previous lemma for $\beta = 0$. $\square$

Now, we present the proof of the main theorem.

Proof of 3.2: $(i) \Rightarrow (ii)$. By induction on α. (1) For $\alpha = 0$, take $Y \subseteq X$ such that $\mid Y \mid < \mid X \mid$, then Y is finite and the norm and the pointwise topology coincide on $C_b(Y)$. Hence $\Psi_0^X \leq t_p$ and from lemma 3.4(ii) we obtain $\Psi_0^X = t_p$.

(2) Suppose $\alpha = \delta + 1$ and let $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$, then $\mid Y \mid \leq \aleph_\alpha$. By the inductive hypothesis we have that $\Psi_\delta^Y = t_p$ thus $\Psi_\alpha^X \leq t_p$ and by lemma 3.4(ii) we get that $\Psi_\alpha^X = t_p$.

(3) Suppose α is a limit ordinal. If $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$ then there is $\eta < \alpha$ such that $\mid Y \mid \leq \aleph_\eta$. Then by the inductive hypothesis we have $\Psi_\delta^Y = t_p$ for every $\delta > \eta$. Therefore $\bigcap_{\eta < \alpha} \Psi_\eta^Y = t_p$ and by lemma 3.5 we obtain $\Psi_\alpha^X = t_p$.

$(ii) \Rightarrow (iii)$. If $\Psi_\alpha^X = t_p$ then $B_1(X)$ is Ψ_α^X-compact by the Tychonoff theorem.

$(iii) \Rightarrow (i)$. The proof also goes by induction on α. (1) For $\alpha = 0$. Take $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$, then $B_1(Y) = T_Y(B_1(X))$. Hence $B_1(Y)$ is $\| . \|$-compact, so by lemma 3.3 Y is finite. Therefore $\mid X \mid \leq \aleph_0$.

(2) If $\alpha = \delta + 1$, let $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$. As before $B_1(Y)$ is Ψ_δ^Y-compact and by the inductive hypothesis we know that $\mid Y \mid \leq \aleph_\delta$, therefore $\mid X \mid \leq \aleph_{\delta+1}$.

(3) Suppose $\alpha = \lambda$ is a limit ordinal and suppose towards a contradiction that there is $Y \subseteq X$ with $\mid Y \mid < \mid X \mid$ and $\mid Y \mid = \aleph_\lambda$. As before we know that $B_1(Y)$ is $\bigcap_{\eta < \lambda} \Psi_\eta^Y$-compact. We will show that this is not possible.

By corollary 3.8 we know that $\bigcap_{\eta < \lambda} \Psi_\eta^Y$ implies uniform convergence over countable subsets of Y. We will define a sequence on $B_1(Y)$ such that for some countable set Z every subsequence does not converge uniformly on Z.

Let $Z \subseteq Y$ be a countable set. Let $\{x_n\}$ be an enumeration of Z and define for every natural number n a function $f_n \in B_1(Y)$ by

$$f_n(x) = \begin{cases} 1 & , \text{if } x = x_n. \\ 0 & , \text{otherwise.} \end{cases}$$

Clearly $f_n \to 0$ pointwise. Since $Sup\{\mid f_n(x) \mid : x \in Z\} = 1$, it follows that $f_n \notin N(Y, Z, 1/2)$ for every n. $\square$

Remark: If $\Re$ is given the discrete topology, then the continuum Hypothesis can be rephrased as follows: CH holds if and only if the unit ball in $C_b(\Re)$ is $\Psi_1^{\Re}$-compact.

As we said in the introduction Wheeler has characterized the discrete spaces as follows:

Theorem 3.9 *(Wheeler [11]) Let X be a completely regular space, then X is discrete if and only if $B_1(X)$ is β_0-compact.* $\square$

It seems natural to determine the relationship between β_0 and the cardinal topologies we have defined. When X is discrete it is easy to prove the following:

Lemma 3.10 *Let X be a discrete space and (f_α) be a net on $C_b(X)$. Then $f_\alpha \to 0$ in β_0 if and only if for every sequence $(x_n) \in X$ and every sequence $(r_n) \in \Re$ which converges to zero it holds that $Sup\{\mid r_n f_\alpha(x) \mid: n \geq 0\} \to 0$*

$\square$

As a corollary we get

Lemma 3.11 *Let X be a discrete space and τ a topology on $C_b(X)$ such that for every net $(f_\alpha) \in C_b(X)$ with $(f_\alpha) \xrightarrow{\tau} 0$, it holds that for every countable $Z \subseteq X$ $f_\alpha \mid Z \xrightarrow{\|\cdot\|} 0$. Then $\beta_0 \leq \tau$* $\square$

Now we have the following comparison theorem

Theorem 3.12 *Let X be a discrete space. For every α one and only one of the following holds:*
(a) $t_p = \Psi_\alpha^X$
(b) $\beta_0 < \Psi_\alpha^X$
In particular β_0 and Ψ_α^X are always comparable.

Proof: If $\mid X \mid \leq \aleph_\alpha$ then by theorem 3.2 we have $\Psi_\alpha^X = t_p$ and (a) holds. On the other hand, if $\mid X \mid = \aleph_\beta > \aleph_\alpha$, then by lemma 3.8 the hypothesis of the previous lemma are satisfied, hence $\beta_0 \leq \Psi_\alpha^X$. From 3.9 we get $\beta_0 < \Psi_\alpha^X$: otherwise $B_1(X)$ would be Ψ_α^X-compact and therefore by theorem 3.2 $\mid X \mid \leq \aleph_\alpha$ which is a contradiction. $\square$

4 Real and Ulam Measurable Cardinals

Following the general pattern of the main theorem, we present two results which characterizes Real and Ulam measurable cardinals in terms of the compactness of the unit ball in $C_b(X)$. The next lemmas will be used toward that purpose. Let us recall first that a cardinal κ is called **Real measurable** if there is a non-trivial σ-additive positive measure defined on the power set of κ assigning value zero to every singleton set, if the measure is two-valued then κ is called **Ulam measurable**.

Lemma 4.1 *(Koumoullis [6]) Let X be a metric space, then X is real compact if and only if $M_p(X) = M_t(X)$.* □

A subset Y of a topological space X is said to be d-**discrete** if there is a continuous pseudometric on X and $\varepsilon > 0$ such that $d(x, y) > \varepsilon$ for every x and y in X. A topological space X is said to be a D-**space** if whenever Y is a d-discrete subspace then Y has a non-real measurable cardinality. If X is discrete then it is a D-space iff $\mid X \mid$ is not real measurable.

Lemma 4.2 *(Sentilles and Wheeler [7], Haydon [3]) Let X be a completely regular Hausdorff space, then X is a D-space if and only if $M_\sigma(X) = M_\mu(X)$.*

The next theorem is the general fact behind the results of this section.

Theorem 4.3 *Let X be a discrete space and β_z a locally convex topology on $C_b(X)$. Then $B_1(X)$ is $\sigma(C_b(X), M_z(X))$-compact if and only if $B_1(X)$ is β-compact for any topology β consistent with the duality $< C_b(X), M_z(X) >$.*

Proof: Since $\sigma(C_b(X), M_z(X)) \leq \beta$, it follows that the β-compactness of $B_1(X)$ implies the $\sigma(C_b(X), M_z(X))$-compactness of $B_1(X)$. Conversely, if $B_1(X)$ is $\sigma(C_b(X), M_z(X))$-compact, since $t_p \leq \sigma(C_b(X), M_z(X))$ then $\sigma(C_b(X), M_z(X))$ coincides with the pointwise topology on the uniformily bounded sets. Therefore $\sigma(C_b(X), M_z(X)) \leq \beta_0$ since β_0 is the finest locally convex topology with that property.

This last inequality implies that $\tau(C_b(X), M_z(X)) \leq \beta_0$, for β_0 is the Mackey topology of the duality $< C_b(X), M_t(X) >$ (see [8]). Therefore, from 3.9 we get that $B_1(X)$ is $\tau(C_b(X), M_z(X))$-compact and the result follows.

□

Corollary 4.4 *Let X be a discrete space. $\mid X \mid$ is non-Ulam measurable if and only if $B_1(X)$ is β_p-compact.*

Proof: Since X is discrete, then X is real compact if and only if $\mid X \mid$ is non-Ulam measurable (see [2]). Then by lemma 4.1, it is equivalent to saying that $M_t(X) = M_p(X)$. Therefore on $C_b(X)$ we have that $\sigma(C_b(X), M_p(X)) = \sigma(C_b(X), M_t(X))$ and since $\sigma(C_b(X), M_p(X)) \leq \beta_0$ then we obtain that $B_1(X)$ is $\sigma(C_b(X), M_p(X))$-compact. Hence by 4.3 it is β_p-compact. Conversely, if $B_1(X)$ is β_p-compact then $\beta_p = \beta_0$ which implies that $M_p(X) = M_t(X)$. □

Remark: The previous result was proved by the second author in a different way (see [9], [10]) and was the original motivation for starting this research.

Corollary 4.5 *Let X be a discrete space. Then $\mid X \mid$ is non-real measurable if and only if $B_1(X)$ is β_σ-compact.*

Proof: Since X is discrete, then $\mid X \mid$ is non-real measurable if and only if X is a D-space. Therefore by lemma 4.2 it is equivalent to say that $M_\sigma(X) = M_\mu(X)$. But on discrete spaces, $M_t(X) = M_\tau(X) = M_\mu(X)$, then by a similar argument as in the proof of corollary 4.4 we get that it is equivalent to say that $B_1(X)$ is β_σ-compact. □

Finally we present a corollary that includes a well known result of Ulam.

Corollary 4.6 *The following are equivalent:*

(1) Real measurable and Ulam measurable cardinals are the same.

(2) For every discrete space X with $\mid X \mid$ non-Ulam measurable we have that on $C_b(X)$ $\beta_p = \beta_\sigma$.

(3) For every discrete space X with $\mid X \mid$ non-Ulam measurable we have that $M_p(X) = M_\sigma(X)$.

(4) The continuum is not Real measurable.

Proof: That (1) is equivalent to (4) is a well known result of Ulam. That (2) and (3) are equivalent follows from 4.1 and 4.2.

It is clear from corollary 4.4 and corollary 4.5 that if $\beta_p = \beta_\sigma$ then Real measurability and Ulam measurability are equivalent. Conversely, suppose that Real measurability and Ulam measurability are equivalent. We will show that $\beta_p = \beta_\sigma = \beta_0$ on $C_b(X)$ for every X with $\mid X \mid$ non-Ulam measurable. In this case $B_1(X)$ is both β_p and β_σ-compact. Since β_0 is the finest locally convex topology that makes $B_1(X)$ compact, we get that β_p and β_σ are $\leq$ than β_0. But this implies that $\beta_0 = \beta_p = \beta_\sigma$. □

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