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RESEARCH ARTICLE

Self-similar and charged radiating spheres: an anisotropic approach

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Abstract Considering charged fluid spheres as anisotropic sources and the diffusion limit as the transport mechanism, we suppose that the inner space–time admits self-similarity. Matching the interior solution with the Reissner–Nordström–Vaidya exterior one, we find an extremely compact and oscillatory final state with a redistribution of the electric charge function and non zero pressure profiles.

Keywords Charged spheres · Anisotropy · Compact objects

1 Introduction

It is well known that astrophysical objects are not significantly charged, For this reason charged bodies have limited interest (for an historical overview see

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Sect. I in [1]). Observed stars, like the Sun or a neutron star, cannot support a great amount of charge. This is so basically because the particles that compose a star have a huge charge to mass ratio, as is the case for a proton or an electron. For cold stars the electric charge is about 100 Coulombs per solar mass [2]. Nevertheless, in compact stars the equilibrium conditions are different, allowing some more net charge [3]. For highly relativistic stars near to form a black hole, the huge gravitational pull can be balanced by huge amounts of net charge. On this kind of configurations we are concerned in this paper and have been considered by other authors [1,3–7].

To include the electric charge a number of authors make additional assumptions such as an equation of state or a relationship between metric variables [8–11]. Bonnor and Wickramasuriya [12] have studied electrically charged matter, with electrostatic repulsion balancing the gravitational attraction. Most works have been done under static conditions, including the ones by Ivanov [13], who exhaustively surveyed static charged perfect fluid spheres in general relativity, and Ray et al. [1], who studied the effect of electric charge in compact stars and its consequences on the gravitational collapse. Bekenstein [3] found that for highly relativistic stars, whose radius is on the verge of forming an event horizon, the large gravitational pull can be balanced by large amounts of net charge.

In this paper we study an electrically charged matter distribution as an anisotropic fluid. It is well known that different energy–momentum tensors can lead to the same space–time [14–19]. For instance, under spherical symmetry viscosity can be considered as a special case of anisotropy [20–22]. To illustrate our approach to obtain dynamical models we consider the diffusion approximation as the transport mechanism, and a self-similar space–time for the inner region. Our treatment allows different scenarios for the gravitational collapse, including the reported some years ago [23]. In particular we are now interested in non stationary initial conditions to explore the fate of the gravitational collapse. The set problem has some physical and geometrical features that we review briefly, these are: anisotropy, dissipation and self-similarity.

For certain density ranges, local anisotropy in pressure can be physically justified in self-gravitating systems, since different kinds of physical phenomena may take place, giving rise to local anisotropy and in turn relaxing the upper limits imposed on the maximum value of the surface gravitational potential [24]. The influence of local anisotropy in general relativity has been studied mostly under static conditions (see [8] and references therein). Herrera et al. [25] have reported a general study for spherically symmetric dissipative anisotropic fluids with emphasis on the relationship among the Weyl tensor, the shear tensor, the anisotropy of the pressure and the density inhomogeneity. In another context, a scalar field with non-zero spatial gradient is an example of a physical system where the pressure is clearly anisotropic. Boson stars, hypothetical self-gravitating compact objects resulting from the coupling of a complex scalar field to gravity, are systems where anisotropic pressure occurs naturally. Similarly, the energy–momentum tensor of both electromagnetic and fermionic fields are anisotropic [26]. We want to stress that in order to have isotropy we need an extra assumption on the behavior of the fields or on the fluid describing interiors.

The proposed dissipative process is suggested by the fact that they provide the only plausible mechanism to carry away the bulk of binding energy, leading to a neutron star or black hole [27]. Furthermore, in cores of densities close to $10^{12}\, g/cm^3$ the mean free path for neutrinos becomes small enough to justify the use of the diffusion approximation [28,29]. Here we do not discuss the temperature distribution during diffusion; we consider this transport mechanism because it provides an efficient way to radiate energy. Additional studies are necessary to explore the consequences of incorporating a hyperbolic theory of dissipation to overcome the difficulties inherent to parabolic theories.

Few exact solutions to the Einstein–Maxwell equations are relevant to gravitational collapse. For this reason, new collapse solutions are very useful, even if they are simplified ones [31]. It is well known that the field equations admit homothetic motion [32–34]. Applications of self-similarity range from modeling black holes to producing counterexamples to the cosmic censorship conjecture [35–41]. It is well established that in the critical gravitational collapse of an scalar field the space–time can be self-similar [47–49]. We can expect on physical grounds that this scenario remains similar for charged matter [50].

In this work the Darmois–Lichnerowicz junction conditions at the surface of the distribution are satisfied. We match the self-similar interior solution with the Reissner–Nordström–Vaidya exterior one. We obtain a model that resembles the Seidel–Suen solitonic star formed by a real and massive scalar field [51].

Section 2 contains the field equations, written in such form that the electrically charged fluid can be considered anisotropic. In this section we also discuss the junction conditions to match the dissipative interior space–time with the exterior space–time, a Reissner–Nordström–Vaidya one, and write the generalized Tolman–Oppenheimer–Volkoff equation. Section 3 contains a review of self-similarity for the sake of completeness. Then we propose a simple solution to include electric charge by means of the active mass. We show two example solutions in Sect. 4 and finally discuss our results in Sect. 5.

2 The field equations and junction conditions

Let us consider a non static distribution of matter which is spherically symmetric and consists of charged fluid of energy density ρ, pressure p, electric energy density ϵ and radiation energy flux q diffusing in the radial direction, as measured by a local Minkowskian observer comoving with the fluid (with velocity $-\omega$). In radiation coordinates [52] the metric takes the form

$$ds^2 = e^{2\beta}\left(\frac{V}{r}du^2 + 2du\, dr\right) - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

where β and V are functions of u and r. As it is well known for the spherical symmetry $F^{ur} = -F^{ru}$ are the only non vanishing electromagnetic field tensor components. Now, defining the function $C(u,r)$ by the relation

$$F^{ur} = \frac{C}{r^2} e^{-2\beta} \tag{2}$$

the inhomogeneous Maxwell equations become

$$C_{,r} = 4\pi r^2 J^u e^{2\beta}, \tag{3}$$

and

$$C_{,u} = -4\pi r^2 J^r e^{2\beta}, \tag{4}$$

where the comma subscript represents partial derivative with respect to the indicated coordinate; J^u and J^r are the temporal and radial components of the electric current four vector, respectively. Thus, the function $C(u,r)$ is naturally interpreted as the charge within the radius r at the time u. Thus, the inhomogeneous Maxwell field equations entail the conservation of charge

$$U^\mu C_{,\mu} = 0, \tag{5}$$

where the four-velocity is given by

$$U^\mu = e^{-\beta}\left[\sqrt{\frac{r}{V}}\left(\frac{1-\omega}{1+\omega}\right)\delta^\mu_u + \sqrt{\frac{V}{r}}\frac{\omega}{(1-\omega^2)^{1/2}}\delta^\mu_r\right]. \tag{6}$$

We can write the conservation equation in a more suitable form, that is

$$C_{,u} + \frac{dr}{du} C_{,r} = 0, \tag{7}$$

where

$$\frac{dr}{du} = e^{2\beta}\frac{V}{r}\frac{\omega}{1-\omega} \tag{8}$$

is the matter velocity. It is clear that inside a sphere comoving with the fluid the charge is conserved.

Introducing the mass function by means of

$$\mu = \frac{1}{2}\left(r - V e^{-2\beta}\right), \tag{9}$$

we can write the Einstein field equations as follows

$$\frac{\hat{\rho} + p_r\omega^2}{1-\omega^2} + \frac{2\omega q}{1-\omega^2} = -\frac{\mathrm{e}^{-2\beta}\mu_{,u}}{4\pi r(r-2\mu)} + \frac{\mu_{,r}}{4\pi r^2}, \tag{10}$$

$$\bar{\rho} = \frac{\mu_{,r}}{4\pi r^2}, \tag{11}$$

$$\bar{\rho} + \bar{p} = \frac{\beta_{,r}}{2\pi r^2}(r-2\mu), \tag{12}$$

$$p_t = -\frac{1}{4\pi}\beta_{,ur}\mathrm{e}^{-2\beta} + \frac{1}{8\pi}(1-2\mu/r)(2\beta_{,rr} + 4\beta_{,r}^2 - \beta_{,r}/r) + \frac{1}{8\pi r}[3\beta_{,r}(1-2\mu_{,r}) - \mu_{,rr}], \tag{13}$$

where

$$\bar{p} = \frac{p_r - \omega\hat{\rho}}{1+\omega} - \frac{1-\omega}{1+\omega}q,$$

and

$$\bar{\rho} = \frac{\hat{\rho} - \omega p_r}{1+\omega} - \frac{1-\omega}{1+\omega}q,$$

with $\hat{\rho} = \rho + \epsilon, p_r = p - \epsilon, p_t = p + \epsilon$ and the electric energy density $\epsilon = E^2/8\pi$, where $E = C/r^2$ is the local electric field intensity. Observe that we have introduced the mass function μ instead the usual total mass $\tilde{m} = (r - V\mathrm{e}^{-2\beta})/2 + C^2/2r$.

If we define the degree of local anisotropy induced by charge as $\Delta = p_t - p_r = 2\epsilon$, the electric charge determines such a degree at any point.

It can be shown that the Tolman–Oppenheimer–Volkoff equation reduces to

$$\frac{\partial\bar{p}}{\partial r} + \frac{\bar{\rho}+\bar{p}}{1-2\mu/r}\left[4\pi r\bar{p} + \mu/r^2\right] - \mathrm{e}^{-2\beta}\left(\frac{\bar{\rho}+\bar{p}}{1-2\mu/r}\right)_{,u} = \frac{2}{r}\left(p_t - \bar{p}\right). \tag{14}$$

This generalized equation is the same as for an anisotropic fluid [54].

The most general junction conditions for our system can be written as [23]

$$\mu_a = \tilde{m}_a - \frac{C_T^2}{2a}, \tag{15}$$

$$\beta_a = 0, \tag{16}$$

and

$$-\beta_{,u}|_a + (1-2\mu_a/a)\beta_{,r}|_a - \frac{\mu_{,r}|_a}{2a} + \frac{C_T^2}{4a^3} = 0, \tag{17}$$

where a subscript a indicates that the quantity is evaluated at the surface, $\tilde{m}_a$ and C_T are the total mass and the total charge, respectively. Remarkably, the last equation is equivalent to

$$p_a = q_a. \tag{18}$$

We have obtained a system of equations which are equivalent to the anisotropic matter case (see [8] and references therein). In the next section we will see how this approach lead us to an extra simplification.

Up to this point all the written equations are general. We can try to solve the system using numerical techniques or a seminumerical approach to gain some insight. The last method is worked out in the next section.

3 The self-similar space–time: a simple solution

We shall assume that the spherical distribution admits a one–parameter group of homothetic motion. A homothetic vector field on the manifold is one that satisfies $\pounds_\xi \mathbf{g} = 2n\mathbf{g}$ on a local chart, where n is a constant on the manifold and $\pounds$ denotes the Lie derivative operator. If $n \neq 0$ we have a proper homothetic vector field and it can always be scaled so as to have $n = 1$; if $n = 0$ then ξ is a Killing vector on the manifold. So, for a constant rescaling, ξ satisfies

$$\pounds_\xi \mathbf{g} = 2\mathbf{g}, \tag{19}$$

and has the form

$$\xi = \Lambda(u,r)\partial_u + \lambda(u,r)\partial_r. \tag{20}$$

The homothetic equations reduce to

$$\xi(\chi) = 0, \tag{21}$$

$$\xi(\upsilon) = 0, \tag{22}$$

where $\lambda = r$, $\Lambda = \Lambda(u)$, $\chi = \mu/r$ and $\upsilon = \Lambda e^{2\beta}/r$. Therefore, $\chi = \chi(\zeta)$ and $\upsilon = \upsilon(\zeta)$ are solutions if the self–similar variable is defined as

$$\zeta \equiv r\,e^{-\int du/\Lambda}. \tag{23}$$

We have reported [23,55] a simple homothetic solution which can be written for the present case as

$$\mu = \mu_a(u)(r/a)^{k+1}, \tag{24}$$

and

$$e^{2\beta} = (r/a)^{l+1} \tag{25}$$

with k and l constants. It can be shown that for a given k, l must be a root of a complicated seventh degree polynomial in order to satisfy the additional symmetry equations. It is obvious that these solutions satisfy the continuity of the first fundamental form, and to satisfy the continuity of the second fundamental form, the radial velocity at the surface must be

$$\omega_a = 1 - \frac{2(l+1)(1-2\mu_a/a)}{2(k+1)\mu_a/a - C_T^2/a^2}. \tag{26}$$

3.1 Surface equations

Now, to completely determine the metric we have two differential equations at the surface, Eq. (8) and the field equation (10) which evaluated at the surface become, respectively,

$$\dot{a} = (1-2\mu_a/a)\frac{\omega_a}{1-\omega_a} \tag{27}$$

and

$$\dot{\mu}_a = -Q(1-2\mu_a/a) + \dot{a}\frac{C_T^2}{2a^2}, \tag{28}$$

where a dot over a variable denotes the derivative with respect to time. The quantity Q is defined as

$$Q \equiv 4\pi a^2 q_a \left(\frac{1+\omega_a}{1-\omega_a}\right), \tag{29}$$

and can be explicitly determined using Eq. (14) evaluated at the surface, resulting in

$$Q = \left\{(1-2\mu_a/a)\left[2(l+1)^2 + k(k+1+3(l+1))\right] - 2(C_T/a)^2 - k(k+1+3(l+1))\right\} \times \left\{k+1-(1-2\mu_a/a)(k+l+2) - (C_T/a)^2\right\} \Big/ \left[4(1-2\mu_a/a)(l+1)a^2\right]. \tag{30}$$

The surface equations (27) and (28) together with Eqs. (26) and (30) can be integrated numerically (using Runge–Kutta for instance) for initial conditions $a(0)$ and $\mu_a(0)$ and some k and C_T. The only restrictions on these conditions and parameters come from the need to get a physically acceptable model.

For $k = 0$ (which implies $l = 0$) we obtain the same results of reference [23]. As we have mention before, we gain some simplification with the anisotropic approach of this paper, that is, one of the similarity equations (Eq. (23) in [23]) was decoupled, leading us to less restrictive models.

3.2 Integrating the conservation equation

Once the surface equations are integrated we have to integrate the conservation equation (7) to obtain all the physical variables inside the source. To do that we

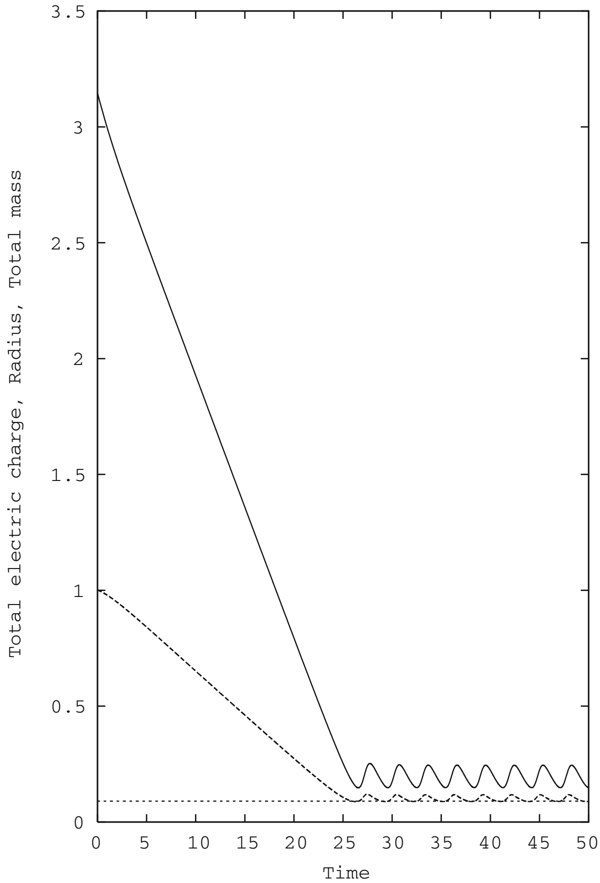

Fig. 1 Total electric charge (*short dashed line*) and the evolution of radius a (*solid line*) and the total mass m_a (*dashed line*)

use comoving spatial markers $x = r/a$. Thus the conservation equation can be written as

$$C_{,u} = -\frac{\mathrm{d}x}{\mathrm{d}u} C_{,x} \tag{31}$$

which is a wave–like equation that can be integrated numerically using the Lax method (with the appropriate Courant–Friedrichs–Levy (CFL) condition). The evolution of the conservation equation is restricted by the surface evolution and was implemented as follows:

$$C_j^{n+1} = \frac{1}{2}\left(C_{j+1}^n + C_{j-1}^n\right) + \frac{\delta u}{2\delta x}\left(\frac{\mathrm{d}x}{\mathrm{d}u}\right)_j^n \left(C_{j+1}^n - C_{j-1}^n\right). \tag{32}$$

The superscript n indicates the hypersurface $u = n\delta u$ and the subscript j the spatial position for a comoving observer at $x = j\delta x$. We typically used $\delta u = 10^{-2}$

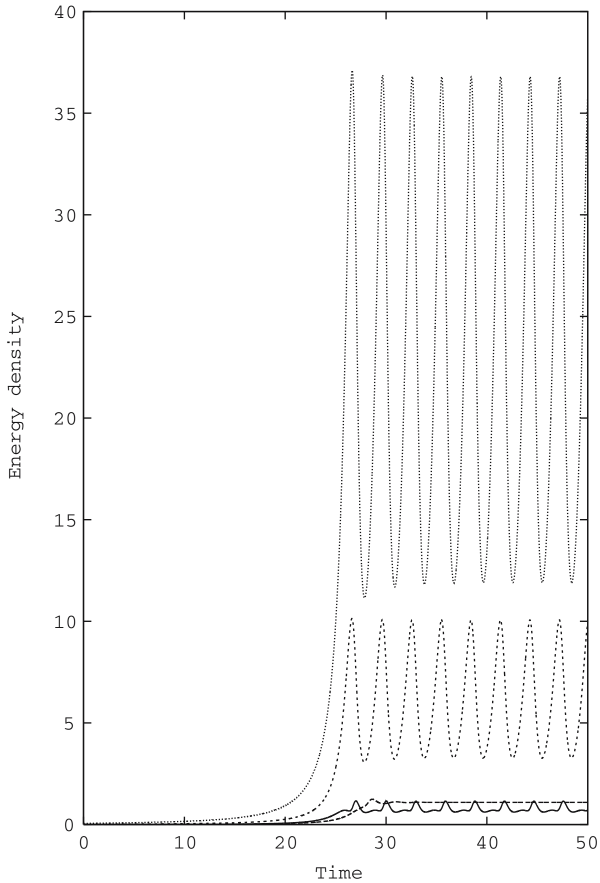

Fig. 2 Energy density as a function of time for different pieces of the distribution: $x = 0.25$ (*short dashed line*) $x = 0.5$ (*large dashed line*); $x = 0.75$ (*solid line*) and $x = 1$ (*dotted line*)

with a CFL condition $\delta u = 2\delta x$. In order to integrate the conservation equation we need to specify a boundary condition and an initial condition. For this work we have used the boundary condition

$$C(x = 0, u) = 0, \tag{33}$$

with an initial condition

$$C(x, u = 0) = C_T\, x^{\mathcal{P}}, \tag{34}$$

where the power $\mathcal{P}$ allows us to test the sensitivity to the initial conditions of our results.

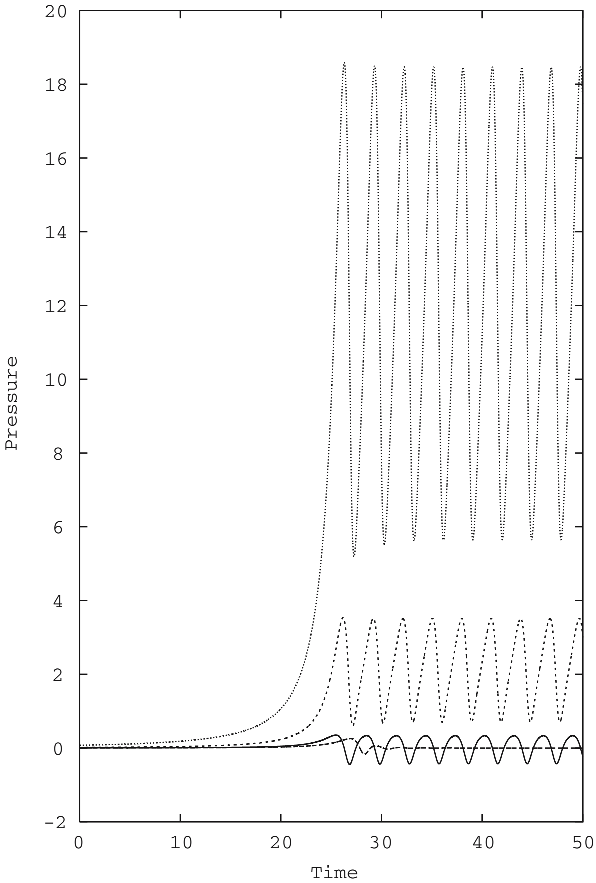

Fig. 3 Pressure as a function of time for different pieces of the distribution: $x = 0.25$ (*short dashed line*); $x = 0.5$ (*large dashed line*); $x = 0.75$ (*solid line*) and $x = 1$ (*dotted line*)

4 Modeling

4.1 Evolving towards dust: $k = 0$

As a test of our approach to dealing with electric charge as anisotropy is to reproduce the results reported in [23]. For this scenario $k = l = 0$, $a(0) = 3.5$, $\mu_a(0) = 1, \mathcal{P} = 1$ and $C_T = 0.09$. Even when the initial mass $m_a(0)$ is not equal to one, we obtain the same results, that is, the collapse is halted and the boundary oscillates when the total electric charge asymptotically approximates the total mass m_a and the radius equals asymptotically to twice the total electric charge. The ratio P/ρ as a function of time is the same for any region. Also, as the distribution evolves, becomes dust asymptotically with damped oscillations and the pressure at the surface coincides with the heat flow. The matter velocity profiles at all regions have damped oscillations that evolve to the static regime.

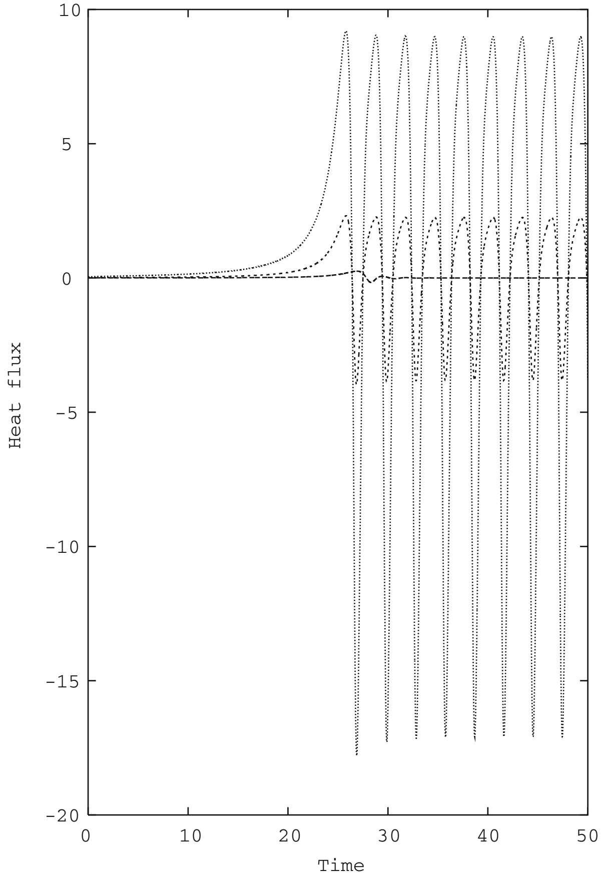

Fig. 4 Heat flux as a function of time for different pieces of the distribution: $x = 0.25$ (*short dashed line*); $x = 0.5$ (*large dashed line*); $x = 0.75$ (*solid line*) and $x = 1$ (*dotted line*)

A striking result not reported before is the ability to obtain any interior profile for matter velocity or electric charge function using the simple rule

$$\mathcal{F}(u,x) = x\mathcal{F}(u,1), \tag{35}$$

where $\mathcal{F}$ could be either $\mathrm{d}r/\mathrm{d}u$ or C, that is, once the surface profile is determined we can go without additional work inside the distribution. Another interesting result is that we can get the interior profiles for pressure and energy density by

$$\mathcal{G}(u,x) = \mathcal{G}(u,1)/x^2, \tag{36}$$

where $\mathcal{G}$ could be ρ or p.

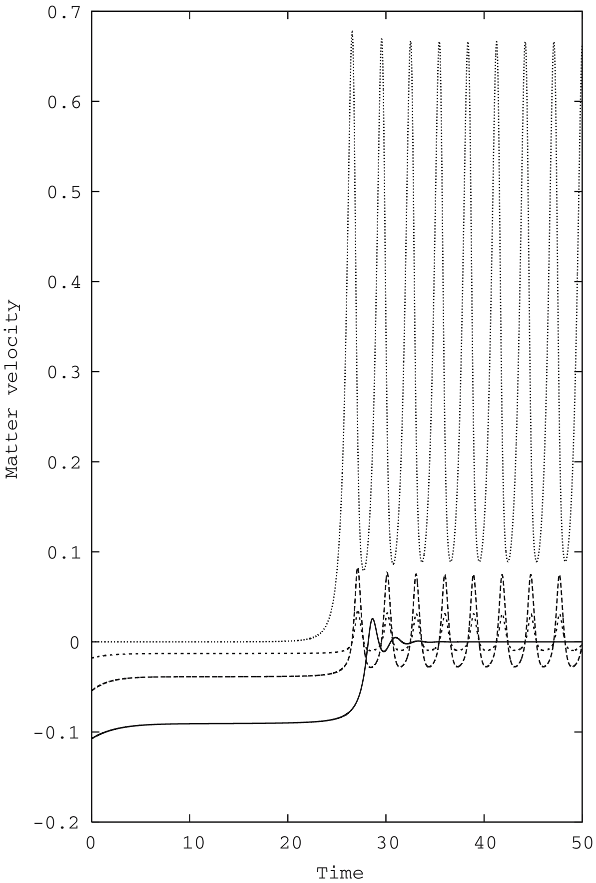

Fig. 5 Matter Velocity as a function of time for different pieces of the distribution: $x = 0.25$ (*short dashed line*); $x = 0.5$ (*large dashed line*); $x = 0.75$ (*solid line*) and $x = 1$ (*dotted line*)

4.2 Redistribution of the electric charge: $k = 0.1$

Now that we have successfully tested our new approach, we explore another physical scenario, one with redistribution of the electric charge. For this case we choose $k = 0.1$, $a(0) = 3.5$, $\mu_a(0) = 1$ and $C_T = 0.09$ to obtain (among other roots) $l = 0.741632$. For $\mathcal{P} = 1$ (power of the initial condition to integrate the conservation equation) the results are shown in Figs. 1, 2, 3, 4, 5 and 6. The system collapses and losses mass until it reaches a stable oscillatory state. When the distribution is in oscillatory regime the mass behaves in the same way, that is, oscillates between two extremal values. This oscillatory behavior extends to all physical variables at all pieces of the material. The interior redistribution of electric charge responds to the surface dynamics. Electric charge (repulsively) let the distribution expand with absorption of energy up to some maximum point in which gravitation compensates electric repulsion. Therefore

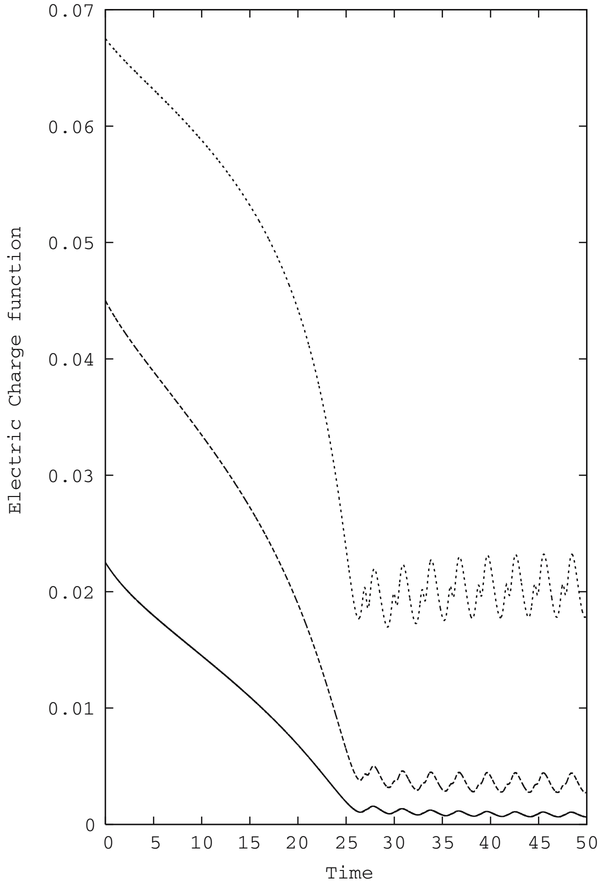

Fig. 6 Electric charge function versus time for different pieces of the material: $x = 0.25$ (*solid line*); $x = 0.5$ (*large dashed line*); $x = 0.75$ (*short dashed line*)

the collapse undergoes with emission of energy, up to some minimum point in which electric repulsion is again dominant and so on indefinitely. This model seems to be independent of the initial conditions, as shown in Fig. 7. The dependence of the radius on the total electric charge is shown in Fig. 8. Analysis of this last result indicates scale invariance with respect to the total electric charge. In fact, once the stationary state (oscillatory) is reached, the amplitude and period depend on the total electric charge. Thus, if we know the oscillatory regime for some specific value of electric charge, we can reproduce the oscillatory regime for any other, non zero, value of electric charge. In the limit of zero charge the distribution shrink completely, forming a very small region with very high energy density and pressure. This last result was found in another context (see Sect. IV. B in [55]).

The energetic at the boundary surface in conjunction with the pull of gravity and push of electric charge, explain the dependence between the amplitude

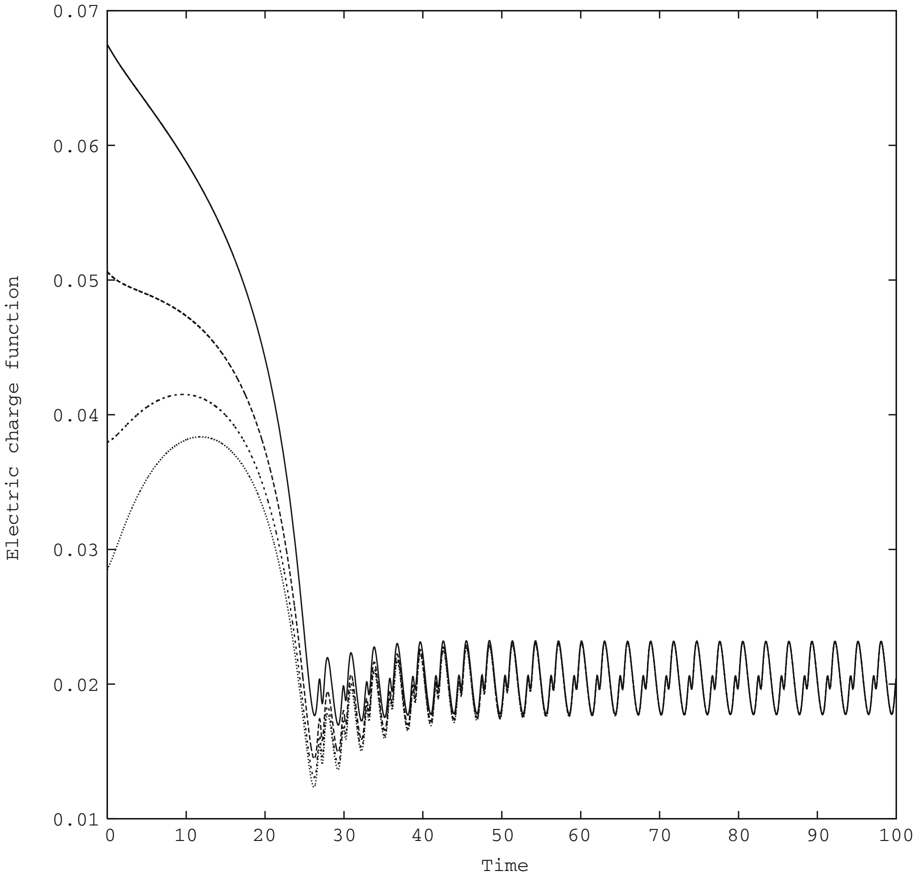

Fig. 7 Electric charge function versus time for $x = 0.75$ and different values of power $\mathcal{P}$ in the initial value of C to integrate the conservation equation. Curves correspond to: $\mathcal{P} = 1$ (*continuous line*); $\mathcal{P} = 2$ (*large dashed line*); $\mathcal{P} = 3$ (*short dashed line*) and $\mathcal{P} = 4$ (*dotted line*). Many values of $a(0)$ were used with essentially the same results

and period with the total electric charge. These features are analogous to the reported by Brady et al. [53] for a real and massive scalar field. In this context the mass destroys the scale invariance of the Einstein-scalar field equations, but after the nearly critical phase the oscillation period depends on the inverse of scalar field mass, i.e. the Compton wavelength. In our case the situation is analogous respect to the electric total charge. Parenthetically, the Seidel–Suen oscillaton [51] is the stable solution far from the critical behavior of a massive and real scalar field. Our solution behaves like that stable configuration from quite simple considerations.

5 Discussion

In this paper we have shown that electrically charged matter can be considered as anisotropic matter. We exemplify the approach obtaining a dynamical model

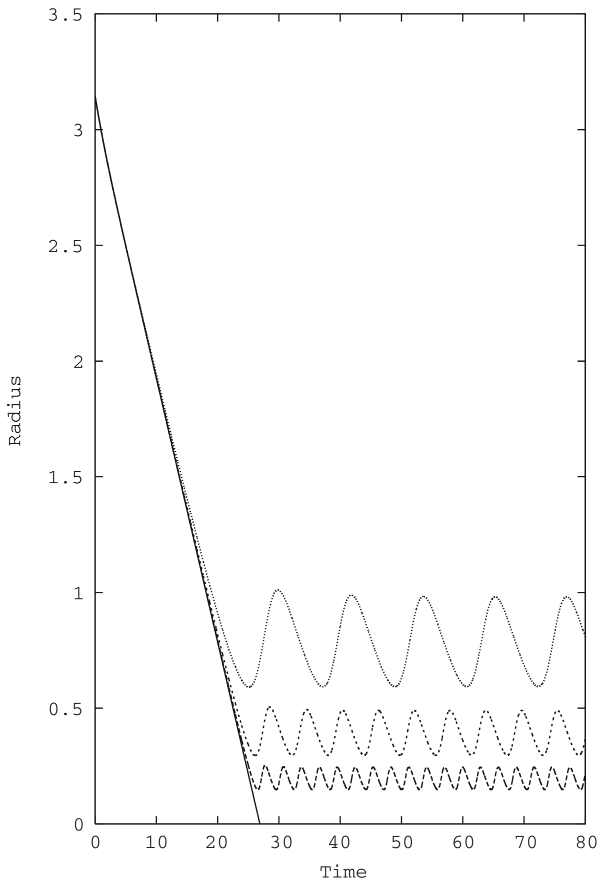

Fig. 8 Radius as a function time for $C_T = 0.00$ (*solid line*); $C_T = 0.09$ (*large dashed line*); $C_T = 0.18$ (*short dashed line*); $C_T = 0.36$ (*dotted line*)

under the diffusion approximation and self-similarity within the source. The showed example is representative of many others varying only the initial conditions, k and C_T, and imposing the following physical conditions: $-1 < \omega < 1$, $p < \rho$ and $\rho > 0$.

Some general considerations concerning our results are listed next,

- Extremely compact distribution: The collapse is halted when the total electric charge is about the total mass (in geometrical units). If the initial radius of the electrically charged distribution is 5.2 km, the radius of the final stable source is $\approx$266 m. Also the final pressure (if not zero) is about 1.5×10^{39} Pa and the energy density 1.8×10^{19} g/cm^3.
- High electric field: The electric charge used in our models can be as high as 1.5×10^{19} C. Thus, the electric field for the stable configuration is about 2.2×10^{19} V/m [3,1]. The sign of net charge does not affect our results.

- High powered: If the initial total mass is $1M_\odot$ the final is $\approx 0.1M_\odot$, that is, almost 90% of the mass is radiated in 0.15 ms.

A more general solutions to the homothetic equations (21) and (22) could be necessary to explore how general our results are. However, the results found using the anisotropic approach are consistent with previously reported results when the static regime is the final state [56–60]. If the solution reaches stationarity it oscillates. For this case we did not investigate the stability of the system, but independence of initial conditions is apparent. It is clear that the presence of some finite electric charge (or anisotropy) avoids singularity formation. Finally, we would like to conclude by posing a question. If the spherically symmetric source has an upper limit for the total net electric charge that carries [1,61], is it relaxed by the transport mechanism or by the additional homothetic symmetry? Work in this direction is in progress.

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