3. Some Simple Models

1. Introduction

What do we mean by a model? Here is a very general definition:

Definition 1. *A model is a set of simultaneous equations such that the number of independent equations is equal to the number of variables. The remaining symbols in the model are parameters.*

The word *independent* in this definition means, essentially, that each equation brings new information into the system. One equation cannot be a multiple of another or the sum of other equations in the model. There are more complicated ways of defining independence, but they all relate to this basic idea of new information.

The simplest macro model has consumption, investment and savings and one aggregate good. The first chapter discussed the pitfalls of aggregating the economy into one commodity, but the advantages of simplicity are so overwhelming, we begin our investigation here.

The most essential components of a model are

1. The SAM
2. Behavioral equations

We could even dispense with behavioral equations and still be consistent with the definition of a model above. Indeed this is sometimes done in structural SAM analysis [**?**]. These models assume very little or no behavior and manipulate SAMs as whole. Our simplest one sector model will assume a consumption function along with the SAM equations. The SAM equations are the income-expenditure balances for each agent *plus* the savings-investment balance. Thus if there are four agents in the model, firms, households, government and foreign, there are five SAM equations. As we have seen in Chapter 2, these equations are *not* indepedent. Walras's law says that if all agents are in income-expenditure balance, then the sum of savings is equal to total investment. Thus with four agents, there are only four independent equations, with three agents only three agents, only three and so on.

Despite its redundancy, the savings-investment balance will still be very useful, however. With n agents there are n SAM equations, but the savings-investment balance can be substituted for any one of these n equations, if it is more convenient to solve and or explain how the model works. Think of the savings-investment balance in reserve, there to be

TABLE 3.1. A Republican Paradise

	A	B	C	D	E
1		Firms	Households	Investment	Total
2	Firms		C	I	Y
3	HH	V_A			Y
4	Savings		S		S
5	Total	Y	Y	I	

used when needed, but always as a pinch hitter for some other more troublesome equation in the model.

SAM equations are *bona fide* equations. Each can solve for one variable and are no better or worse than any other equation in them model. Moreover, not all SAMequations need be used in any given model. It is only essential to the same number of independent equations as endogenous variables to have a model.

EXAMPLE 1. *The SAM for the Republican Paradise in which there are is no government and no foreign trade has two agents. Therefore there are two independent SAM equations, one for firms and one for households. The SAM of Chapter 2 is repoduced in Table 3.1.The income-expenditure balance for firms is*

$$Y = C + I$$

where Y is equal to value added by the factors of production; C = consumption by households, I = investment by both firms and households. Value added is paid to households in the form of income who in turn consume C and save S.

$$Y = C + S$$

Subtracting the second of these two equations from the first, we have

$$0 = I - S$$

that is, the savings-investment balance. Since we ony have two independent equations, we can only solve for two variables. We indentify which ones in the continuation of this example below.

2. CALIBRATING BEHAVIORAL EQUATIONS

Behavior equations are used to show how the cells of the SAM relate to one another. The Keynesian consumption function, for example,

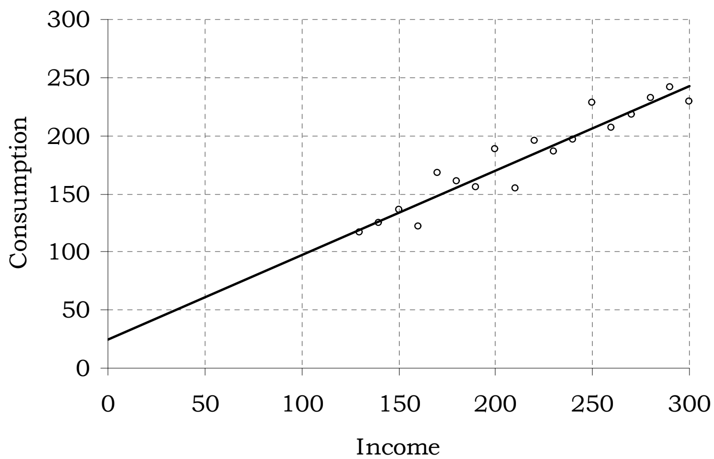

FIGURE 3.1. The consumption function

shows how household income determines consumption

$$C = \bar{C} + cY \tag{3.1}$$

where $\bar{C}$ is *autonomus consumption* and c is the *marginal propensity to consume*. Autonomous consumption is autonomous or independent of income while the marginal propensity to consume indicates how marginal changes in income affect consumption. Graphically, $\bar{C}$ is the y-intercept of the consumption function and c is the slope as shown in the diagram.

In this diagram, we assume that we have time-series observations on income and consumption for our country. We plot them and use the Excel function *trendline* to extract an estimate of the consumption function.[1]

$$C = 24.78 + 0.73Y$$

where the $R^2 = .91$

Now that we have the consumption function, we must adapt it to the SAM. There are a number of problems. First, the consumption function

[1]Go to *Chart, Trendline, Options* and check the box for *Display equation on chart.* No matter how the variables are named, Excel will call the independent variable x and the dependent variable y.

is a product of a time series, while the SAM is for one year. Moreover, when the SAM income is inserted into the consumption function, the result for C must agree with the SAM. This means that we must drop the estimate in Figure 3.1 of *one* of the two parameters, $\bar{C}$ and c. We then resolve the consumtpion function with the SAM value of C on the right-hand side and solve for one of the two parameters on the left. The consumption function fits the SAM and is thereby calibrated.

Figure 3.2 gives an indication as to which one we choose. Observe that the range of income goes from 130 to 300. In particular, income *never falls to zero, or anywhere even close.* This implies that the interpretation of the y-intercept as *autonomous* consumption is somewhat misleading. In fact, $\bar{C}$ is better thought of as the y-intercept and nothing more. We tend to take the national accounts as given data, but in fact the are really statistical samples based on a sampling procedure. Had another sample been taken, shown in Figure 3.1 by the $x's$, and another consumption function estimated, chances are good that the intercept would be different, possibly quite different. On the other hand, the estimate for the slope is more stable. It does not jump around as much as the intercept. As Figure 3.2 shows, because the range of income is limited, the estimate for $\bar{C}$ is in fact quite unstable. With a small sample variance inside the probability elipse as shown, the intercept jumps from 35 to 60, an increase of more than 100 percent while the change in the slope is only -15 percent.

This suggests that in the calibration procedure, we drop the intercept and retain the slope from any available econometric study of the consumption function. We then calculate the $\bar{C}$ from the consumption function

$$C_{SAM} = \bar{C}_{calibrated} + c_{econometric} Y_{SAM} \tag{3.2}$$

where C_{SAM} is the consumption of the SM, $c_{econometric}$ is the slope of the consumption function, as determined from the time series regression, and Y_{SAM} is the income in the SAM.

Example 2. *Calibrate a consumption function with* $c = 0.75$ *to the RP SAM in Table 3.2. Write equation 3.2 with the data of SAM inserted,* $C_{SAM} = 160$, $Y_{SAM} = 200$ *and* $c = 0.8$

$$160 = \bar{C} + 0.75(200)$$

The solution is: $\bar{C} = 10$. *The calibrated consumption function is then*

$$C = 10 + 0.75Y$$

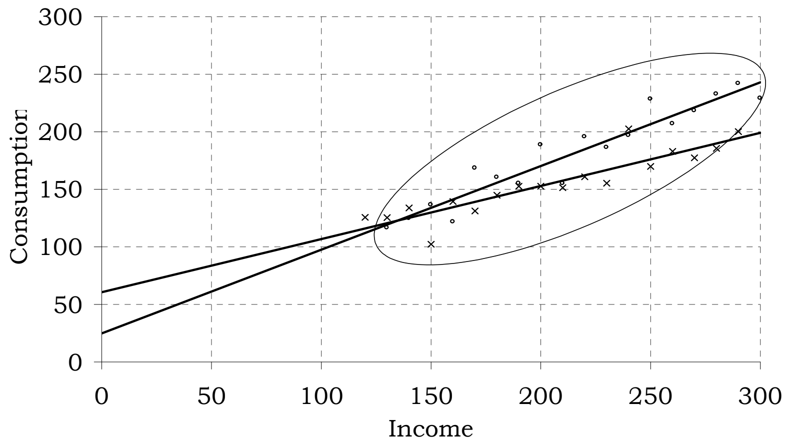

FIGURE 3.2. Instability of the $y-$ intercept

TABLE 3.2. Calibrating the consumption function

	A	*B*	*C*	*D*	*E*
1		Firms	Households	Investment	Total
2	Firms		160	40	200
3	HH	200			200
4	Savings		40		40
5	Total	200	200	40	

3. Model Statement

We now have in hand a full model, calibrated to the data base 3.2. It consists of equations, variables and parameters. Let us write it down:

$$\begin{aligned} Y &= C + I0 \\ S &= Y - C \\ C &= \bar{C} + cY \end{aligned}$$

The first two are SAM equations and the last is, of course, the consumption function. These are three equations in the unknowns Y, S

and C. The rest of the symbols in the model are *parameters* determined exogenously, either from the SAM or other sources such as our consumption-income time series. Note that every *symbol* must be classified as either a parameter or a variable, but how does one know which are which? There is no simple answer to this question, such as "the variables are always listed on the left" or any rule. In this case, we have used the Keynsian consumption function, but even that is not enough to be able to say which symbols are variables. There is no way to tell which are the variables; to know a model is to know the *variable list* $V(.)$ and *parameter list* $P(.)$. In this case we have $V(Y, S, C)$ and $P(I, \bar{C}, c)$.

Example 3. *Write the complete calibrated model for the Republican Paradise model.*

$$\begin{aligned} Y &= C + I \\ S &= Y - C \\ C &= \bar{C} + cY \end{aligned}$$

$$\begin{aligned} &V(Y, S, C) \\ &P(40, 10, 0.75) \end{aligned}$$

Substituting the parameters into the equations must give a solution for the variables that agree (perfectly) with the underlying SAM, in this case $V(200, 40, 160)$.

To emphasize the point that we have must know the variable and parameter list in order to know the model, consider a switch between a variable and parameter of the Keynesian model. With the *exact same equations,* we could conherently write the variable and parameter lists as

$$\begin{aligned} &V(Y, I, C) \\ &P(S, \bar{C}, c) \end{aligned}$$

where the savings and investment variables have been reveresed. Since savings is equal to investment, it might be thought that this change would be of no consequence. But nothing could be further from the truth. Rexpress the model as

$$\begin{aligned} S &= I \\ S &= Y - C \\ C &= \bar{C} + cY \end{aligned}$$

where now the savings-investment balance has now been substituted for the first SAM equation. Substitute the last equation, the consumption function, into the second SAM equation and rewrite the system as

$$\begin{aligned} S &= I \\ S &= -\bar{C} + (1-c)Y \\ C &= \bar{C} + cY \end{aligned}$$

Now if the level of savings is given, so is the income, Y, in the second equation. With Y known, so are C and I from the third and first equations respectively. The character of the model is totally different; we must now ask what determines the level of savings, rather than investment. Is it retirement, education for children? Clearly additional detail is needed here to make a convincing model. In any case, it should be clear that the fundamental character of the model has changed from one in which "investment drives savings," to one in which "savings drives investment." This change is the product of the simple substitution of a parameter for a variable in the parameter and variable lists.

Finally, there is one detail that must be addressed. We have argued that the number of independent SAM equations is equal to the number of agents. Adding another variable like value added, V_A, does change the SAM equation count, but not in an essential way. Formally speaking, it would acceptable to define the model as:

$$\begin{aligned} Y &= C + I \\ S &= Y - C \\ Y &= V_A \\ C &= \bar{C} + cY \end{aligned}$$

with $V(Y, C, I, V_A)$ and $P(I, \bar{C}, c)$. Nothing has changed with the definition of this intermediate variable, V_A. It is always possible to add new intermediate variables and corresponding equations, without changing the basic structure of the model. We will often find it convenient to do so in solving models, especially analytically, but we shall retain the basic idea that the number of independent SAM equations is equal to the number of agents in the model.

4. Comparative Statics in the SAM Framework

Now that we have a formal model, calibrated to a base SAM, the next step is to employ it to do some basic policy analysis. Comparative static experiments within the SAM framework are straightforward and to the extent that the underlying model is linear, will correspond closely to formal comparative static analysis. In this section, we provide and example of the procedure. Consider the following definitions:

Definition 2. *Comparative statics: the change in the equilibrium values of the variables with respect to a change in one and only one parameter.*

There is much to say about this definition. First, note that the causality in models runs from the parameters to the variables as a whole. It follows that it is not generally possible to change a parameter without affecting more than one variable and often, the entire set of variables. There are special cases of course, in which the model is said to be *decomposable,* such that changes in a parameter only affect a subset of the variables.

A second point is that comparative statics requires that one and only one parameter be changed at a time. This is necessary for analytical clarity since if we changed more than one parameter, it would generally not be possible to apportion causality among the changed parameters. This is not to say that it is *impossible* to change more than one parameter at a time; clearly it is not. If more than one parameter is changed, it is not comparative statics, but rather simulation modeling that we are undertaking.

Definition 3. *Simulation: the change in the equilibrium values of the variables with respect to a change in more than one parameter at a time.*

Simulation modeling attempts to replicate the path of the economy by setting the values of a range of parameters simultaneously. This is a very different exercise from comparative statics in many ways and can be of great practical value in setting multiple policies simultaneously, despite its lack of precise analytical content in deciding issues of cause and effect.

Consider then a comparative static change in investment in the simple Republican paradise model

$$\begin{aligned} Y &= C + I \\ S &= Y - C \\ C &= \bar{C} + cY \end{aligned}$$

with $V(Y, S, C)$ and $P(I, \bar{C}, c)$. Since all three variables can change with respect to each of the three parameters, there are a total of *nine* comparative static results, or multipliers, available for this model.

Here are the steps

1. Indentify a parameter of interest.
2. Differentiate the entire system of equations with respect to the parameter choosen.

3. Solve for the three derivatives of the variables with respect to the choosen parameter.

   In this example, we choose I as the parameter of interest.

$$\begin{aligned}\frac{dY}{dI} &= \frac{dC}{dI} + \frac{dI}{dI} \\ \frac{dS}{dI} &= \frac{dY}{dI} - \frac{dC}{dI} \\ \frac{dC}{dI} &= \frac{d\bar{C}}{dI} + c\frac{dY}{dI}\end{aligned}$$

   but note immediately that $\frac{dI}{dI} = 1$ and $\frac{d\bar{C}}{dI} = 0$ since we have selected a different parameter, I for the comparative static analysis. Out next task is to solve the system

$$\begin{aligned}\frac{dY}{dI} &= \frac{dC}{dI} + 1 \\ \frac{dS}{dI} &= \frac{dY}{dI} - \frac{dC}{dI} \\ \frac{dC}{dI} &= c\frac{dY}{dI}\end{aligned}$$

Example 4. *Calculate all the comparative static derivatives dY/dI and dC/dI for the RP model of Example 3. Solving the system of equaitons above, we have $dY/dI = 1/(1-c)$; $dS/dI = 1$; $dC/dI = cdY/dI = c/(1-c)$ which are all familiar results. For the data of Example 3, we have $dY/dI = 4$ and $dC/dI = 3$.*

5. Multipliers in Excel

Let us see how these result correspond to the Republican Paradise problem in Table 3.3 To solve the model in Excel, it is necssary that we first go to *Tools/Options/Recalculation/Iteration* menu and be sure that box *iteration* is checked. The next step is to insert the consumption function into cell $C2$. Here we have written in hard numbers, 10 and 0.75 into the spreadsheet directly. This is not the best practice technique and is done here only for convenience. It would be much better to write the parameters a separate cell so they can be easily changed; this is a matter of taste and housekeeping, of course, and has no other significance.

A more complete spreadsheet is shown in Table 3.4. Here we have defined a base SAM and a second matrix in which the model is installed. Finally a third matrix is shown in rows 20-23.

Table 3.3. Multipliers in Excel

	A	B	C	D	E
1		Firms	Households	Investment	Total
2	Firms		$= 10 + 0.75 * C5$	40	$= sum(C2 : D2)$
3	HH	$= B5$			$= B3$
4	Savings		$= C5 - C2$		$= C4$
5	Total	$= E2$	$= E3$	$= D2$	
6					

The advantages of this structure are many in that since all the paratmers are explicitly shown in the first few rows, none is buried in the equations below. This is a better technique than employed in Table 3.3 since it allows for direct comparison of the effects of changed parameters. Note that there are no hard numbers below the 4th row; every other entry is a formula (or a text label). This structure not only minimizes embedding errors, that is an odd invisible hard number, itself perhaps a vestige of a previous experiment, but also allows us to use the SAM structure as a further consistency check. Not only must the forecast SAM balance, but also the ΔSAM, the difference between the base and the forecast SAM must balance as well.

TABLE 3.4. Multipliers in Excel, con't

	A	*B*	*C*	*D*	*E*	*F*
1	*Parameters*			Base	Forecast	
2	Investment			40	50	
3	Autonomous Consumption			10	10	
4	Marginal propensity to consume			0.75	0.75	
5						
6	*Base SAM*					
7		Firms	Households	Investment	Total	
8	Firms		$= D3 + D4 * C11$	$= D2$	$= SUM(C8 : D8)$	
9	Households	$= B11$			$= B9$	
10	Savings		$= C11 - C8$		$= C10$	
11	Total	$= E8$	$= E9$	$= D8$		
12						
13	*Forecast SAM*					
14		Firms	Households	Investment	Total	
15	Firms		$= E3 + E4 * C18$	$= E2$	$= SUM(C15 : D15)$	
16	Households	$= B17$			$= B16$	
17	Savings		$= C12 - C9$		$= C17$	
18	Total	$= E15$	$= E16$	$= D15$		
19						
20	*Changes from Base*					
21		Firms	Households	Investment	Total	
22	Firms		$= C15 - C8$	$= D15 - D8$	$= E15 - E8$	
23	Households	$= B16 - B9$			$= E16 - E9$	
24	Savings		$= C17 - C10$		$= E17 - E10$	
26	Total	$= B18 - B11$	$= C18 - C11$	$= D18 - D11$		

Table 3.5 confirms the effect of raising investment as it would be seen in Excel. Raising investment produces a balanced forecast SAM as well as a balanced SAM for the changes from the base.

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The agreement with the analytical solution is perfect since the underlying model is linear in the parameter I.as shown in Table 3.6

Example 5. *Consider next the effect on the marginal propensity to consume:*

$$\begin{aligned}\frac{dY}{dc} &= \frac{dC}{dc}\\ \frac{dS}{dc} &= \frac{dY}{dc}-\frac{dC}{dc}\\ \frac{dC}{dc} &= c\frac{dY}{dc}+Y\end{aligned}$$

Solving$\frac{dY}{dc} = Y/(1-c)$ *and* $\frac{dS}{dc} = 0$, *so that* $dY = Y dc/(1-c)$. *For an experiment in which c rises from* 0.75 *to* 0.8 *gives a predicted change in output of*

$$\frac{dY}{dc} = \frac{200(.05)}{0.25} = 40.0$$

In fact the change in the level of income is: $\frac{\Delta Y}{\Delta c} = 250 - 200 = 50$ *as shown in the Table 3.7.*

This example shows that at least some multipliers depend on the level of income and is therefore tied to the SAM and the structure of the economy.

Comparative statics is useful for understanding how a model works and what one might expect when its parameters change. But the goal of model building is not comparative statics, but simulation, in which several parameters can change at the same time. Even in this simple model, both investment and consumption might well change, in different proportions, while at the same time the marginal propensity to consume could also vary. This gives a richer menu of course, but what is missing is the ability to make analytical statements, such as “what is the effect of a unit change in investment?”

Table 3.8 shows the effect of raising investment by 10% while at the same time increasing autonomous consumption by 5% and reducing the marginal propensity to consume by 0.01 (in absolute terms). Both the increases in autonomous consumption and investment will, of course, drive GDP higher, but the effect on the consumption is partially offset

TABLE 3.5. Multipliers in Excel, con't

	A	B	C	D	E
1	*Parameters*			Base	Forecast
2	Investment			40	50
3	Autonomous Consumption			10	10
4	Marginal propensity to consume			0.75	0.75
5					
6	*Base SAM*				
7		Firms	Households	Investment	Total
8	Firms		160	40	200
9	Households	200			200
10	Savings		40		40
11	Total	200	200	40	
12					
13	*Forecast SAM*				
14		Firms	Households	Investment	Total
15	Firms		190	50	240
16	Households	240			240
17	Savings		50		50
18	Total	240	200	50	
19					
20	*Changes from Base*				
21		Firms	Households	Investment	Total
22	Firms		30	10	40
23	Households	40			40
24	Savings		10		10
25	Total	40	40	10	
26					

TABLE 3.6. Comparative static derivatives

$$
\begin{array}{rlrl}
\frac{dY}{dI} = & 4 & \frac{\Delta Y}{\Delta I} = & 40 = 240 - 200 \\
\frac{dS}{dI} = & 1 & \frac{\Delta S}{\Delta I} = & 10 = 50 - 40 \\
\frac{dC}{dI} = & 3 & \frac{\Delta C}{\Delta I} = & 30 = 190 - 160
\end{array}
$$

TABLE 3.7. Change in the MPC

	A	*B*	*C*	*D*	*E*
1	*Parameters*			Base	Forecast
2	Investment			40	40
3	Autonomous Consumption			10	10
4	Marginal propensity to consume			0.75	0.8
5					
6	*Base SAM*				
7		Firms	Households	Investment	Total
8	Firms		160	40	200
9	Households	200			200
10	Savings		40		40
11	Total	200	200	40	
12					
13	*Forecast SAM*				
14		Firms	Households	Investment	Total
15	Firms		210	40	250
16	Households	250			250
17	Savings		40		40
18	Total	250	200	40	
19					
20	*Changes from Base*				
21		Firms	Households	Investment	Total
22	Firms		50	0	50
23	Households	50			50
24	Savings		0		0
25	Total	50	50	0	

by the rise in the MPC. Note the complexity of the change, even in such a small model. The elasticity of consumption with respect to income can always be expressed as the ratio of the slope of the curve to the slope of the chordline, which is drawn from the origin to the point on the curve at which the elasticity is to be computed. The ratio of the marginal to average consumption has fallen in this example, as the intercept increases, but the slope falls. Table 3.8 shows that the net effect of these various changes in this simple model amounts to a GDP growth of 5%, whereas consumption increases by 4%.

Table 3.8. A simulation model

	A	*B*	*C*	*D*	*E*
1	*Parameters*			Base	Forecast
2	Investment			40	44
3	Autonomous Consumption			10	10.5
4	Marginal propensity to consume			0.75	0.74
5					
6	*Base SAM*				
7		Firms	Households	Investment	Total
8	Firms		160	40	200
9	Households	200			200
10	Savings		40		40
11	Total	200	200	40	
12					
13	*Forecast SAM*				
14		Firms	Households	Investment	Total
15	Firms		166	44	209.6
16	Households	209.6			250
17	Savings		44		44
18	Total	209.6	210	44	
20					
21	*Changes from Base*				
22		Firms	Households	Investment	Total
23	Firms		6	4	9.6
24	Households	9.6			9.6
25	Savings		4		4.0
26	Total	9.6	10	4	
27					

6. Productivity, Labor Markets and Income distribution

The tables above report some important features of the small economies there, but there is much more that can be added. There is, for example, no labor market in this model. We know nothing about income distribution from the base SAM since the needed detail is subsumed in the row of value added. With some additional information on the distribution between wages and profits, we can make the model more realistic.

The first step is to break down value added, V_a, into labor remuneration, L and nonwage income Π

$$V_a = L + \Pi$$

Dividing by gross value of production, X, we can write

$$v_a = l + \pi$$

where v_a is unit value added, that is divided by X, l is the direct labor coefficient and π is total nonwage income per unit of output.

Taking the wage rate, w, we have unit cost equals price, or:

$$wl + \pi = p$$

where p is now the GDP deflator. We set both the wage and the deflator equal to one for the base year for which the SAM is constructed. The real wage, w^r,

$$w^r = \frac{w}{p}$$

is then also equal to one.[2] In the simple model above, the direct labor coefficient gives the level of emplyment, $L = lY$. This implies that employment rises in proportion to output. If there is techological change is involved, employment per unit of output could fall exogenously, according to

$$l_t = l_{t-1}(1 - \hat{\rho})$$

where $\hat{\rho}$ is the rate of growth of productivity and t indicates the time period.[3] Labor productivity can be estimated from time series data and is usually about 1 to 2% per year [**?**].

The implications of productivity growth needs to be absolutely clear. In order for employment to rise, the rate of growth of GDP must be exceed rate of growth of productivity. In Table 3.8 above, GDP is

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[2] Incidentally, we need not worry in this case about whether we are referencing the real or the nominal wage since they are the same in the base SAM.

[3]Productivity is just the inverse of the labor coefficient

$$\rho = \frac{Y}{L}$$

By the rules of "hats" we have

$$\hat{\rho} = \hat{Y} - \hat{L}$$

But

$$l = L/X$$

which gives

$$\hat{l} = \hat{L} - \hat{X}$$

and thus $\hat{\rho} = -\hat{l}$; that is, the negative of the growth rate of the labor coefficient is the growth rate of productivity.

growing at just less than 5%. If productivity growth is 1.5%, then employment will only be less than 3.5%. Employment growth can only accelerate if wages fall and more labor is used per unit of output at the same rate of technological change. This may be difficult to achieve and if achieved difficult to defend to some political constituencies.

Standard microeconomic theory requires the real wage to be set equal to the marginal product of labor for profit maximization. The marginal product of labor is the derivative of the production function with respect to labor, of course, and for a Cobb-Douglas production function of the form

$$Q = K^{\beta} L^{(1-\beta)}$$

we can write, using the exponent rule for derivatives:

$$\frac{\partial Q}{\partial L} = (1-\beta) K^{\beta} L^{(1-\beta-1)}$$

where $\frac{\partial Q}{\partial L}$ is written as a partial derivative to indicate that K is treated as a constant in the process of differentiation. Although this appears to be a complicated expression, the Cobb-Douglas function is used so frequently largely becauses its marginal products can be expressed simply. The key step is to substitute the production function itself back into the expression for the marginal product. First note that we can write:

$$K^{\beta} L^{(1-\beta-1)} = \frac{Q}{L}$$

so that we can eliminate the complicated expression as in the marginal product on the right-hand side of the previous equation. Note further that Q/L is the inverse of the labor coefficient, L/Y, so long, that is, that output Q is equal to Y. We shall have much more to say on this issue below, but for right now, let us assume that they are indeed the same. We then can write the condition that the marginal product equals the real wage as:

$$\frac{\partial Q}{\partial L} = \frac{(1-\beta)}{l} = w^r \tag{3.3}$$

Labor demand must be consistent with this equation so we can solve for the *demand for labor*

$$L = \frac{(1-\beta)}{w^r} Q \tag{3.4}$$

where β is a parameter to be calibrated. The labor coefficient is inversely proportional to the real wage for the Cobb-Douglas technology and so to offset a 1% decline in the labor coefficient requires, approximately, a 1%

reduction in the real wage. This will be a useful fact to keep in mind in the continuation.

Equation 3.3 is the first-order condition for profit maximization and embodies the relationship between the real wage, w^r, labor productivity (the inverse of the labor coefficient) and the wage share $\sigma = (1 - \beta)$. With $\rho = 1/l$ for labor productivity and "hats" as growth rates of the variables, the equation can be written as

$$\begin{aligned} \frac{w^r}{\rho} &= \sigma \\ \hat{w}^r - \hat{\rho} &= \hat{\sigma} \end{aligned}$$

so that if real wages are constant, $\hat{w}^r = 0$, the wage share falls with productivity growth on percent-for-percent basis. On the other hand, if real wages keep up with productivity, then the labor share is constant and we have $\hat{\sigma} = 0$. Real wages growth should and usually is modelled separately from productivity growth depending on local circumstances. In most countries, the share of labor is more or less constant so that real wages do eventually adjust to productivity growth. All that we can say is that there is usually no *trend* in the wage share, since this would eventually lead to its rising above one or falling below zero, and that makes no sense. This does not mean that the wage share resists all change. In the short run it can be very volatile and in the long run can shift from one center of gravity or variation to another as a result of stuctural change.

We can analyze one-shot structrural change easily in the context of the simple models of this chapter. If, for example, globalizaton has caused a permant decline in the share of labor, and a rise in the share of capital, there could be some fairly dramatic macroeconomic effects. The first question is how consumption responds to the wage share? This is an old and debated issue in economics [**?**]. Intuitively, a redistribution from the "rich" to the "poor" shouild increase consumption. The data however, does not fully support this expectation. The issue is frought with estimation problems that can easily obscure the effect. First, wage income is not a good measure of the incomes of the "poor", especially in developing countries in which there is an active informal sector. If wage income measures formal sector workers, redistibution from non-wage to wage income might worsen the distribution of income and cause consumption to fall. Even if it is true that every individual would save more with higher income, raising the wage of a given individual by one dollar and reducing the assoicated profit income *of another individual* by the same amount will only cause consumption to rise the income of the first is less than the income of the second person. This is a "pure" income redistribution, as discussed by Blinder [**?**] and others. But these pure redistributions are difficult to "see" in the aggregate data. A rise in wages may well be offset by a decline in informal sector returns or accrued profits, with dramatically different effects for each.

Consider for example, a model in which there are only three classes: workers, managers and owners, as discussed in Palley (2005). Middle managers are considered to be workers; they earn a wage that may or may not be tied to the profitablity of the firm. Now reduce profits and increase wages by the same dollar amount. The rise in wage income is not likely to be completely offset, if at all, by a reduction in payments to the managerial class. Perhaps dividends paid to owners will fall which might well affect the rich, but it might also affect retired households. If dividends are not cut, them perhaps some other component of cost might be reduced, including casual low wage labor or even pensions payments. Identifying who ultimately "pays" for the wage increase could be quite complex. The point is that any attempt to empirically isolate the effect of distribution on aggregate consumption will be muddied by these complications. This does not mean that no predictable relationships exist. It is rather that policymakers must have a clear view of winners and losers before implementing any proposal designed to simulate the economy.

There are a few clear cases in which redistribution would have an impact on consumption. Pick the poorest individual and transfer income to that person from *any* other. If the MPC falls with income, consumption will then most likely rise as a share of GDP. This result can be generalized to any individual whose income-savings decision made subject to a binding survival constraint. As income is shifted to these individuals, consumption must increase by definition so long as the constraint continues to bind. Only when the survival constraint ceases to bind, can individuals make a decision to save versus consume.

Another complication arises when extended families jointly maximize [**?**]. A given family may consist of retired and elderly, wage earners and children. A rise in wages might well have a complex impact in this environment. If there is a pure redistribution from some individual with a higher income to this family, aggregate consumption might go up or down. If the family elects to increase consumption then the aggregate will increase as well of course; but, it the family decides to save the addition to income to pay for future educational expenditures, it is easy to see that aggregate demand will in fact *contract* with this redistribution from rich to poor. The question becomes who pays for the redistribution. If workers' wages increase at the expense of even poor segments of the economy, then aggregate demand will fall, under the assumption of an inverse relationhip between income level and the MPC. But note that if profits are indeed insulated from the rise in wages, then there is no immediate reason to believe that investment will fall at all. We shall return to this question in subsequent chapters, but for now all we can confidently conclude is that consumption will rise when income is redistributed from savers (accumulators) to non-savers (consumers).

Table 3.9 continues the simulation of Table 3.8 but adds to it the division between wages and profits. The labor coefficient is taken from other data sources and is set at 0.55. With wages taken as one ($w = 1$) in the base, this implies that $(1 - \beta) = \sigma = 0.55$. The remainder of value added is allocated to profits. We assume that over the period of time for which the comparative statics is relevant there is productivity growth of 1%. This reduces the labor coefficient from 0.55 to 0.5445 as shown in the table. The first observation that can be made concerns employment. In Table 3.8 employment increases by 4.8%, the same rate of growth as the GDP. But as shown in Table 3.10, which is just a continuation of Table 3.9, the growth in productivity has caused a slowdown in hiring; as a result, employment only grows by 3.8%, a percentage point less. Productivity growth has evidently been captured by profits.

As shown in Table 3.9, profits increase by slightly more than 6%. The likely effect of this redistribution is most likely to lower consumption and raise investment even further. But by how much in each case? The answer is not clear and has to be judged according to the current conditions of the economy. Note that the change in these two components of aggregate demand will offset each other and it is a matter of debate which effect will dominate. [**?**] We have already seen that the effect of a small increase in the marginal propensity to consume is significant. Note that without the contractionary effect on the marginal propensity to consume or an expansionary effect on investment, there would be no impact of productivity growth whatsoever. GDP is the same in Table 3.8 and 3.9. So far, the change in distribution is entirely neutral, but in the next period, it could alter the size and composition of GDP. If GDP rises with a shift in the distribution of income toward profits then the economy is usually termed "exhilarationist" while if GDP stagnates (falls) then it is stagnationist.

TABLE 3.9. A model with productivity growth

	A	*B*	*C*	*D*	*E*
1		*Parameters*		Base	Forecast
2	Investment			40	44
3	Autonomous Consumption			10	10.5
4	Marginal propensity to consume			0.75	0.74
5	Labor coefficient			0.55	0.5445
6	Productivity growth			0.01	
7					
8		*Base SAM*			
9		Firms	Households	Investment	Total
10	Firms		160	40	200
11	HH	200			200
12	Profits	90			90
13	Wages	110			110
14	Savings		40		40
15	Total	200	200	40	
16					
17		*Forecast SAM*			
18		Firms	Households	Investment	Total
19	Firms		165.6	44	
20	HH	209.6			209.6
21	Profits	95.5			95.5
22	Wages	114.1			114.1
23	Savings		44		44
24	Total	209.6	209.6	44	
25					

There is no way to say *a priori* which effect will predominate. If GDP is does not change, as in Table 3.9, the two effects exactly cancel. In this case, a 10% increase investment is necessary to offset a one percentage point decline in the marginal propensity to consume. The economy is on the borderline between the two regimes. One may even question the relationship between the *share* of income going to wages and the marginal propensity to consume. The payment to labor, as seen in Table 3.10 has increased, but how that payment is distributed to households is what makes the crucial difference. Consider the case in which real wages are constant. Each employed worker then receives the same real income as in the previous period. If the total wage bill has risen it

Table 3.10. A model with productivity growth, con't

	A	B	C	D	E
26	Changes from Base				
27		Firms	Households	Invest	Total
28	Firms		5.6	4	9.6
29	Households	9.6			9.6
30	Profits	5.5			5.5
31	Wages	4.1			4.1
32	Savings		4		4
33	Total	9.6	9.6	4	
34					
35	Reporting				
36					
37	*Exogenous changes*		*Growth rates*		
38	Investment		0.10		
39	Autonomous Consumption		0.05		
40	MPC		-0.01		
41	*Endogenous changes*				
42	GDP		0.048		
43	Consumption		0.035		
44	Employment		0.038		
45	Profits		0.061		
46					

follows that some workers who were not employed in the past are now employed. If wages of new workers are lower than those previously employed, then the average wage might fall while no one's individual wage is falling. This could cause the marginal propensity to consume out of wage income to rise. Income of profit recipients can be thought of in the same way. If profits rise because new firms are satisfying the extra demand, and the profitability of these firms is lower than average, then the propensity to consume out of profit income might also rise. The asssumption of a falling marginal propensity to consume would be then be off base. If when there is no change in the total GDP cannot safely *assume* that a redistribution of income from rich to poor would lower the marginal propensity to consume, especially when there is an active informal sector. In that case, a redistribution from profits to workers would not necessarily correspond to a redistribution from rich to poor. On the other hand, when the *functional* distribution of income between wages and profits closely corresponds to the *size* distribution of income between households, the assumption of a progressive distribution of income leading to a higher marginal propensity to consume would be valid.

Population growth can make a big difference since young workers can enter the labor market at wages lower than the prevailing. The average wage could then fall with population growth without any individual wage falling. This is one of the structural features of aggregation we have already discusses at some length.

EXAMPLE 6. *In Table 3.9, how much must investment increase to offset a one percentage point decline in the MPC? Solution: There are two ways of accomplishing this task in Excel. The first is very straightforward and is often the easiest in practice. After reducing exogenously the MPC, in* $E4$ *(in Table 3.9) we adjust the level of investment in cell* $E2$ *until the rate of growth of GDP in cell* $C42$ *in Table 3.10 is zero This can also be done by way of Solver, the built in equation solver in Excel. Solver works nicely, when it works, but it can easily fail if the solution to the problem is far away from the initial starting point. Solver will not necessarily find a solution even when the initial guess is very close, but most of the time it is successful. On the Excel menu, choose Tools/Solver and a dialogue box will open to allow us to set the parameters. Go to "Set Target Cell" and choose* $C42$*; click on "Value of" and enter zero in the space provided. Go to "By Changing Cells" and enter* $E2$*. Solver is now ready to go. Click on "solve" and hope that it finds a solution. If it does not, raise the level of investment in* $E2$ *to something higher, say 41, and try solver again.*[4] *It will eventually find that new investment should be 41.5, a 3.75 percent increase. Confirm that if there is no increase in autonomous consumption, then the required increase in investment is 5%.*

While this simulation of this example is not formally a comparative statics excercise, it is nonetheless somewhat analytical in that it is not attempting to track the real history of the economy in question. It serves rather to focus debate on the critical issues: will productivity growth affect employment and therefore consumption? And by how much? If it does, will the contractionary impact be offset by rising investment? If autonomous consumption does not increase with investment, profits rise by only 1.2% and this had to have generated an 5% increase in investment to maintain a level GDP. This is an elasticity of more than

[4]Do not be afraid to try it several times. Also check to see that there are actually Excel expressions in each cell.

4.1, relatively high by empirical standards. Our best guess is that productivity growth in this model, with this data, would *not* be self-correcting through increases in investment.

If there were no difference in the spending behavior out of labor versus profit income, then there would be no loss in consumption to make up for. The positive impact of productivity on investment would always give rise to an increase inoutput, more or less powerfully depending on the elasticity of investment with respect to profits. It is possible that a shift from wages to profits could have no impact on consumption? In other words, are these conclusions a product of the kind of model we have sketched? A more classical model, in which savings drives investment, might yield different conclusions. Let us explore this possibility in the following environment. First, let the capital stock be fixed and say that the labor supply is $\bar{L}$. From the definition of labor productivity, ρ, we have.

$$Y = \rho\bar{L} \tag{3.5}$$

If somehow productivity were known, fixed and given, the level of labor supply $\bar{L}$ would determine income Y. If income is known, then consumption follows from equation 3.1, the consumption function. With both income and consumption known, savings is determined as a residual in the SAM. Remarkably, equation 3.5 implies that output cannot change unless productivity changes. Since productivity is just the inverse of the labor coefficient, technological change which lowers the level of labor input per unit of out will *automatically raise output* in the savings-driven or classical model. Morerover, when the change in the labor coefficient produces a proportional change in output, that is, with an elaticity of one. If that is not the case, then labor productivity must itself change according to equation 3.5.

Table 3.11 shows the stucture of the classical savings-driven model in the SAM. Follow the sequence of causality. Output is determined by the product of labor supply and labor productivity in cell $B10$. Under the assumption that the both real and nominal wages are equal to one, the labor supply determines the entry in $B12$. Profits are then the residual of output and are calculated that way in $B11$, under the assumption of no firm savings. Household income is then determined as the sum of wages and profits and serves as the budget constraint in $C14$. The consumption function is installed in cell $C9$ and savings in cell $C13$ is then determined as a residual. Now look across row 9. With the total already know, *investment becomes a residual.* This is the essence of the classical model.

TABLE 3.11. A classical model

	A	*B*	*C*	*D*	*E*
1		*Parameters*		Base	
2	Autonomous Consumption			10	
3	Marginal propensity to consume			0.75	
4	Productivity ($\rho = 1/l$)			1.82	
5	Labor supply			110	
6					
7		*Base SAM*			
8		Firms	Households	Investment	Total
9	Firms		$= D2 + D3 * C14$	$= E9 - C9$	$= B14$
10	HH	$= D5 * D4$			$= B10$
11	Profits	$= B14 - D5$			$= B11$
12	Wages	$= D5$			$= B12$
13	Savings		$= C14 - C9$		$= B13$
14	Total	$= B10$	$= E10$	$= D9$	

There is no room for an independent investment function once income has already been determined by productivity. Usually we say that income is determine in the *factor markets* and this is certainly true. Just think about what this means. In order to determine aggregate labor demand, one must have a production function since labor demand is a *derived demand*, derived from the demand for goods. In any given period the level of capital is fixed so if the labor market equilibrium gives the total amount of labor hired, output can be found by inserting the equilibrium level of labor into the production function. No need to worry about consumption or investment; it is the job of the SAM to make sure that they are consistent with the level of output determined in this way.

EXAMPLE 7. *Analyze the same a 1% increase in labor productivity of Table 3.9 in a model in which savings drives ivestment. Solution: Table 3.12 shows the effect of increased productivity. Note that the 1% increase in output per worker, ρ, causes a 1.25% increase in investment and a 1% increase in GDP. In the model of Table 3.9 a 1.25% in investment would also cause 1% increase in output (assuming no change in the consumption parameters). But there is a very important difference. A*

TABLE 3.12. A classical model with productivity growth

	A	B	C	D	E
1	*Parameters*			Base	Forecast
2	Autonomous Consumption			10	10
3	Marginal propensity to consume			0.75	0.75
4	Productivity			1.82	1.84
5	Productivity growth			0.01	
6	Labor supply			110	110
7					
8	*Base SAM*				
9		Firms	Households	Investment	Total
10	Firms		160	40	200
11	HH	200			200
12	Profits	90			90
13	Wages	110			110
14	Savings		40		40
15	Total	200	200	40	
16					
17	*Forecast SAM*				
18		Firms	Households	Investment	Total
19	Firms		161.5	40.5	
20	HH	202			202
21	Profits	92			92
22	Wages	110			110
23	Savings		40.5		40.5
24	Total	202	202	40.5	
25					

fall in the margianl propensity to consume could cancel out the effect of rising investment, causing GDP to stagnate. In Table 3.12 a fall in the marginal propensity to consume would have no effect on output growth. It would cause investment to increase, by 6.25% if the MPC falls from 0.75 to 0.74. We will have more to say about this problem in Chapter 4 on dynamics.

7. Slightly More Complex Models

We now abandon the RP model and add in government and the foreign sector. We can augment the s the simple model with governent and a foreign sector as seen in the SAM of Table3.13

TABLE 3.13. A SAM with Government and Foreign Sectors

	A	B	C	D	E	F	G
1	*Parameters*			Base			
2	Autonomous Consumption			10			
3	Marginal propensity to consume			0.75			
4	Labor coefficient			0.55			
5	Marginal propensity to import			0.08			
6	Tax rate			0.1			
7	Transfers			10			
8							
9	*Base SAM*						
10		Firms	Households	Invest	Govt	Foreign	Total
11	Firms	0	172	40	26	12	250
12	Households	230			10		240
13	Profits	92.5			4		96.5
14	Wages	137.5			6		143.5
15	Savings		44		-12	8	40
16	Govt		24				24
17	Foreign	20	0				20
18	Total	250	240	40	24	20	
19							
20	*Reporting*						
21	GVP		250				
22	GDP		230				
23	Personal income		240				
24	Personal disposable income		216				
25			0				
26	Current Account Surplus		0				
27	Fiscal		-12				
28	Foreign		-8				
29							

We can make several observations about this SAM which will be important for what follows.

Consider an open economy in which we have the SAM equation for firm income, Y_f:

$$Y_f = C + I + G + N_f \tag{3.6}$$

where C = consumption demand, I = investment demand, G = demand by government and N_f is net firm exports. These latter are exports of firms less competitive imports, that is imports that compete directly with the production of firms. The current accoun surplus, is difference between net firm exports and all other imports, including transfers and interest payments. Since in this SAM there are no foreign transfers or interest payments, the current account, shown in the reporting section is just the trade balance T.

$$T = N_f - eM \tag{3.7}$$

where e is the nominal exchange rate.

The behavioral equations of the model are first, the level of consumption given by

$$C = c_0 + cY^d \tag{3.8}$$

where Y^d is disposable income. Assuming that firms pay out all their income we have:

$$Y^d = (1 - t)(Y_f + T^r) \tag{3.9}$$

where t is the income tax rate on households and T^r is government transfers. Transfers in this SAM include government wages, interest payments as well as ordinary transfer payments and are taxed at the same rate as ordinary income. There are no indirect or sales taxes in the simple model.

Note that the exact equations are not installed in Excel, as above, since it is not necessary. As a relational data base, the program makes the calculations in a natural way, processing (and saving) intermediate steps that that are not precisely expressed by the analytical equations.

TABLE 3.14. A SAM with Government and Foreign Sectors

	A	*B*	*C*	*D*
1	*Parameters*			Base
2	Autonomous Consumption			10
3	Marginal propensity to consume			0.75
4	Labor coefficient			0.55
5	Marginal propensity to import			0.08
6	Tax rate			0.1
7	Transfers			10
8				
9	*Base SAM*			
10		Firms	Households	Invest
11	Firms		$= D2 + D3 * (C18 - C16)$	40
12	Households	$= B18 - B17$		
13	Profits	$= B12 - B14$		
14	Wages	$= D4 * B18$		
15	Savings		$= C18 - C16 - C11$	
16	Govt		$= C18 * D6$	
17	Foreign	$= D5 * B18$		
18	Total	$= G11$	$= G12$	$= D11$
19				
20	*Reporting*			
21	GVP		$= G11$	
22	GDP		$= G11 - B17$	
23	Personal income		$= G12$	
24	Personal disposable income		$= C18 - C16$	
25				
26	Current Account Surplus			
27	Fiscal		$= E15$	
28	Foreign		$= -F15$	
25				