Modelado de Sistemas Físicos

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Modelado de Sistemas Físicos

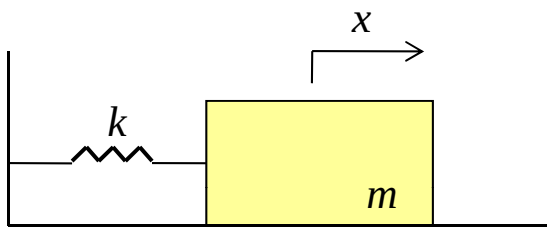
Unidad II: Modelado de sistemas mecánicos y electromecánicos.

Tema 1. (Parte mecánica) Componentes básicos de un sistema mecánico. Leyes de Newton. Modelos matemáticos de sistemas mecánicos.

Tema 2. Analogías. Ecuaciones de movimiento de Lagrange.

Modelado de Sistemas Mecánicos

Sistema masa-resorte



$$\sum F_y = m\,a(t)$$

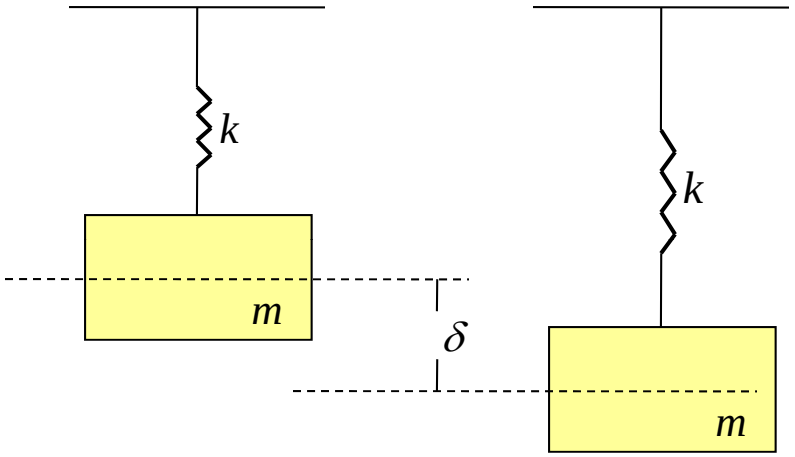
$$N - m\,g = m\,a(t) = 0$$

$$\sum F_x = m\,a(t)$$

$$-F_k = m\,\ddot{x}(t)$$

$$m\,\ddot{x}(t) = -k\,x(t)$$

$$\ddot{x}(t) = -\frac{k}{m}x(t)$$



δ es constante

En equilibrio:

$$m\,g = k\,\delta$$

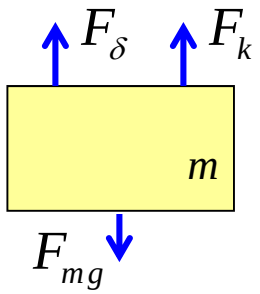
$$\sum F_y = m\,a(t),$$

$$-F_k - F_\delta + F_{mg} = m\,\ddot{y}(t)$$

$$m\,\ddot{y}(t) = -k\,y(t) - k\,\delta + m\,g,$$

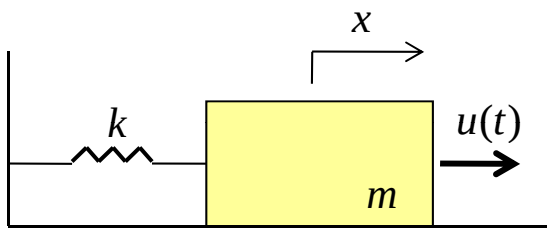
$$\ddot{y}(t) = -\frac{k}{m}y(t)$$

DCL



Modelado de Sistemas Mecánicos

Sistema masa-resorte

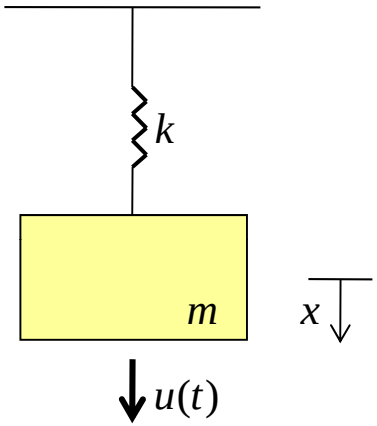


$$\sum F_x = m\,a(t)$$

$$u(t) - F_k = m\,\ddot{x}(t),$$

$$m\,\ddot{x}(t) = -k\,x(t) + u(t)$$

$$\ddot{x}(t) = -\frac{k}{m}x(t) + \frac{1}{m}u(t)$$



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Modelado de Sistemas Mecánicos

Representación Interna

$$\ddot{x}(t) = -\frac{k}{m}x(t) + \frac{1}{m}u(t)$$

Cambio de variable: $x_1(t) = x(t)$

$$x_2(t) = \dot{x}_1(t) = \dot{x}(t)$$

Así: $\dot{x}_1(t) = x_2(t)$

$$\dot{x}_2(t) = -\frac{k}{m}x_1(t) + \frac{1}{m}u(t)$$

Mido la posición

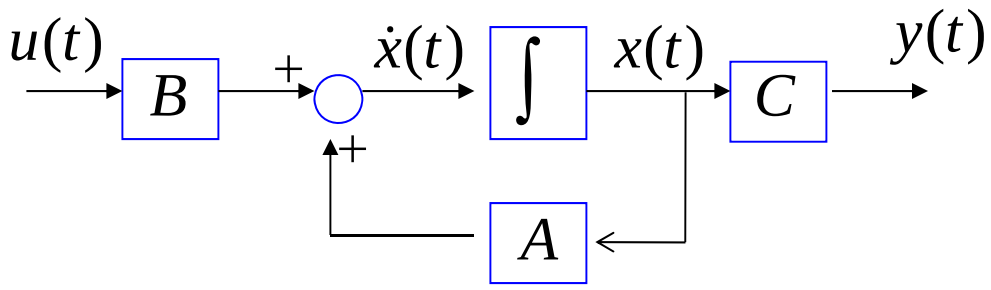
$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Modelado de Sistemas Mecánicos

Diagrama de Bloques

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$



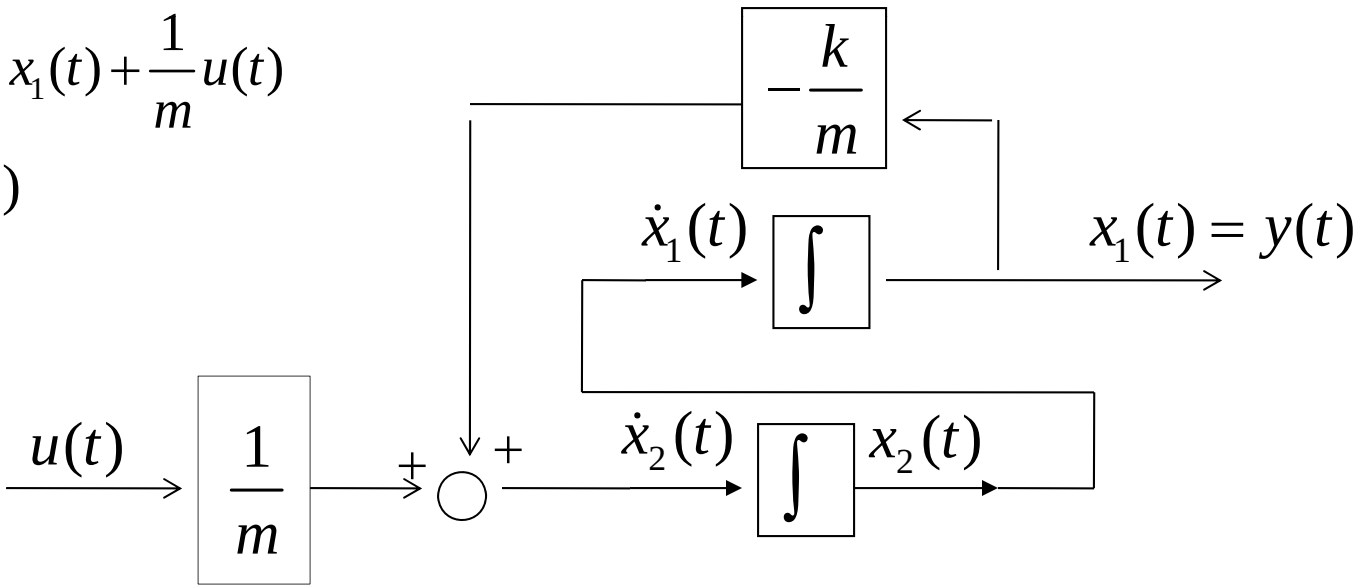
Modelado de Sistemas Mecánicos

Diagrama de Bloques Detallado

$$\dot{x}_1(t) = x_2(t)$$

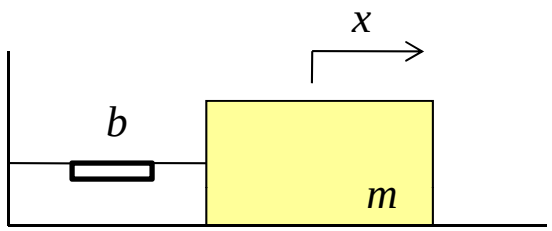
$$\dot{x}_2(t) = -\frac{k}{m}x_1(t) + \frac{1}{m}u(t)$$

$$y(t) = x_1(t)$$



Modelado de Sistemas Mecánicos

Sistema masa-amortiguador



$$\sum F_y = m\,a(t)$$

$$N - m\,g = m\,a(t) = 0$$

$$\sum F_x = m\,a(t)$$

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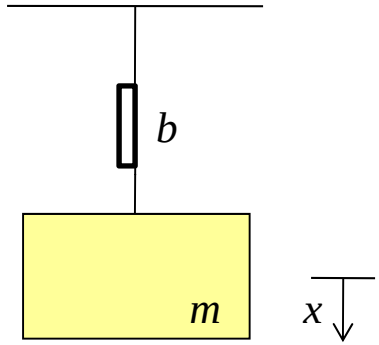
$$m\,\ddot{x}(t) = -b\,\dot{x}(t)$$

$$\ddot{x}(t) = -\frac{b}{m}\,\dot{x}(t)$$



$$\sum F_x = m\,a(t),$$

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Modelado de Sistemas Mecánicos

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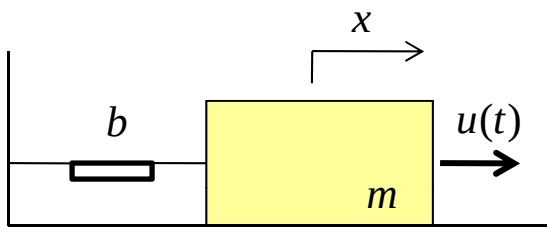

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$$\sum F_x = m\,a(t)$$

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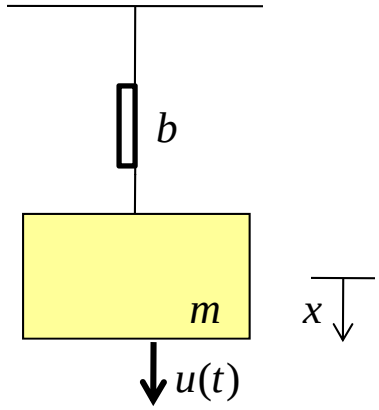
$$\ddot{x}(t) = -\frac{b}{m}\dot{x}(t) + \frac{1}{m}u(t)$$



$$\sum F_x = m\,a(t),$$

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$$m\,\ddot{x}(t) = -b\,\dot{x}(t) + u(t)$$

$$\ddot{x}(t) = -\frac{b}{m}\dot{x}(t) + \frac{1}{m}u(t)$$

Modelado de Sistemas Mecánicos

Representación Interna

$$\ddot{x}(t) = -\frac{b}{m}\dot{x}(t) + \frac{1}{m}u(t)$$

Cambio de variable: $x_1(t) = x(t)$

$$x_2(t) = \dot{x}_1(t) = \dot{x}(t)$$

Así: $\dot{x}_1(t) = x_2(t)$

$$\dot{x}_2(t) = -\frac{b}{m}x_2(t) + \frac{1}{m}u(t)$$

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t),$$

Mido la posición

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Modelado de Sistemas Mecánicos

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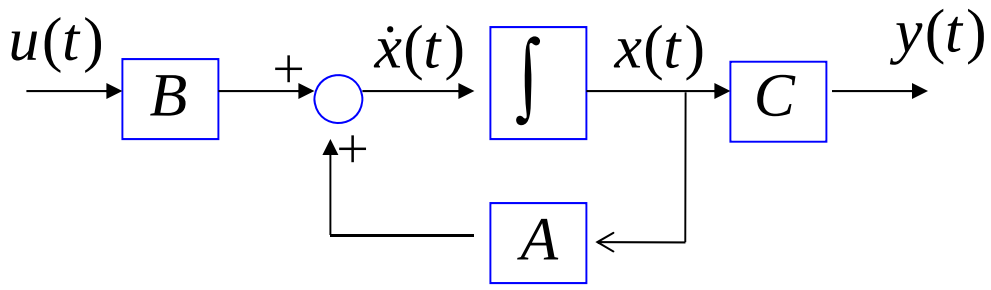

Modelado de Sistemas Mecánicos

Diagrama de Bloques Detallado

$$\dot{x}_1(t) = x_2(t)$$

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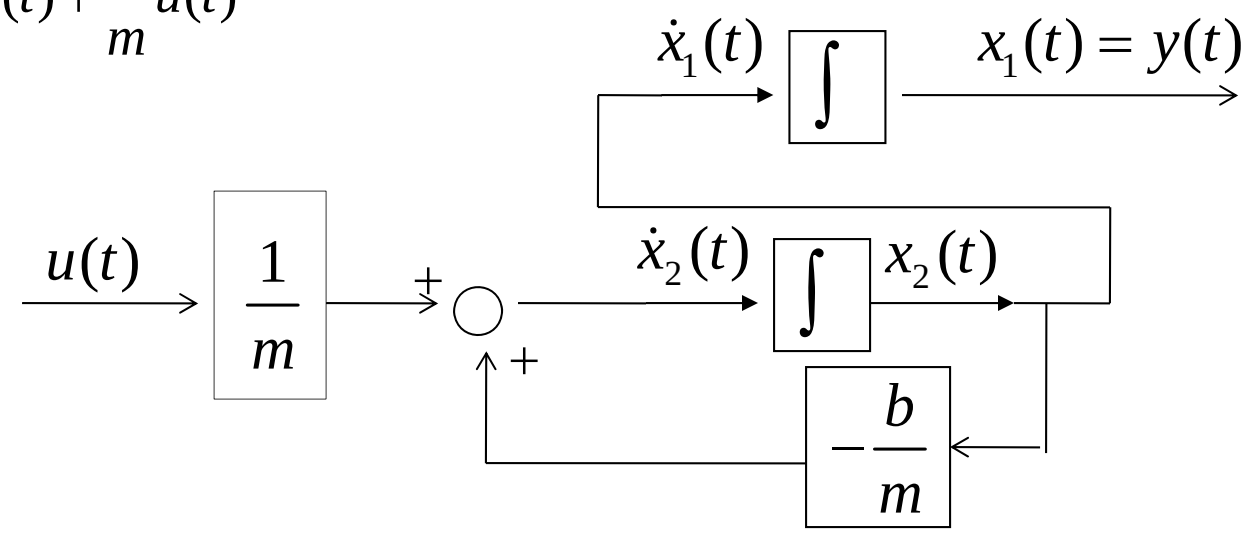
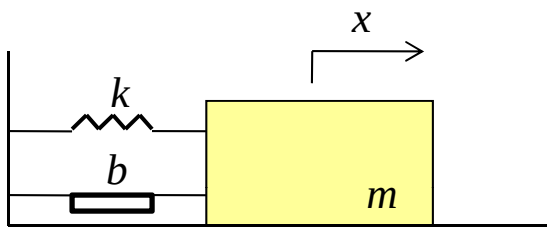


Modelado de Sistemas Mecánicos

Sistema masa-resorte-amortiguador



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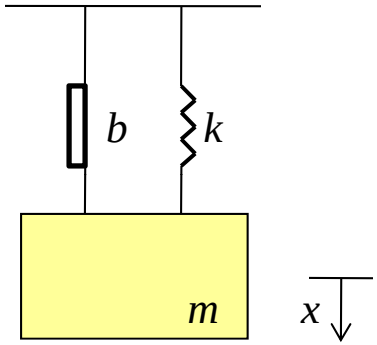

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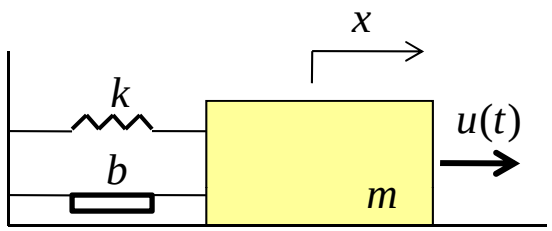

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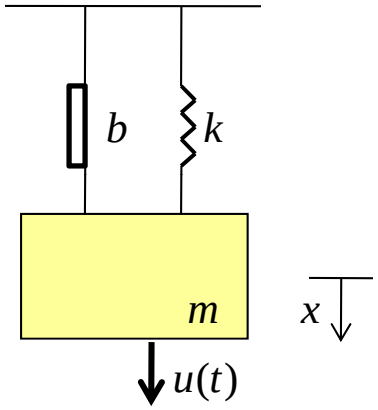

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Modelado de Sistemas Mecánicos

Representación Interna

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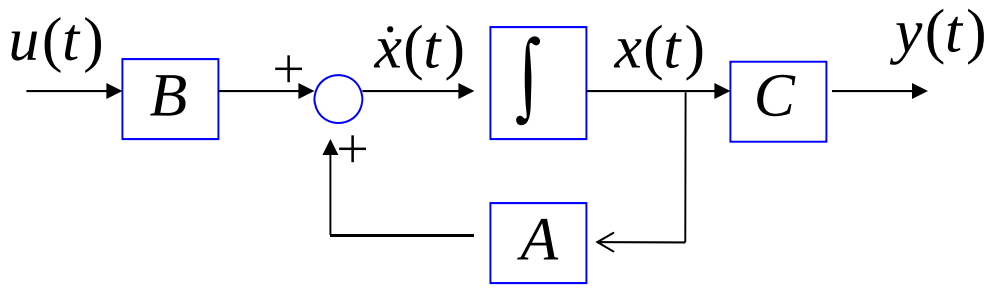

Modelado de Sistemas Mecánicos

Diagrama de Bloques Detallado

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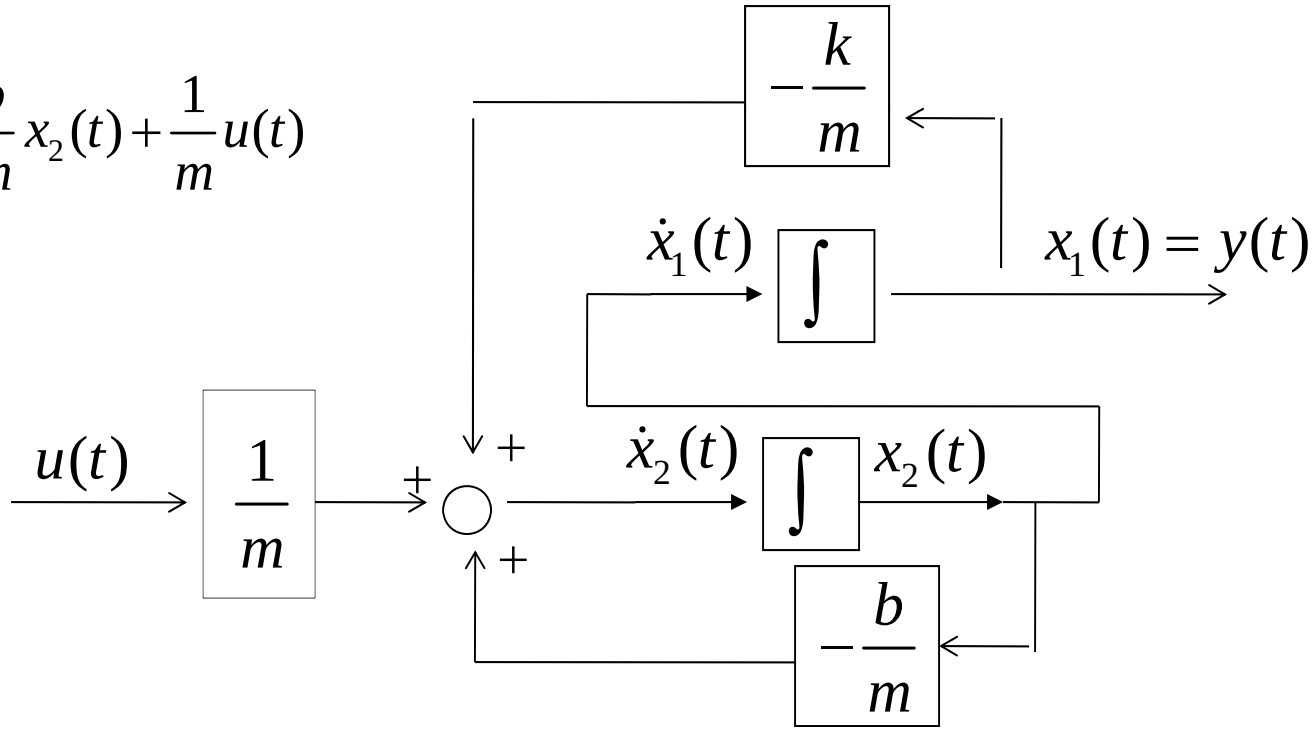

Modelado de Sistemas Físicos

Referencias del material usado para estas diapositivas:

•Material de las diapositivas de la Prof. Mariela Cerrada. Departamento de Control, Facultad de Ingeniería, Universidad de Los Andes, Mérida, Venezuela, 2012.

•Ogata, K. Dinámica de Sistemas, Prentice Hall, 1987.

•Lewis, J. Modelling Engineering Systems, High Text Publications, 1994.