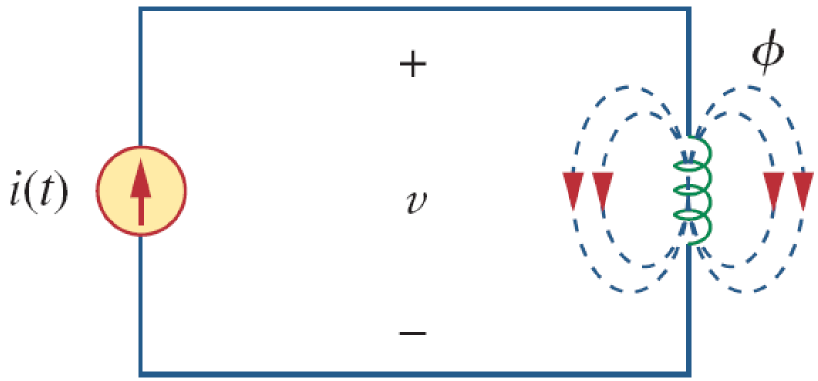
INGENIERIA
UNIVERSIDAD DE LOS ANDES
MÉRIDA VENEZUELA

INGENIERIA ELECTRICA
ULA

Acoplamiento Magnético

Prof. Gerardo Ceballos

Autoinductancia



$$v = N\frac{d\phi}{dt}$$

Faraday

$$v = N\frac{d\phi}{di}\frac{di}{dt}$$

En una bobina el cambio en el flujo se produce por el cambio en la corriente

$$v = L\frac{di}{dt}$$

$$L = N\frac{d\phi}{di}$$

Autoinductancia, viene dada por la configuración geométrica y espacial de las espiras

Autoinductancia

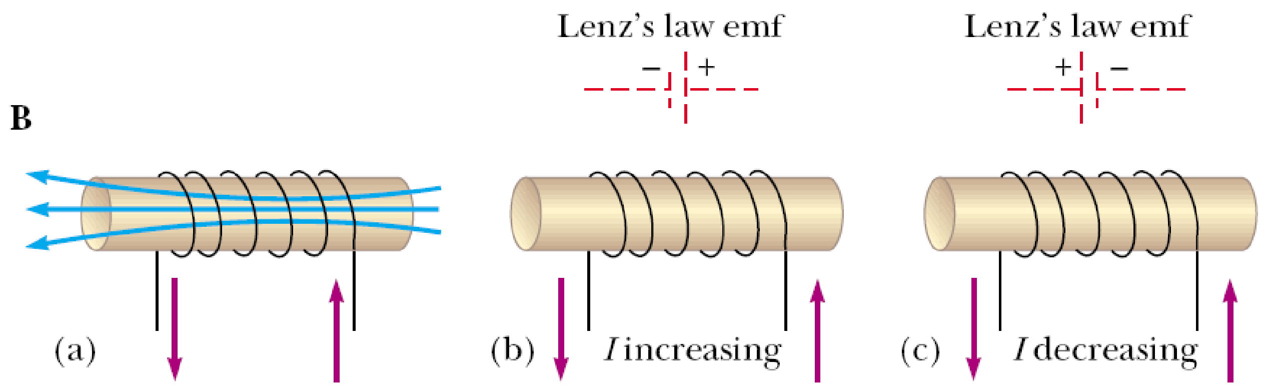
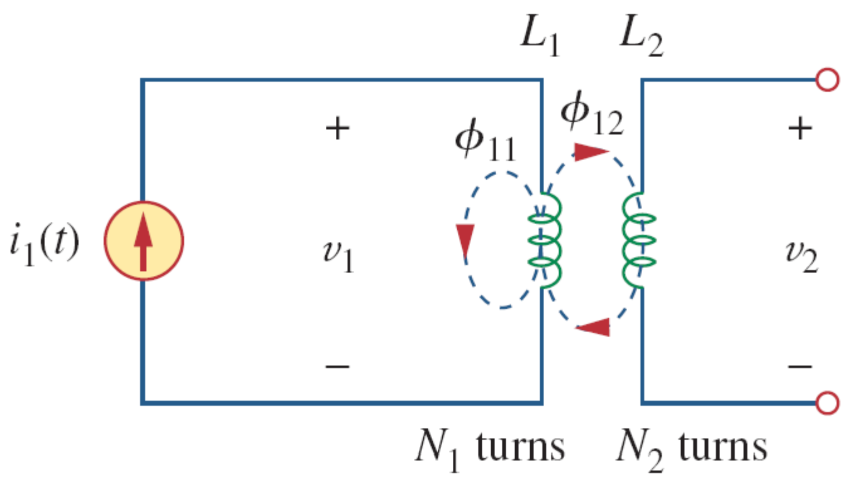
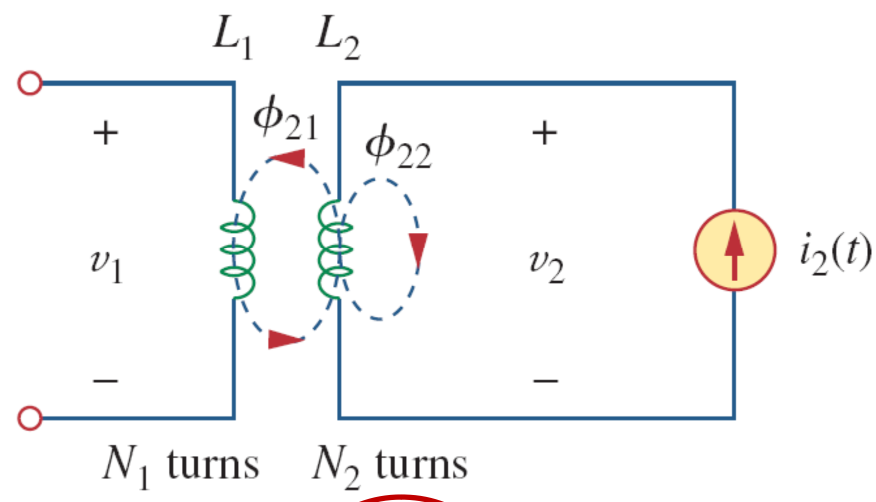


Figure 32.2 (a) A current in the coil produces a magnetic field directed to the left. (b) If the current increases, the increasing magnetic flux creates an induced emf in the coil having the polarity shown by the dashed battery. (c) The polarity of the induced emf reverses if the current decreases.

Inductancia mutua





$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

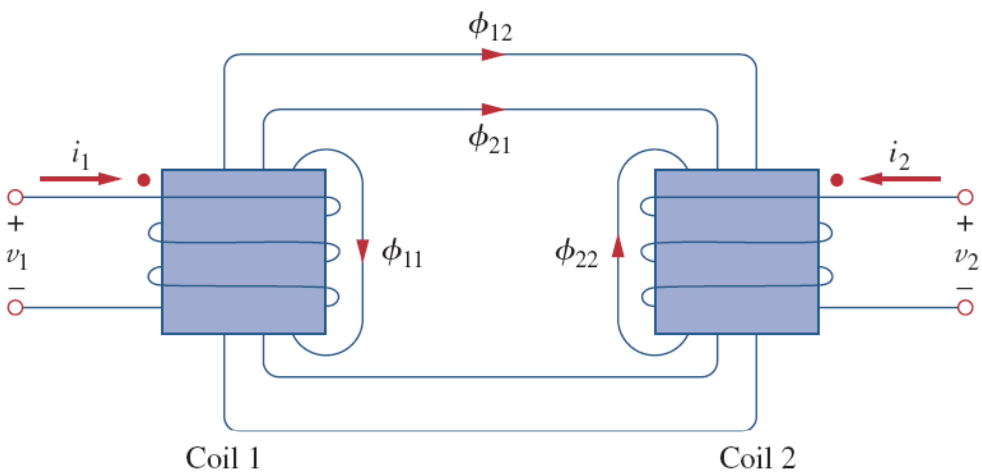
$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

$$v_1 = M_{12} \frac{di_2}{dt}$$

$$v_2 = M_{21} \frac{di_1}{dt}$$

$$M_{12} = M_{21} = M \text{ [H]}$$



$$v_1 = v_{11} + v_{12} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = v_{22} + v_{21} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Con fasores:

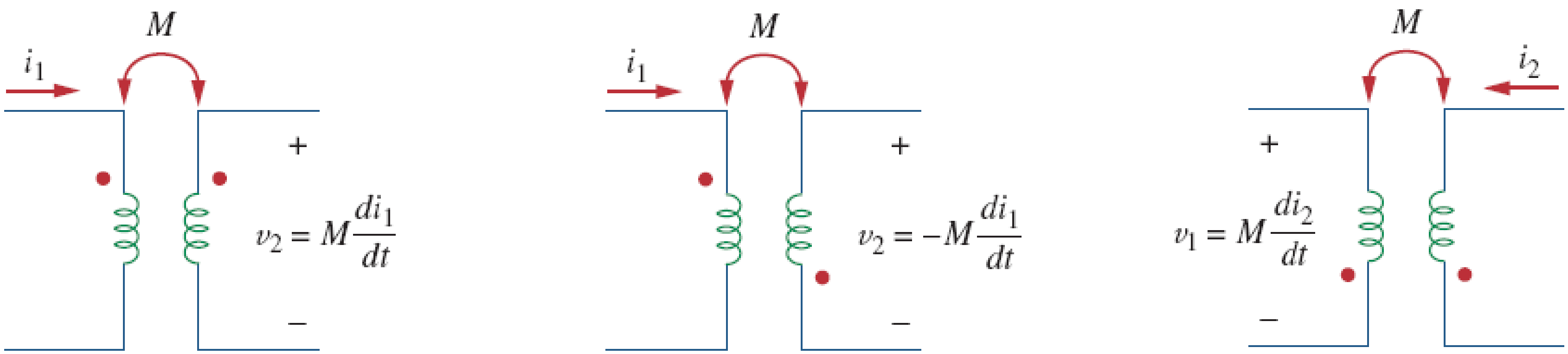
$$V_{L_1} = j\omega L_1 I_{L_1} + j\omega M I_{L_2}$$

$$V_{L_2} = j\omega L_2 I_{L_2} + j\omega M I_{L_1}$$

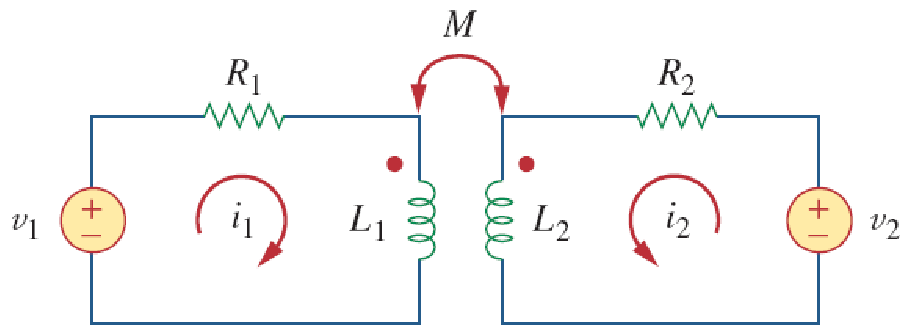
Si el flujo magnético se suma, se suman los voltajes

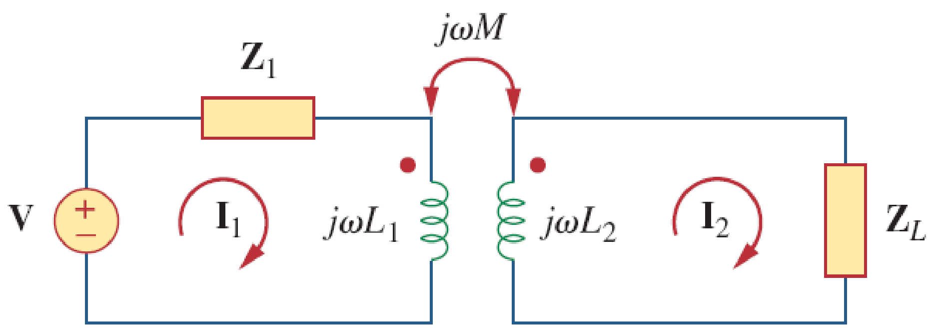
$$v_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

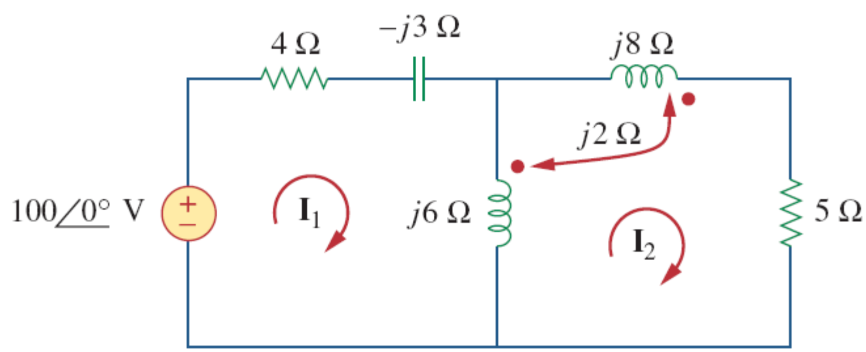
Si no se restan



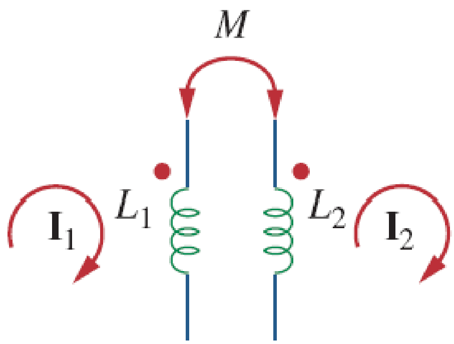
Si se toma como referencia las corrientes que entran por las marcas y el voltaje con el + en las marcas, siempre se suman los efectos

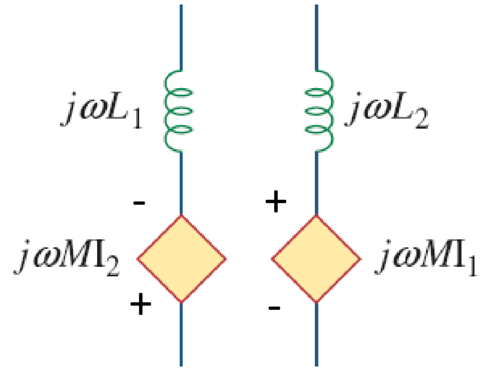

M
R_1
R_2
v_1
i_1
L_1
L_2
i_2
v_2

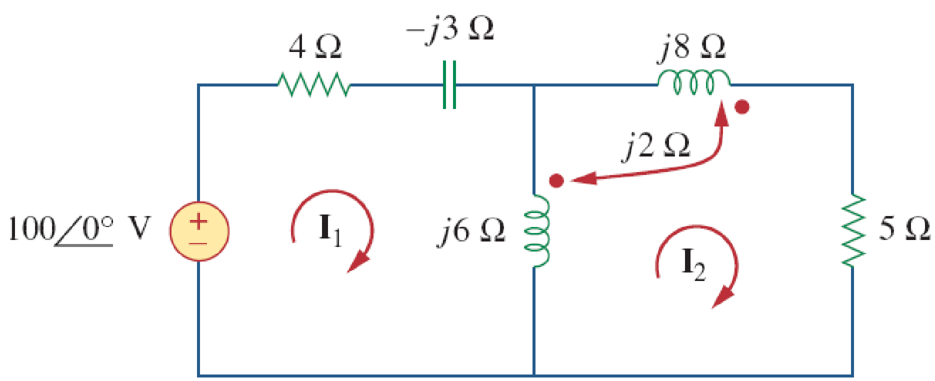

$j\omega M$
$\mathbf{Z}_1$
V
$\mathbf{I}_1$
$j\omega L_1$
$j\omega L_2$
$\mathbf{I}_2$
$\mathbf{Z}_L$

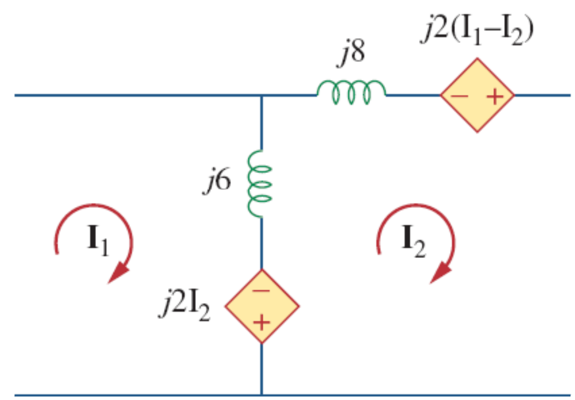

4 Ω
−j3 Ω
j8 Ω
j2 Ω
100∠0° V
$\mathbf{I}_1$
j6 Ω
$\mathbf{I}_2$
5 Ω

Modelado del acoplamiento magnético

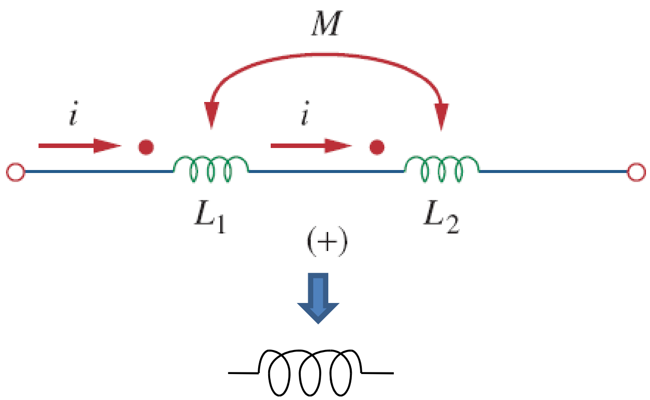






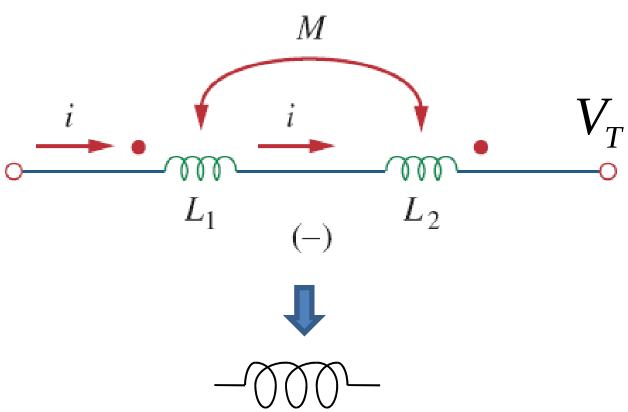


Inductancias en paralelo acopladas magneticamente



$$L_1 + L_2 + 2M$$

$$v_T = L_1 \frac{di}{dt} + M\frac{di}{dt} + L_2 \frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$$

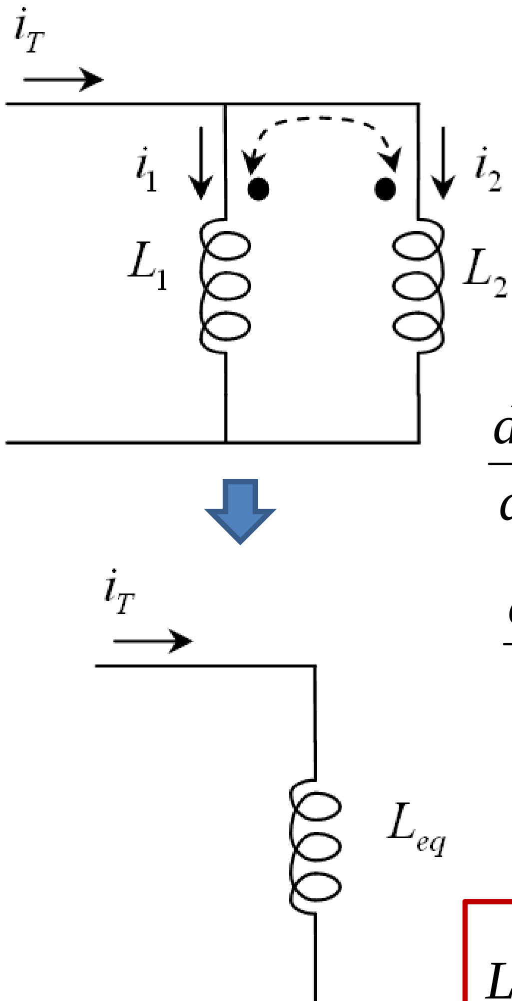


$$L_1 + L_2 - 2M$$

Esta vez con fasores:

$$V_T = j\omega L_1 I - j\omega MI + j\omega L_2 I - j\omega MI = j\omega(L_1 + L_2 - 2M)i$$

Inductancias en serie acopladas magneticamente



$$i_T = i_1 + i_2$$

$$V_T = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_T = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\frac{di_1}{dt} = \frac{1}{\Delta} \begin{vmatrix} V_T & M \\ V_T & L_2 \end{vmatrix} \qquad \Delta = \begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}$$

$$\frac{di_2}{dt} = \frac{1}{\Delta} \begin{vmatrix} L_1 & V_T \\ M & V_T \end{vmatrix}$$

$$\frac{di_1}{dt} = \frac{L_2 - M}{L_1 L_2 - M^2} V_T \qquad \frac{di_2}{dt} = \frac{L_1 - M}{L_1 L_2 - M^2} V_T$$

$$\frac{di_T}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{L_2 - M + L_1 - M}{L_1 L_2 - M^2} V_T$$

$$V_T = \left(\frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M} \right) \frac{di_T}{dt}$$

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M}$$

"-" cuando el acoplamiento es en el mismo nodo

Energía almacenada en un acoplamiento magnético

$$V_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} \qquad V_2 = L_2 \frac{di_1}{dt} \pm M \frac{di_1}{dt}$$

$$\int_0^t v_1 i_1 dt + \int_0^t v_2 i_2 dt = \int_0^t L_1 \frac{di_1}{dt} i_1 dt \pm \int_0^t M \frac{di_2}{dt} i_1 dt + \int_0^t L_2 \frac{di_2}{dt} i_2 dt \pm \int_0^t M \frac{di_1}{dt} i_2 dt$$

$$\int_0^t L_1 i_1 di_1 \pm \int_0^t M i_1 di_2 + \int_0^t L_2 i_2 di_2 \pm \int_0^t M i_2 di_1$$

$$uv = \int u dv + \int v du$$

$$\boxed{E(t) = \frac{1}{2} L_1 i_1(t)^2 + \frac{1}{2} L_2 i_2(t)^2 \pm M i_1(t) i_2(t)}$$

Restricciones en el valor de M

$$E = \frac{1}{2}L_1 i_1{}^2 + \frac{1}{2}L_2 i_2{}^2 \pm M i_1 i_2 \geq 0$$

Completando cuadrados, sumando y restando $i_1 i_2 \sqrt{L_1 L_2}$

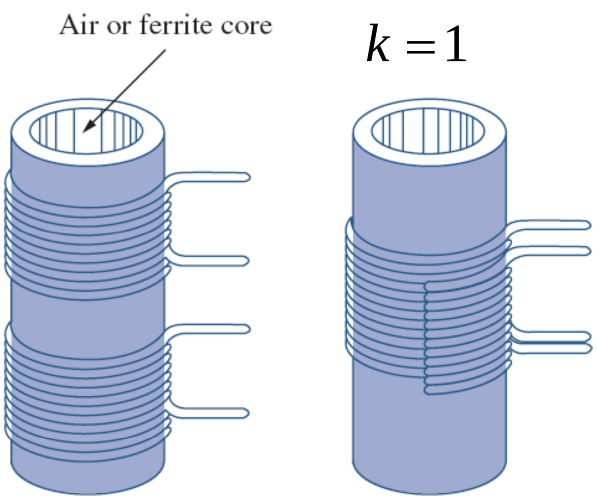
$$\frac{1}{2}L_1 i_1{}^2 - i_1 i_2 \sqrt{L_1 L_2} + \frac{1}{2}L_2 i_2{}^2 - M i_1 i_2 + i_1 i_2 \sqrt{L_1 L_2} \geq 0$$

$$\frac{1}{2}\left(i_1\sqrt{L_1} - i_2\sqrt{L_2}\right)^2 + i_1 i_2\left(\sqrt{L_1 L_2} - M\right) \geq 0$$

$$\sqrt{L_1 L_2} - M \geq 0$$

$$M \leq \sqrt{L_1 L_2}$$

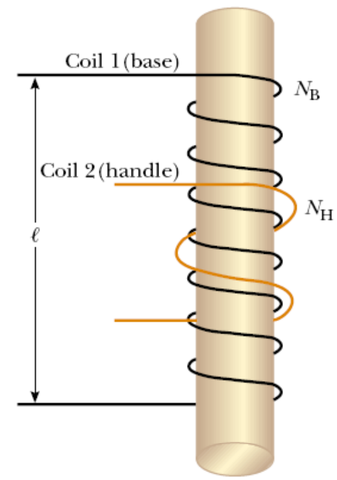
$$M = k\sqrt{L_1 L_2}$$



$k = 1$

Coeficiente de acoplamiento

$0 \leq k \leq 1$



Ejercicio

- Hallar las corrientes I_1 , I_2 e I_3