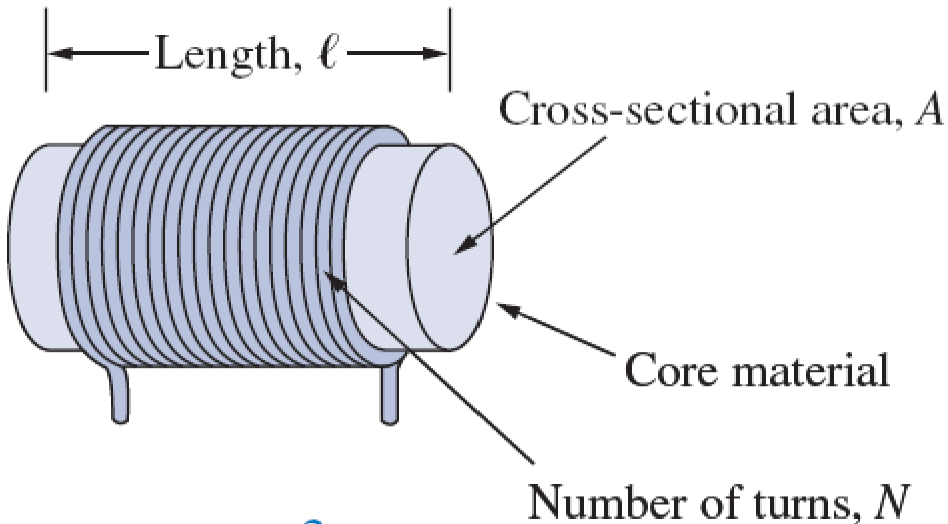
INGENIERIA
UNIVERSIDAD DE LOS ANDES
MÉRIDA VENEZUELA

INGENIERIA ELECTRICA
ULA

Régimen Transitorio Circuito RL de Primer Orden

Prof. Gerardo Ceballos

Inductor



$$L = \frac{N^2 \mu A}{\ell}$$

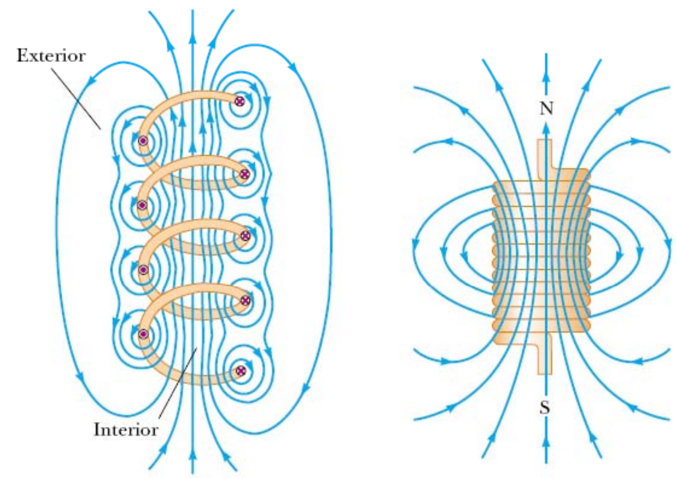
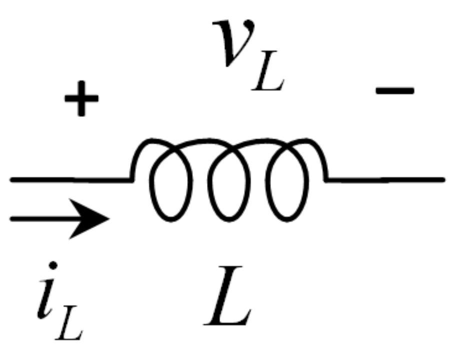


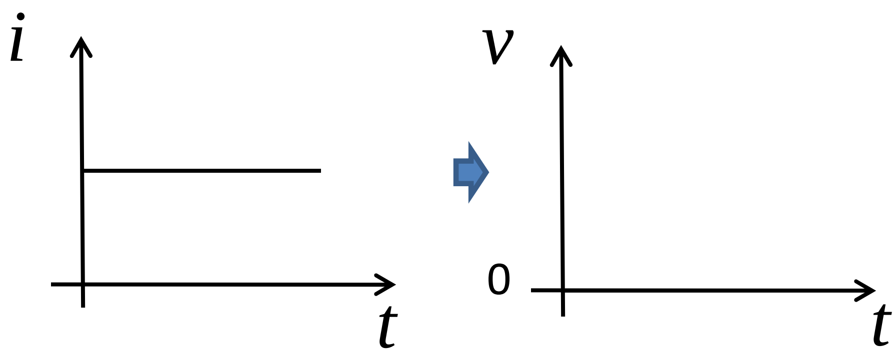
Figure 30.17 The magnetic field lines for a loosely wound solenoid.

Inductor

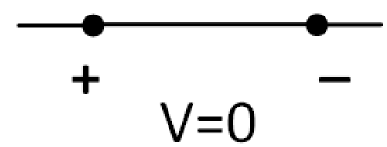


$$v_L = L\frac{di_L}{dt}$$

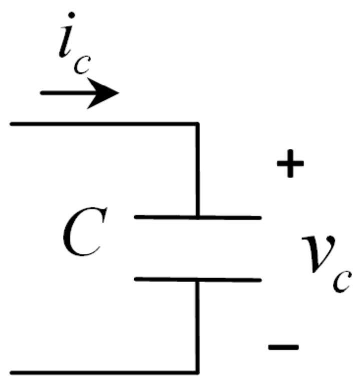
$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau$$

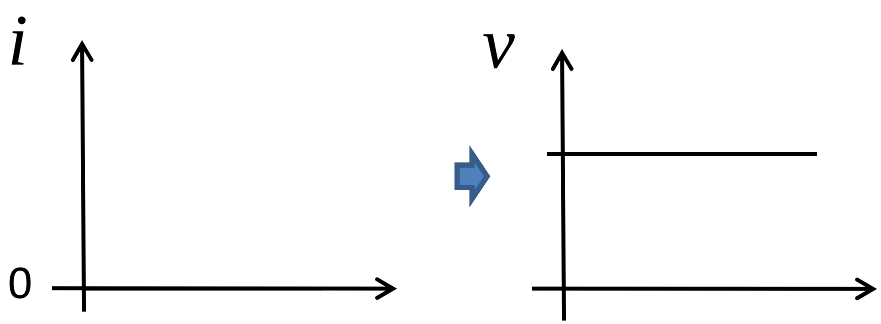


Régimen Permanente



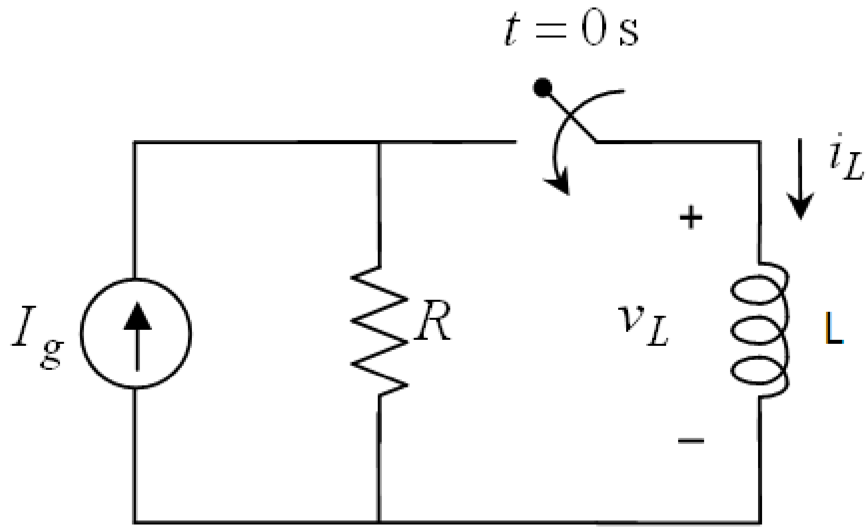
Capacitor vs. Inductor





Régimen Permanente

Análisis del circuito RL de 1er orden



$$I_g = \frac{V_L}{R} + i_L$$

$$I_g = \frac{L}{R}\frac{di_L}{dt} + i_L$$

$$\frac{di_L}{dt} + \frac{R}{L} i_L = \frac{R}{L} I_g$$

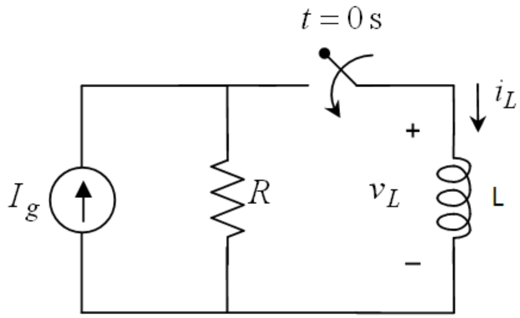
Ec. Diferencial de 1er Orden

$$i_L(t) = i_{Lh}(t) + i_{Lp}(t)$$

Respuesta transitoria (Sol . De la ec. dif. homogénea)

Respuesta forzada o particular (Sol . De la ec. dif. particular)

$$i_L(t)=i_{Lh}(t)+i_{Lp}(t)$$



Sol. Homogénea:

$$\frac{di_L}{dt}+\frac{R}{L}i_L=0 \Rightarrow i_{Lh}(t)=Ae^{mt}$$

$$Ame^{mt}+\frac{R}{L}Ae^{mt}=0 \Rightarrow \begin{matrix} A\neq 0 \\ e^{mt}\neq 0 \\ m=-\frac{1}{L/R} \end{matrix} \Rightarrow i_{Lh}(t)=Ae^{-\frac{t}{L/R}}$$

Sol. Particular: de la misma forma que $\Phi(t)=\frac{R}{L}I_g \Rightarrow i_{Lp}(t)=K$

$$\frac{dK}{dt}+\frac{R}{L}K=\frac{R}{L}I_g \Rightarrow K=I_g \Rightarrow i_{Lp}(t)=I_g$$

Para hallar A se usan las condiciones iniciales:

$$i_L(t)=Ae^{-\frac{t}{L/R}}+I_g \Rightarrow i_L(0^+)=Ae^{-\frac{0}{L/R}}+I_g$$

$$i_L(\infty)=I_g=I_N$$

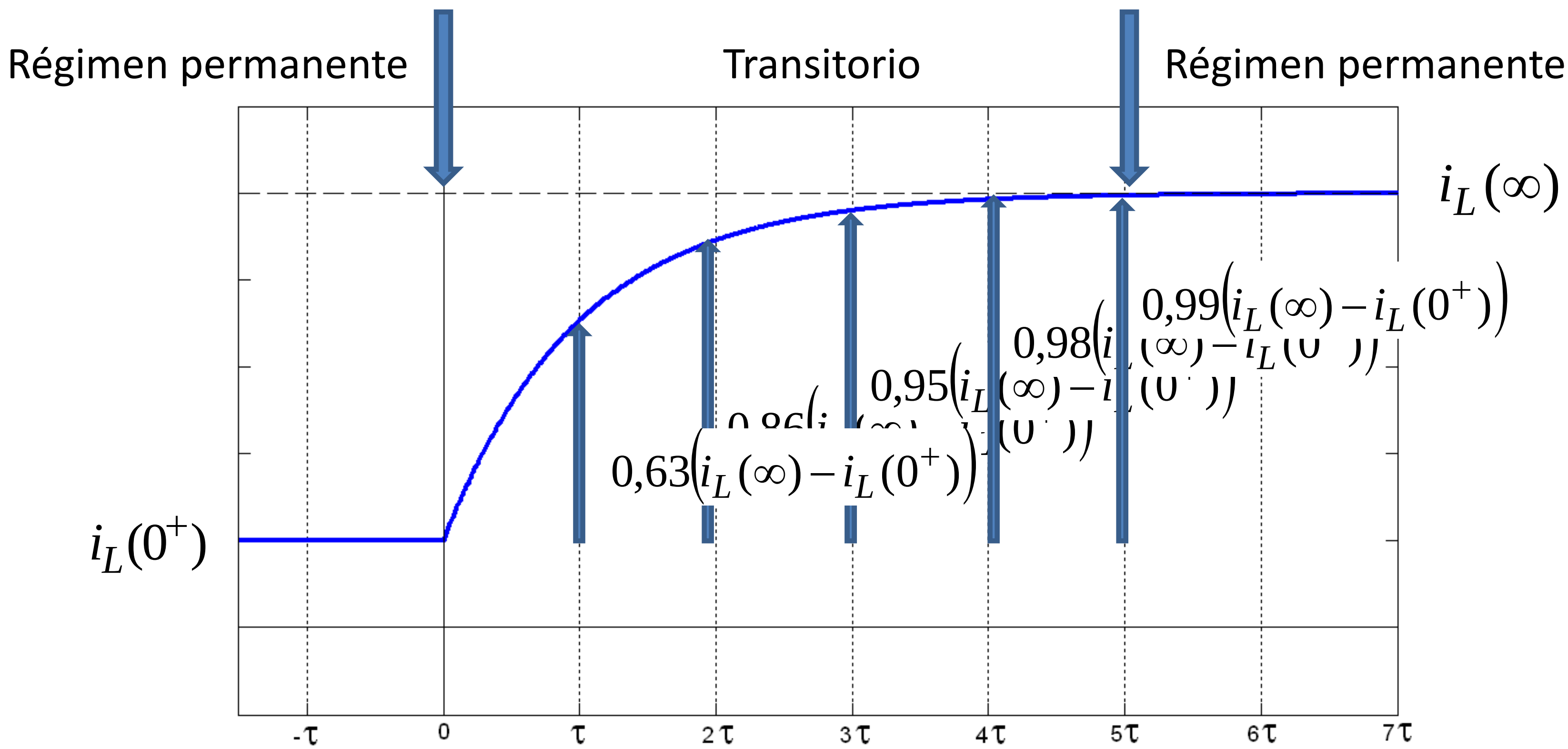
↑ Si R > 0

$$\Rightarrow A=i_L(0^+)-I_g \Rightarrow \boxed{i_L(t)=i_L(\infty)+\left(i_L(0^+)-i_L(\infty)\right)e^{-\frac{t}{L/R}}}$$

Constante de tiempo

$$i_L(t) = i_L(\infty) + \left(i_L(0^+) - i_L(\infty)\right)e^{-\frac{t}{L_{eq}/R_{Th}}}$$

$$\tau = \frac{L_{eq}}{R_{Th}} \qquad t_s = 5\tau$$



Pasos para analizar un circuito RL de primer orden

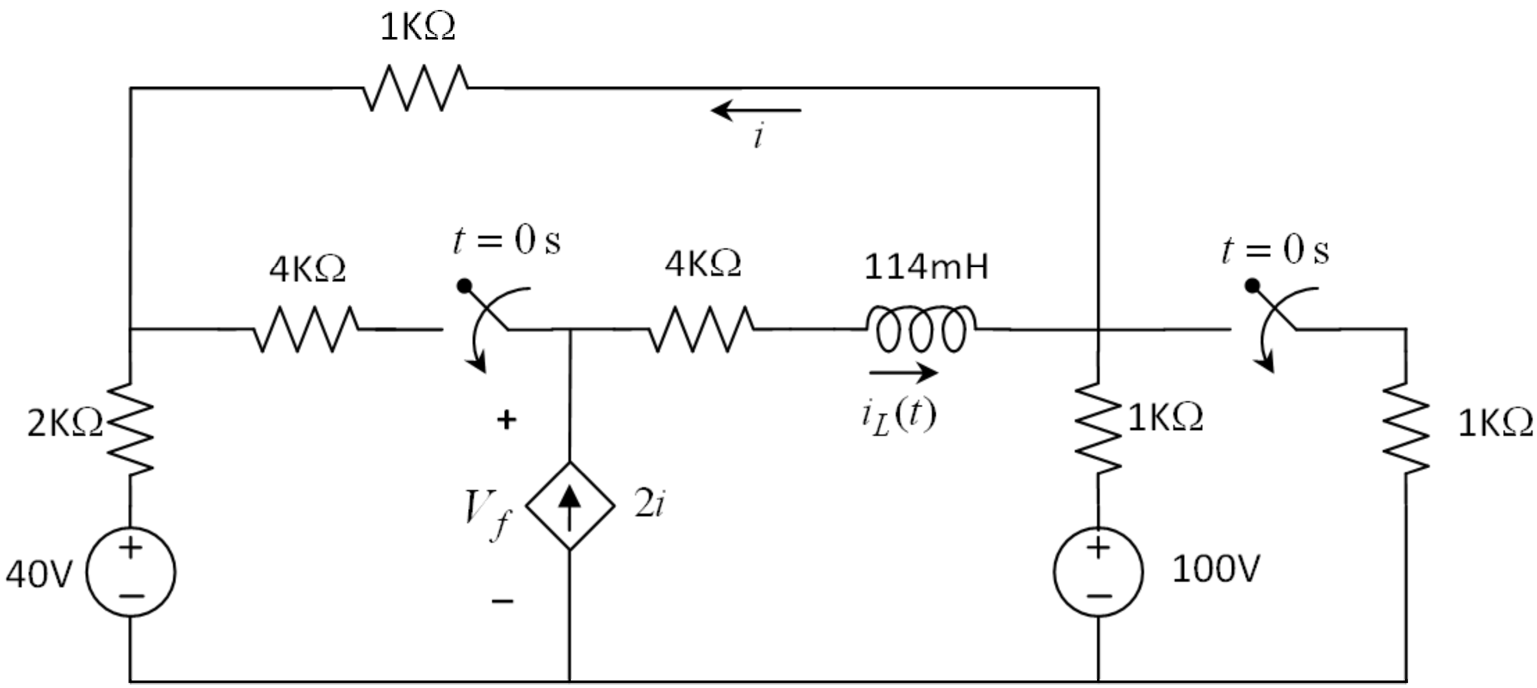
$$i_L(t) = i_L(\infty) + \left(i_L(0^+) - i_L(\infty)\right)e^{-\frac{t}{L_{eq}/R_{Th}}}$$

$$x_c(t) = x_c(\infty) + \left(x_c(0^+) - x_c(\infty)\right)e^{-\frac{t}{L_{eq}/R_{Th}}}$$

- Analizar en $t=0^-$ para hallar $i_L(0^-)$, se puede modelar el inductor como un corto.
- Analizar en $t=0^+$, donde $i_L(0^+)=i_L(0^-)$, se puede modelar al inductor como una fuente de corriente con valor $i_L(0^+)$
- Analizar para $t>0$
  - Equivalente de Norton, $I_N= i_L(\infty)$, $\tau=L_{eq} /R_{th}$

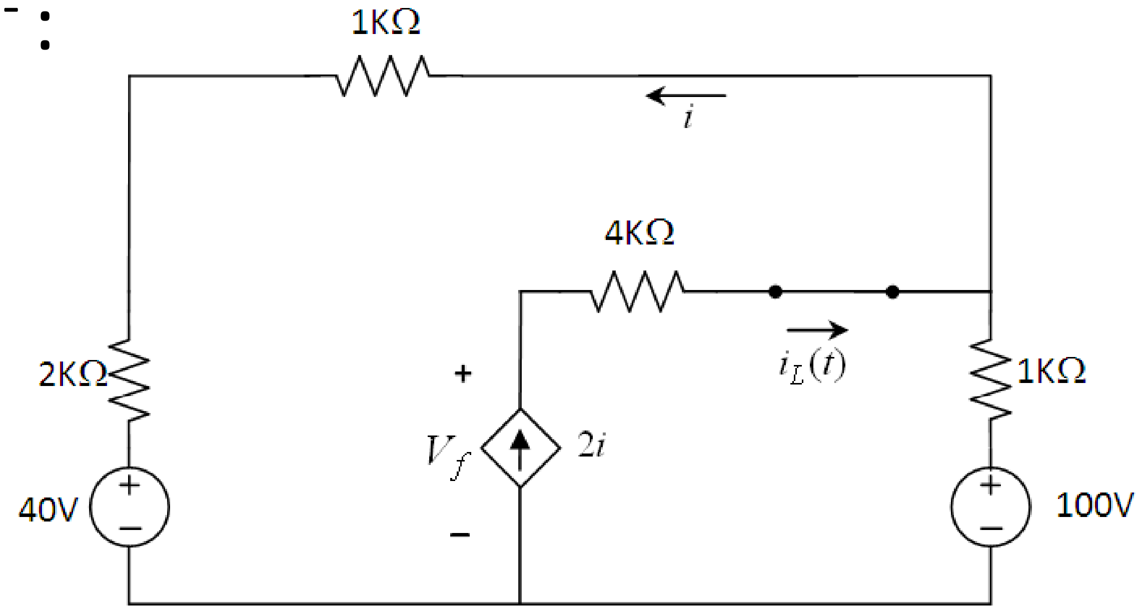
Ejercicio

- Hallar y graficar $i_L(t)$ y $V_f(t)$



Ejercicio

- t=0⁻ :

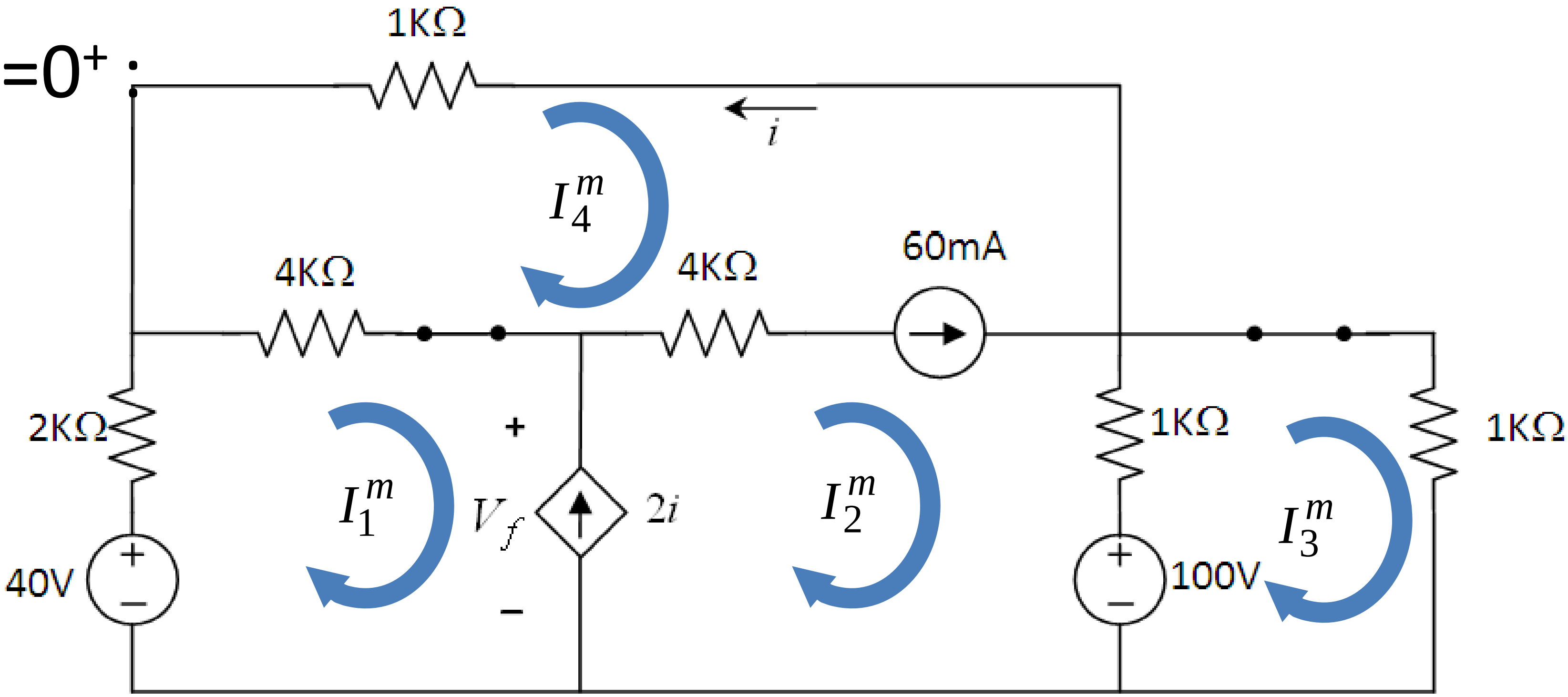


$$+100-40-3ki+1ki=0 \Rightarrow i(0^-)=30mA \Rightarrow i_L(0^-)=60mA$$

$$-V_f+4k(2i)+1ki+100=0 \Rightarrow V_f(0^-)=370V$$

Ejercicio

- $t=0^+$:



$$2kI_1^m + 1kI_2^m - 1kI_3^m + 1kI_4^m = -60$$

$$I_2^m - I_1^m = 2i \Rightarrow -I_1^m + I_2^m + 2I_4^m = 0$$

$$I_2^m - I_4^m = 60mA$$

$$-1kI_2^m + 2kI_3^m = 100$$

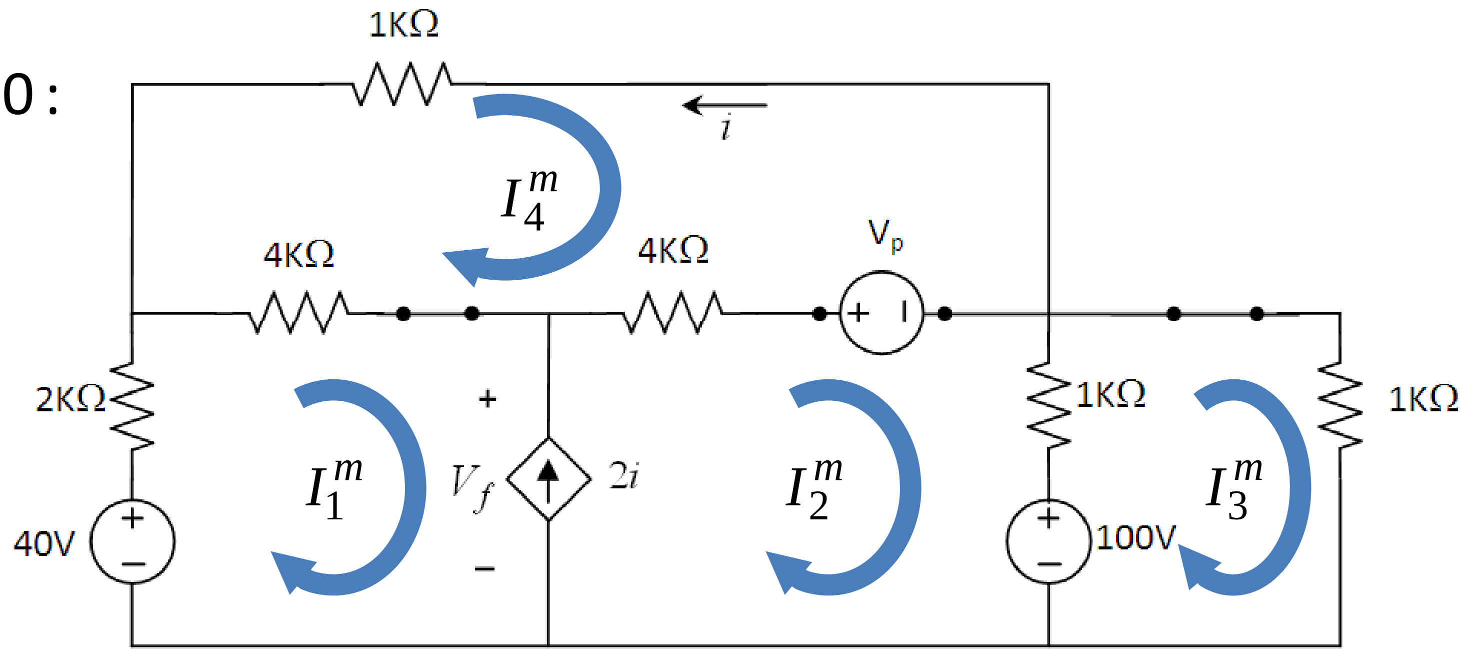
$$V_f(0^+) = 40 - 2kI_1^m - 4k(I_1^m - I_4^m)$$

$$V_f(0^+) = -21{,}3V$$

$$i_L(0^+) = i_L(0^-) = 60mA$$

Equivalente de Norton que ve el inductor:

- t>0 :



$$6kI_1^m + 5kI_2^m - 1kI_3^m - 8kI_4^m = -60 - V_p$$

$$I_2^m - I_1^m = 2i \Rightarrow -I_1^m + I_2^m + 2I_4^m = 0$$

$$-1kI_2^m + 2kI_3^m = 100$$

$$-4kI_1^m - 4kI_2^m + 9kI_4^m = V_p$$

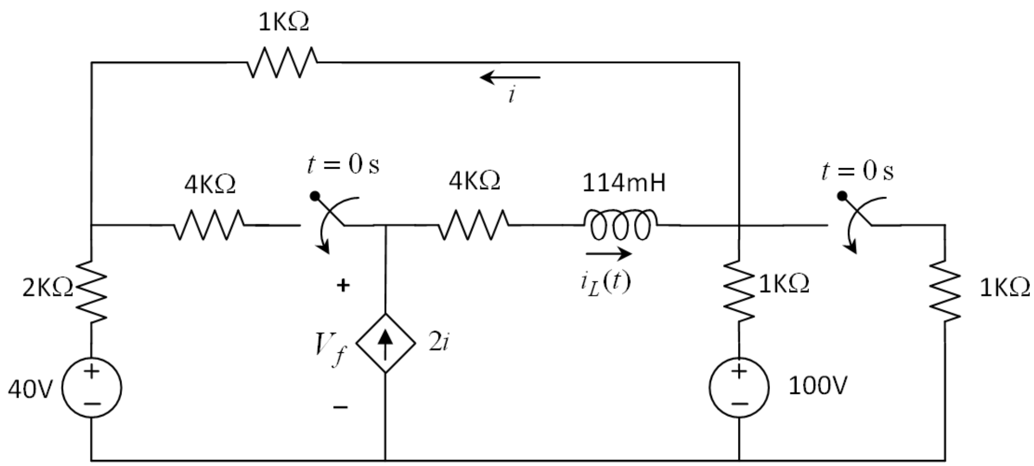
$$R_{Th} = \frac{V_{Th}}{I_N} = 5{,}69k\Omega$$

$$V_f(\infty) = 40 - 2kI_1^m - 4k(I_1^m - I_4^m)\Big|_{V_p=0}$$

$$V_f(\infty) = 56{,}47V$$

$$V_{Th} = V_p\Big|_{I_2^m = I_4^m} = 9{,}33V \qquad I_N = I_2^m - I_4^m\Big|_{V_p=0} = 1{,}64mA$$

Ejercicio



$$i_L(t) = \overset{I_N}{i_L(\infty)} + \left(i_L(0^+) - \overset{I_N}{i_L(\infty)}\right)e^{-\frac{t}{L/R_{Th}}}$$

$$i_L(t) = 1{,}64mA + \underbrace{(60mA - 1{,}64mA)}_{58{,}3mA}e^{-\frac{t}{20\mu}}$$

$$V_f(t) = V_f(\infty) + \left(V_f(0^+) - V_f(\infty)\right)e^{-\frac{t}{L/R_{Th}}}$$

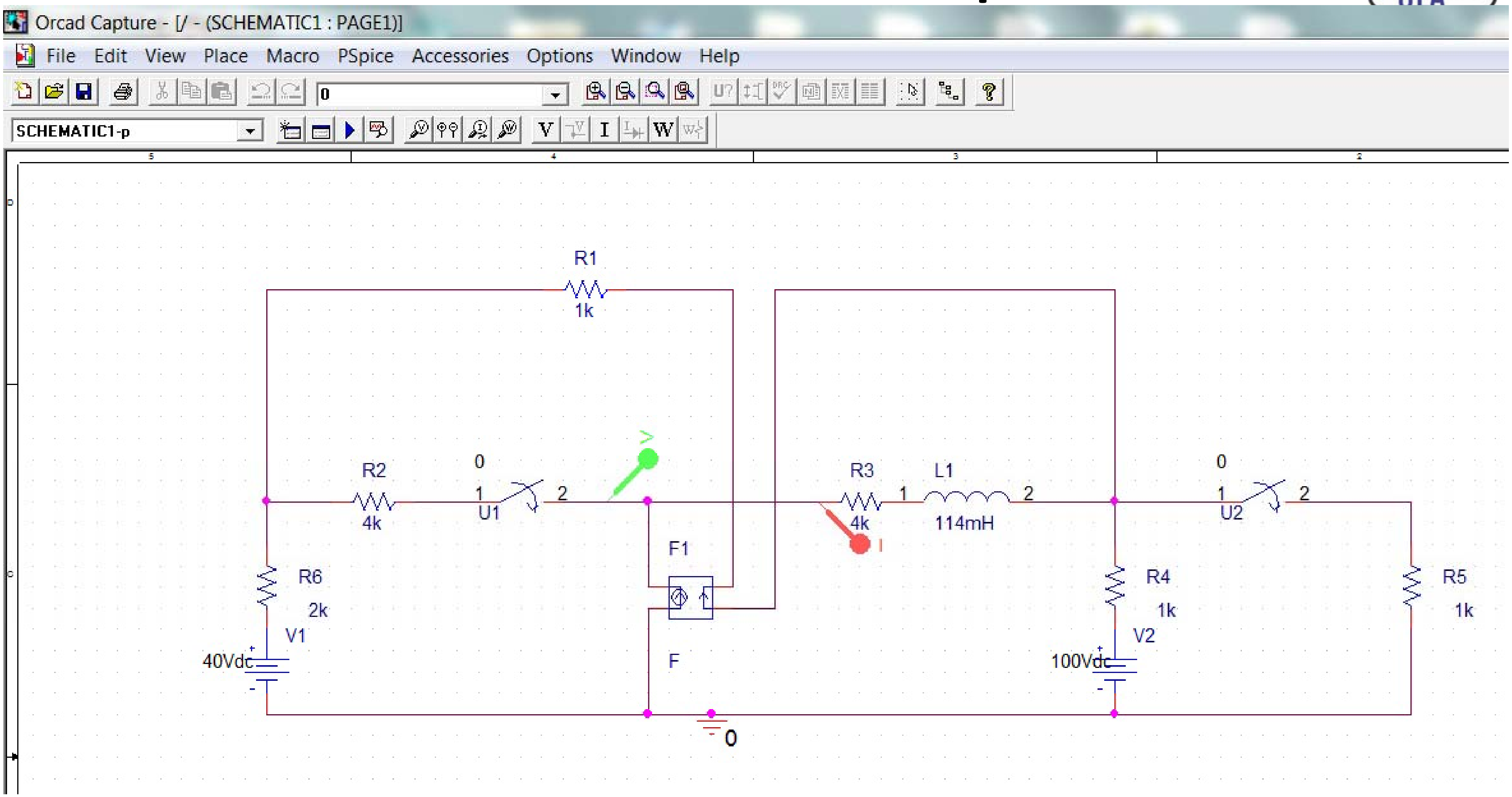
$$V_f(t) = 56{,}47 + \underbrace{(-21{,}3 - 56{,}47)}_{-77{,}7V}e^{-\frac{t}{25\mu}}$$

$$V_f(0^-) = 370V$$

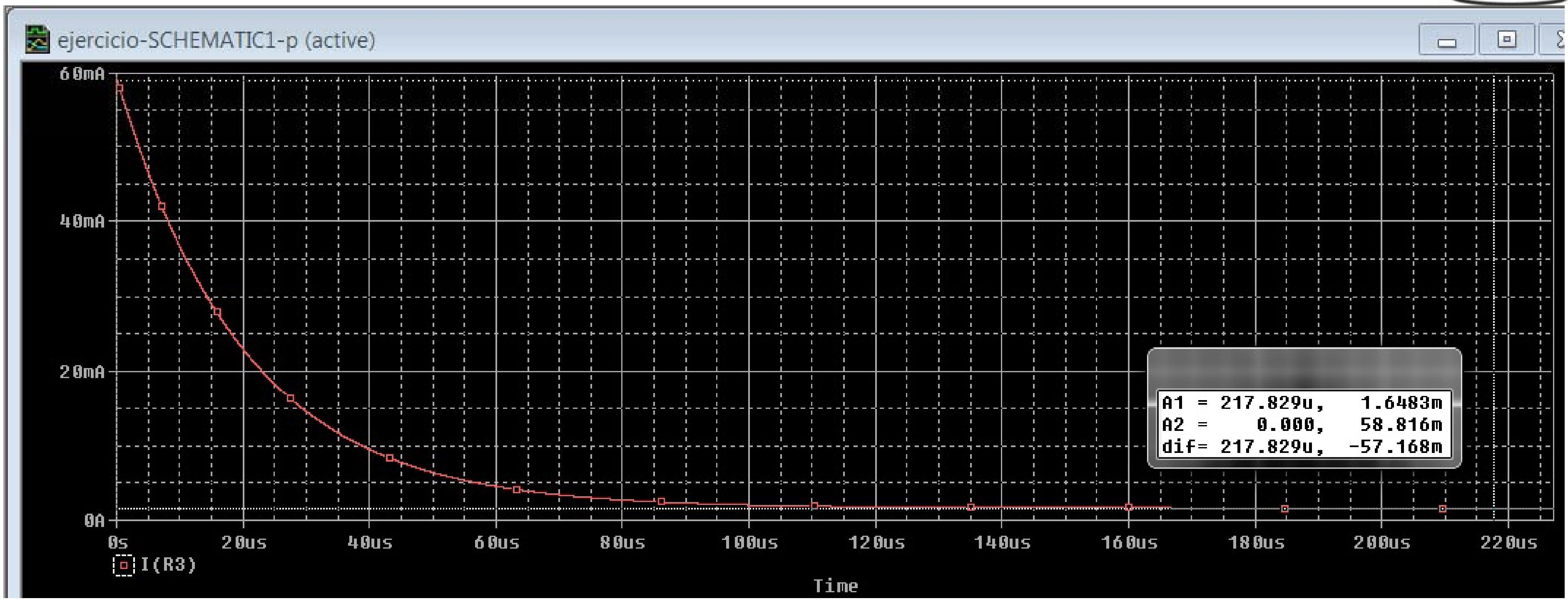
$$\tau = \frac{L}{R_{Th}} = \frac{114mH}{5{,}69k\Omega} = 20\mu s$$

$$t_s = 5\tau = 5.20\mu s = 100\mu s$$

Simulado en Pspice



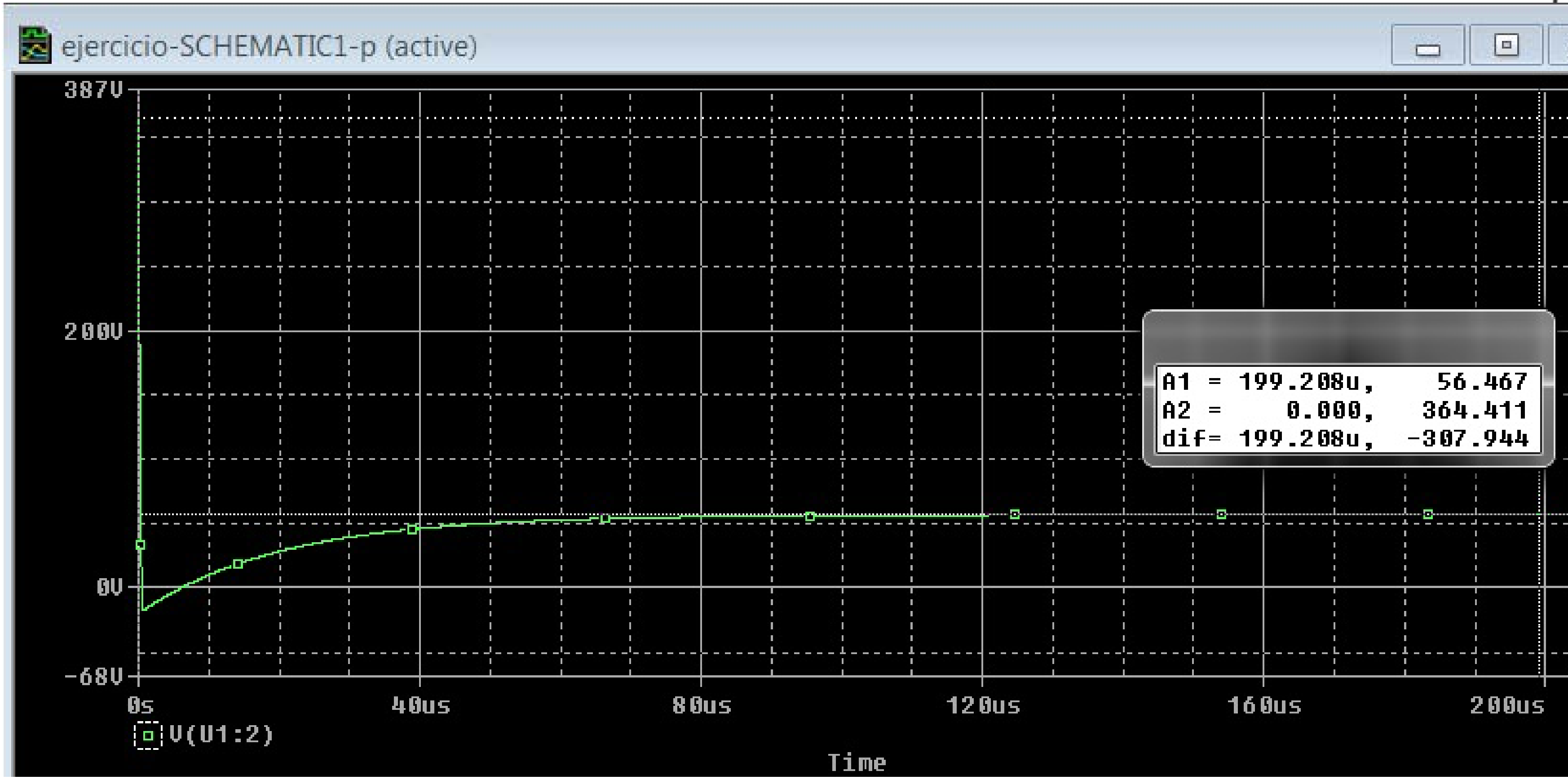
Ejercicio



$$i_L(t) = i_L(\infty) + \left(i_L(0^+) - i_L(\infty)\right) e^{-\frac{t}{L/R_{Th}}} \qquad \tau = 20\mu s$$

$$i_L(t) = 1{,}64mA + \underbrace{\left(60mA - 1{,}64mA\right)}_{58{,}3mA} e^{-\frac{t}{20\mu}} \qquad t_s = 100\mu s$$

Ejercicio



$$V_f(t) = V_f(\infty) + \left(V_f(0^+) - V_f(\infty)\right)e^{-\frac{t}{L/R_{Th}}}$$

$$V_f(t) = 56{,}47 + \left(-\underbrace{21{,}3 - 56{,}47}_{-77{,}7V}\right)e^{-\frac{t}{25\mu}}$$

$$V_f(0^-) = 370V$$

$$t_s = 100\mu s$$

$$\tau = 20\mu s$$

- Ver en la página web ejercicio de resistencia negativa con inductor
- Ver en la página web ejercicio de inductores en paralelo con condiciones iniciales