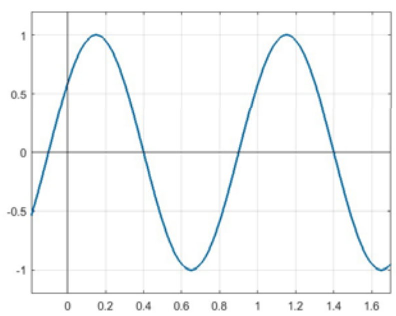
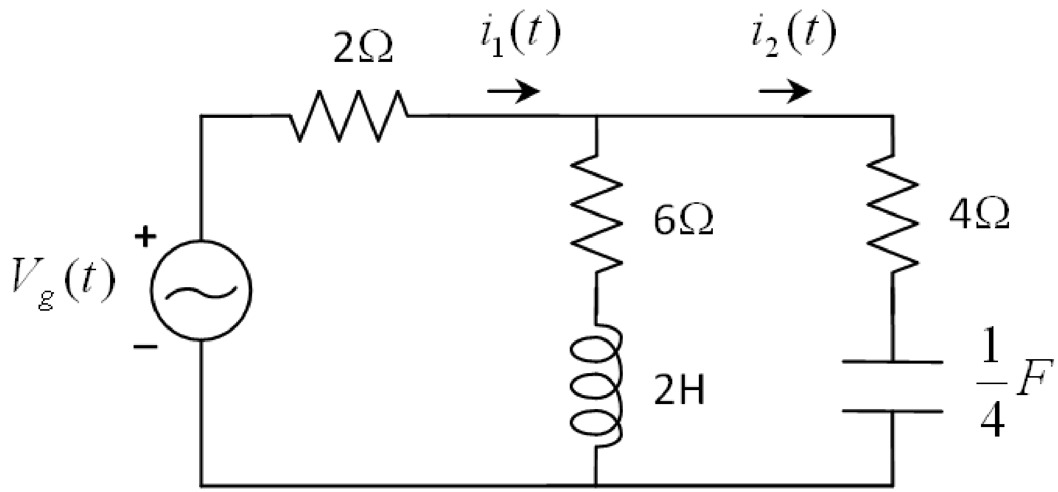
Régimen permanente ante señales sinusoidales

Prof. Gerardo Ceballos

Circuito alimentado por fuente AC

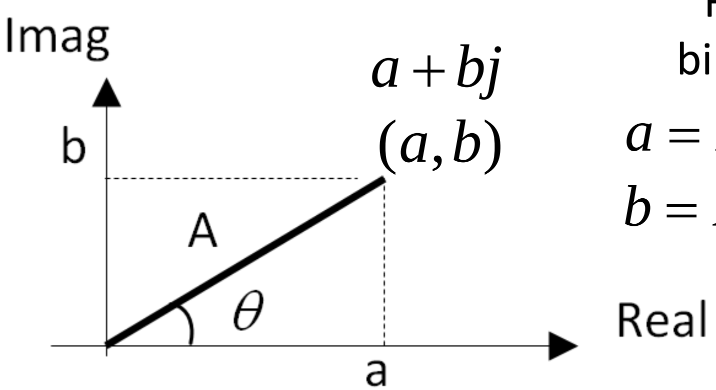




Todas las variables son sinusoidales

La frecuencia se mantiene igual en todas las variables del circuito

Repaso de números complejos



Forma binómica

$$a = A\cos(\theta)$$
$$b = A sen(\theta)$$

Forma polar

$$A\angle\theta$$
$$A = \sqrt{a^2+b^2}$$
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Forma exponencial

$$Ae^{j\theta} = A\cos(\theta) + A sen(\theta)\, j$$

Operaciones:

Suma y resta: recomendable hacerla en forma binómica

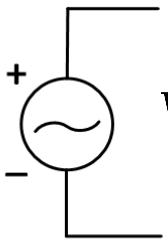
$$(a_1 + b_1 j) \pm (a_2 + b_2 j) \quad = (a_1 \pm a_2) + (b_1 \pm b_2)\, j$$

Multiplicación y división: recomendable hacerla en forma polar

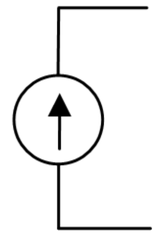
$$A_1\angle\theta_1 \;.\; A_2\angle\theta_2 \quad = A_1 A_2 \angle\theta_1+\theta_2$$

$$\frac{A_1\angle\theta_1}{A_2\angle\theta_2} \quad = \frac{A_1}{A_2}\angle\theta_1-\theta_2$$

Análisis de circuitos de corriente alterna



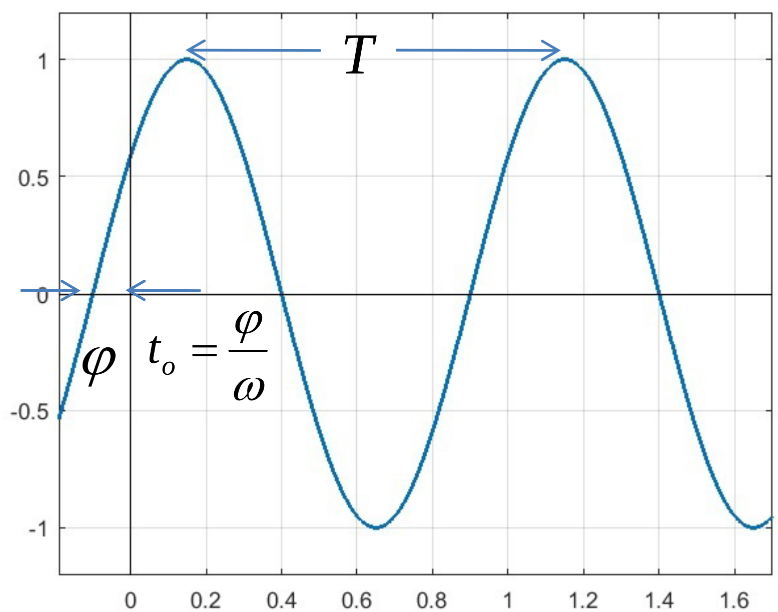
$$v(t) = V_M Sen(\omega t + \varphi)$$



$$i(t) = I_M Sen(\omega t + \varphi)$$

$$Sen(\omega t) = Cos\left(\omega t - \frac{\pi}{2}\right)$$

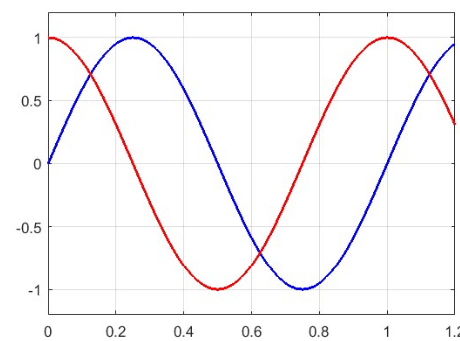
$$Cos(\omega t) = Sen\left(\omega t + \frac{\pi}{2}\right)$$



$$f = \frac{1}{T}$$

$$\omega = 2\pi f$$

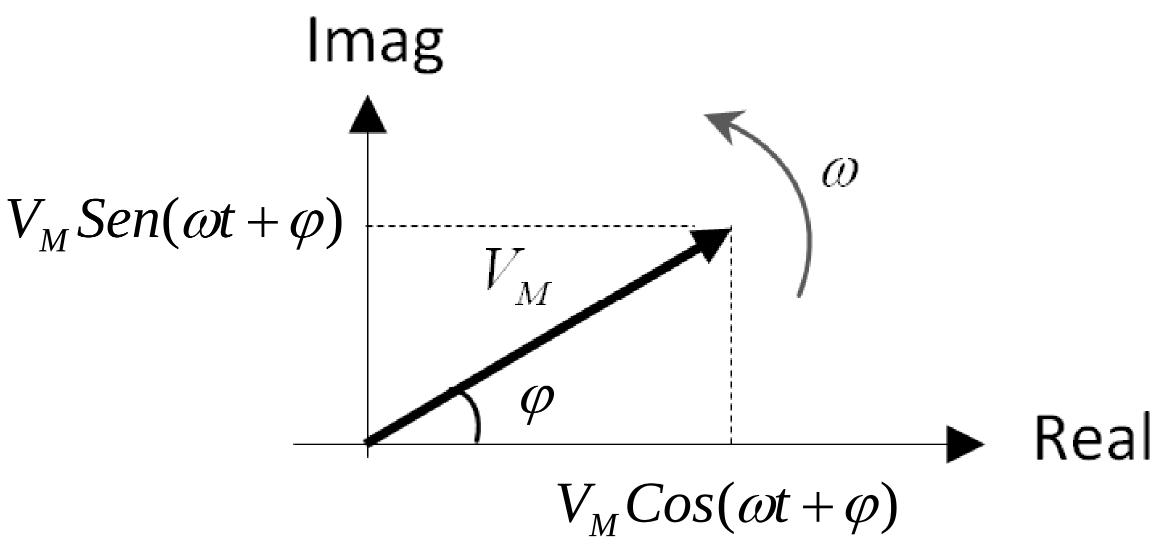
$Cos(\omega t)$ $Sen(\omega t)$

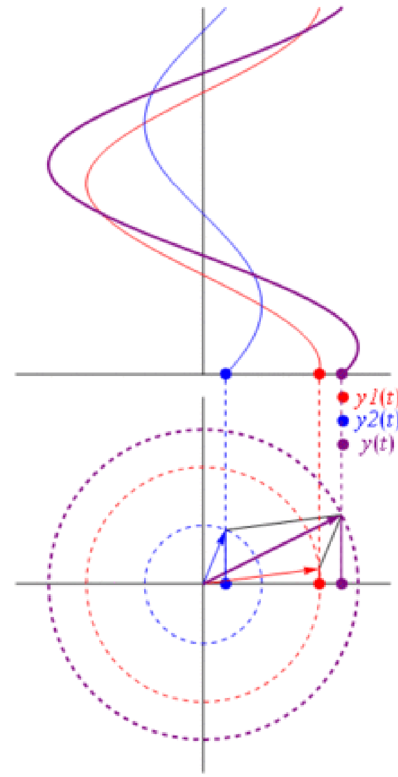


Prof. Gerardo Ceballos

Fasor

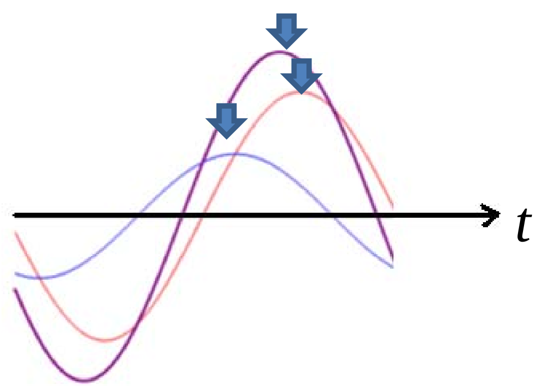
$$v(t) = V_M Sen(\omega t + \varphi)$$



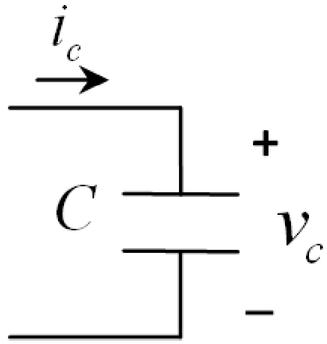


$$\hat{V} = V_M Cos(\omega t + \varphi) + V_M Sen(\omega t + \varphi)\, j$$

$$\hat{V} = V_M e^{j(\omega t + \varphi)} = V_M \underline{\lfloor \varphi}$$



Fasores en un capacitor e Impedancia



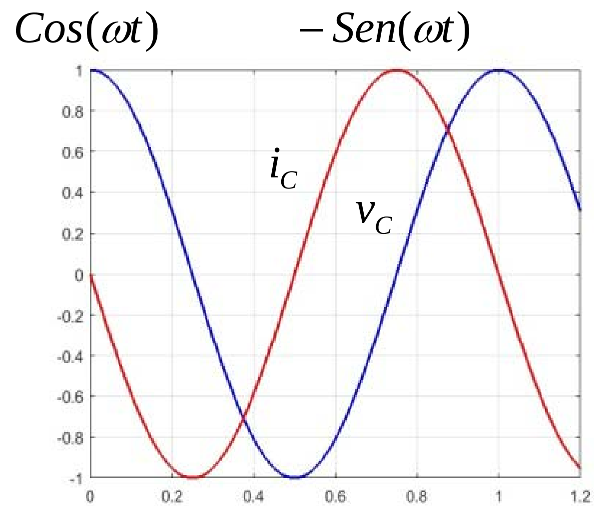
$$i_c = C\frac{dv_c}{dt}$$

Análisis en tiempo:

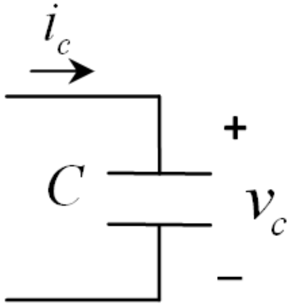
$$v_c(t) = V_M Cos(\omega t)$$

$$i_c(t) = C\frac{dv_c}{dt} = CV_M\,\omega\left[-Sen(\omega t)\right]$$

$$i_c(t) = CV_M\,\omega Cos\left(\omega t + \frac{\pi}{2}\right)$$



Fasores en un capacitor e Impedancia

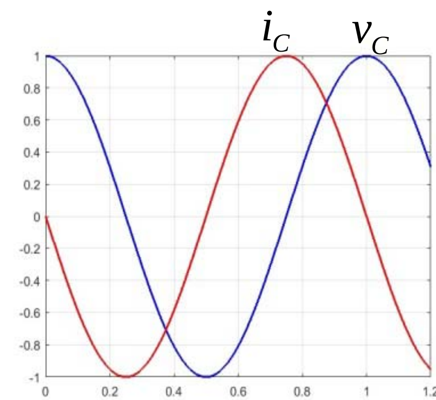


$$i_c = C\frac{dv_c}{dt} \qquad v_c(t) = V_M Cos(\omega t)$$

Análisis con fasores:

$$V_C = V_M Cos(\omega t) + V_M Sen(\omega t)\, j = V_M e^{j\omega t} = V_M \angle 0^{\circ}$$

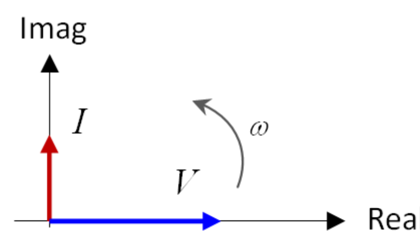
$$I_C = \omega C V_M\, j e^{j\omega t} = \omega C V_M e^{j\frac{\pi}{2}} e^{j\omega t} = \omega C V_M e^{j\left(\omega t + \frac{\pi}{2}\right)} = \omega C V_M \angle 90^{\circ}$$



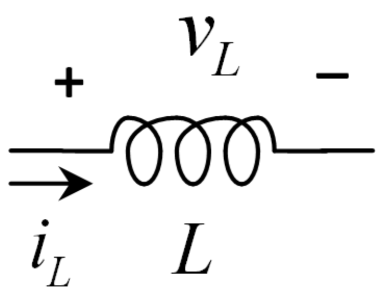
$$Z = \frac{V_C}{I_C} = \frac{V_M e^{j\omega t}}{\omega C j V_M e^{j\omega t}} = -\frac{1}{\omega C}\, j$$

$$Z = \frac{V_M \angle 0^{\circ}}{\omega C V_M \angle 90^{\circ}} = \frac{1}{\omega C}\angle -90^{\circ}$$

$$|V_C| = \left|\frac{1}{\omega C}\right| |I_C|$$



Fasores en un inductor e Impedancia



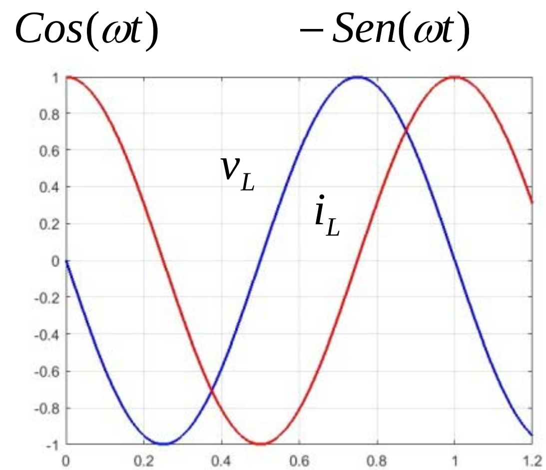
$$v_L = L\frac{di_L}{dt}$$

Análisis en tiempo:

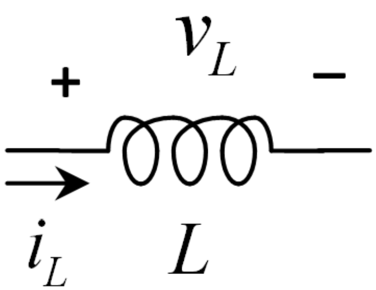
$$i_L(t) = I_M Cos(\omega t)$$

$$v_L(t) = L\frac{di_L}{dt} = LI_M\omega\left[-Sen(\omega t)\right]$$

$$v_L(t) = LI_M\omega Cos\left(\omega t + \frac{\pi}{2}\right)$$



Fasores en un capacitor e Impedancia



$$v_L = L\frac{di_L}{dt} \qquad i_L(t) = I_M Cos(\omega t)$$

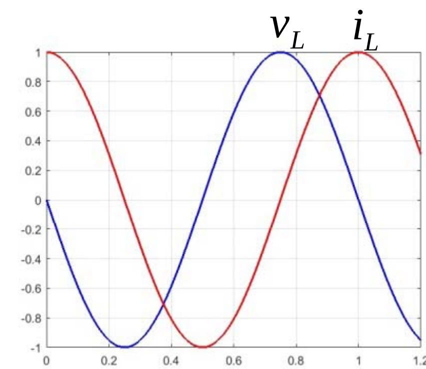
Análisis con fasores:

$$I_L = I_M Cos(\omega t) + I_M Sen(\omega t)\, j = I_M e^{j\omega t} = I_M \angle 0^o$$

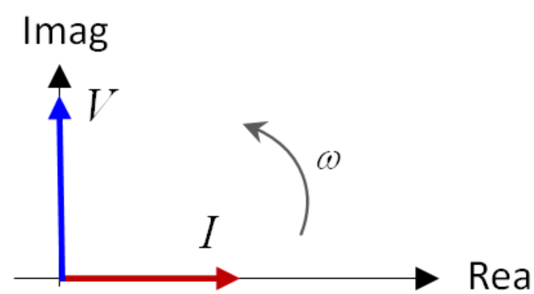
$$V_L = \omega L I_M\, j e^{j\omega t} = \omega L I_M e^{j\frac{\pi}{2}} e^{j\omega t} = \omega L I_M e^{j\left(\omega t + \frac{\pi}{2}\right)} = \omega L I_M \angle 90^o$$

$$Z = \frac{V_L}{I_L} = \frac{\omega L j I_M e^{j\omega t}}{I_M e^{j\omega t}} = \omega L j$$

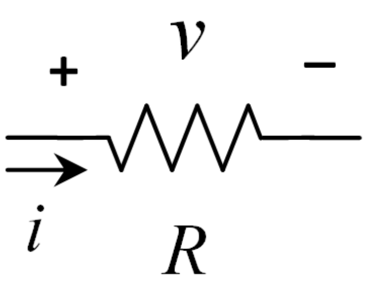
$$Z = \frac{\omega L I_M \angle 90^o}{I_M \angle 0^o} = \omega L \angle 90^o$$



$$|V_L| = |\omega L||I_L|$$



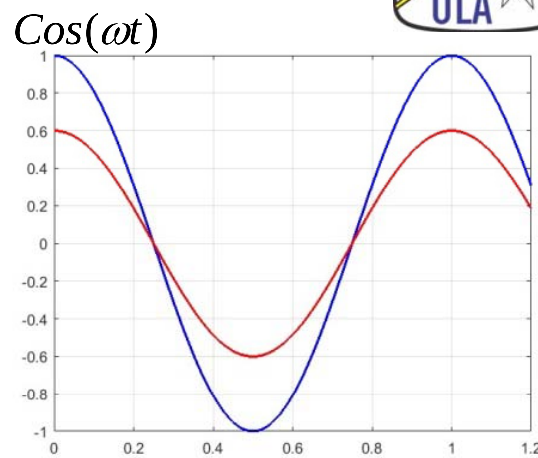
Fasores en un resistor e Impedancia



$$v = R.i$$

$$i_R(t) = I_M Cos(\omega t)$$

$$v_R(t) = RI_M Cos(\omega t)$$

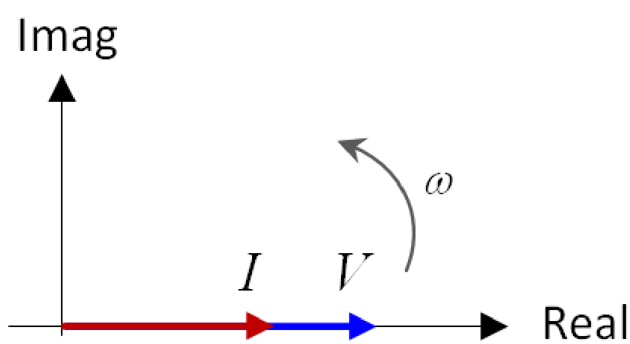


$$I_R = I_M Cos(\omega t) + I_M Sen(\omega t)\, j = I_M e^{j\omega t}$$

$$V_R = RI_M Cos(\omega t) + RI_M Sen(\omega t)\, j = RI_M e^{j\omega t}$$

$$Z = \frac{V_R}{I_R} = \frac{RI_M e^{j\omega t}}{I_M e^{j\omega t}} = R$$

$$|V_R| = R|I_R|$$



Impedancia

<u>Impedancia:</u> Relación voltaje a corriente con fasores

$$Z = R + Xj \quad \Omega$$

Impedancia → Z; Resistencia → R; Reactancia → X

<u>Admitancia:</u> Inverso de la impedancia

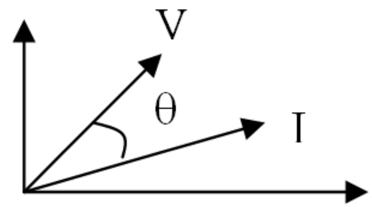
$$Y = G + Bj \quad \mho$$

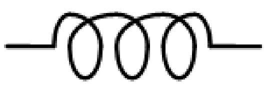
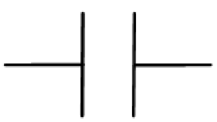
Admitancia → Y; Conductancia → G; Susceptancia → B

$$Y = \frac{1}{Z} = \frac{1}{R + Xj} = \frac{R - Xj}{R^2 + X^2} = \frac{R}{R^2 + X^2} - \frac{X}{R^2 + X^2} j \neq \cancel{\frac{1}{R} + \frac{1}{X}} j$$

Impedancia

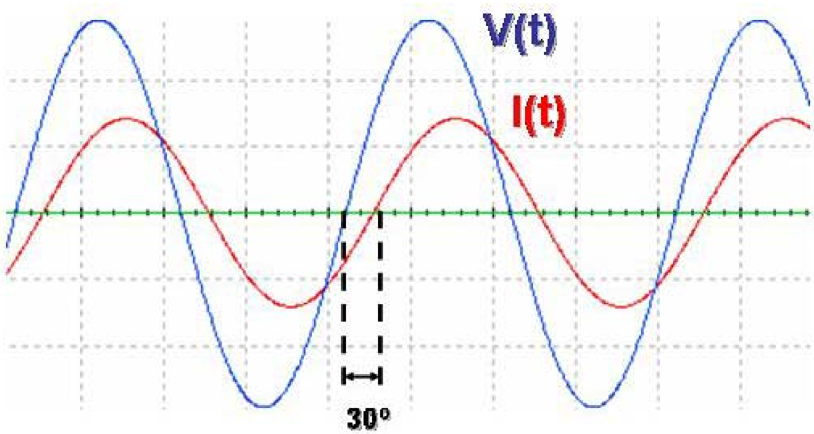
$$Z = |Z|\angle\theta$$

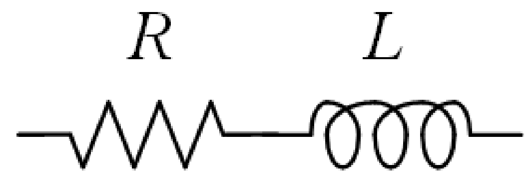


R 	$Z = R$	$Y = \frac{1}{R}$	$X = 0$	$B = 0$	$\theta = 0^{o}$
L 	$Z = \omega L j$	$Y = -j\frac{1}{\omega L}$	$X = \omega L$	$B = -\frac{1}{\omega L}$	$\theta = 90^{o}$
C 	$Z = -\frac{1}{\omega C}j$	$Y = \omega C j$	$X = -\frac{1}{\omega C}$	$B = \omega C$	$\theta = -90^{o}$

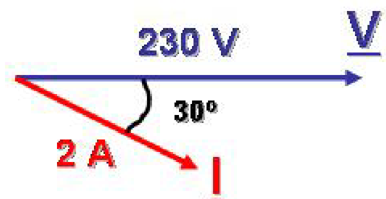
El ángulo de la impedancia es el ángulo con el que el voltaje adelanta a la corriente

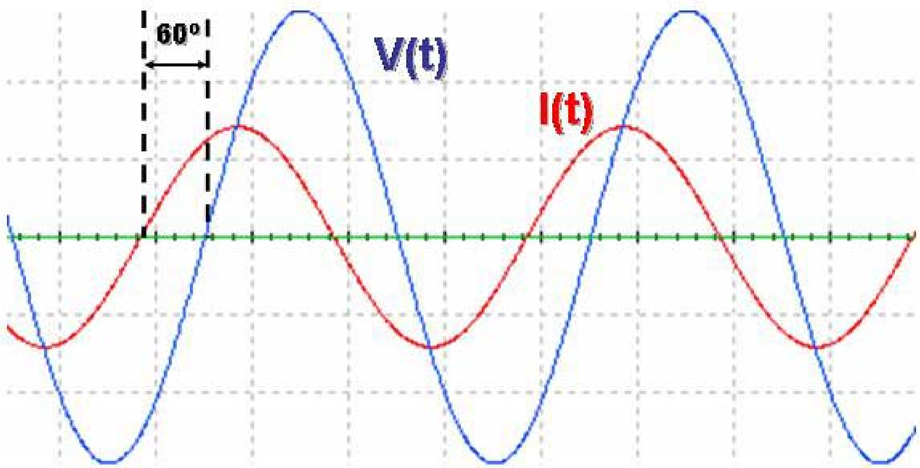
Y la magnitud relaciona las amplitudes del voltaje y la corriente



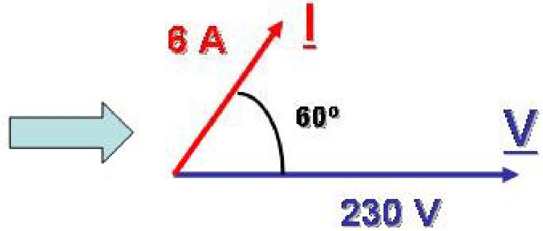


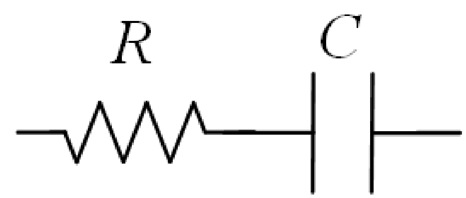
Representación fasorial equivalente

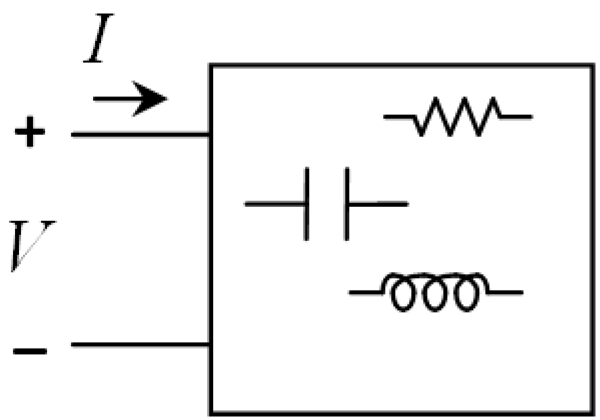


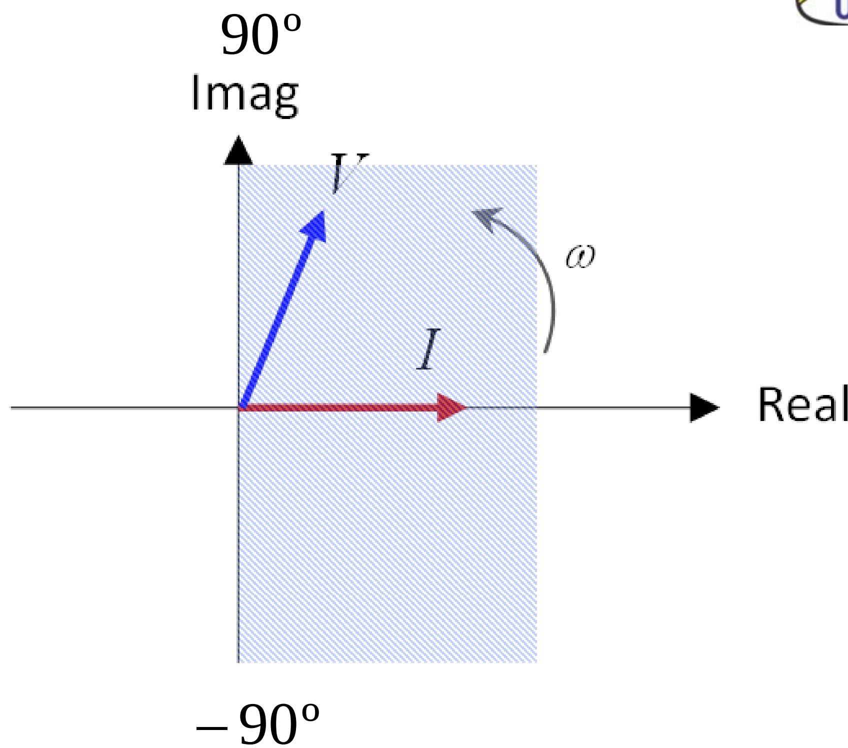


Representación fasorial equivalente






I
+
V
–

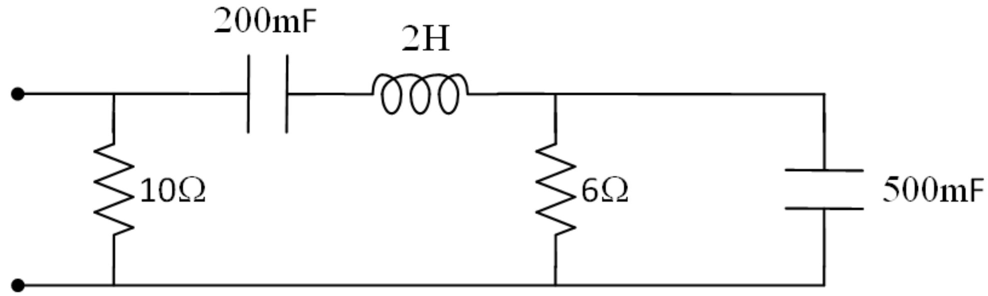

90º
Imag
V
ω
I
Real
– 90º

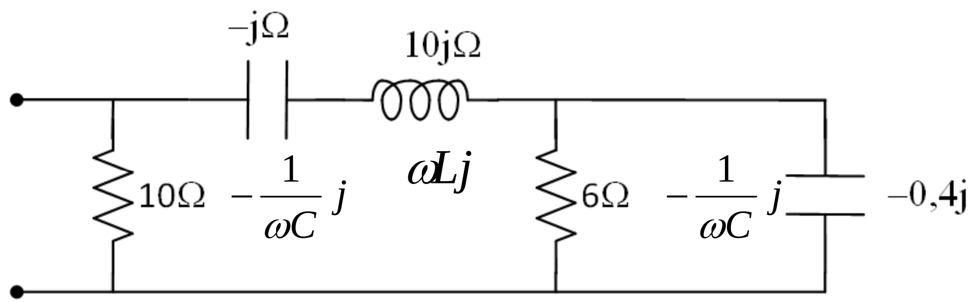
- Conversión de fuentes
- Transformación triangulo-estrella
- Teorema de Thevenin y Norton
- Superposición
- Método de Mallas y de Nodos
- Superposición, etc
- **Máxima Transferencia de Potencia** (no es igual)

Ejemplos

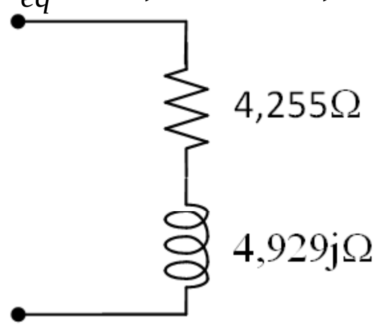
Hallar el circuito equivalente formado por dos elementos en serie y por dos elementos en paralelo

$$\omega = 5\frac{rad}{seg}$$





$$Z_{eq} = 4{,}255 + 4{,}929\,j\ \ \Omega$$



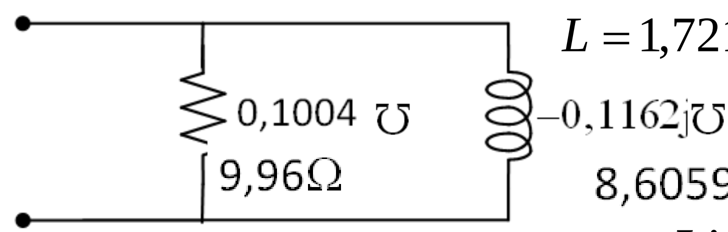
$$\omega Lj$$

$$L = 0{,}9858\,H$$

$$Y_{eq} = \frac{1}{Z_{eq}} = \frac{1}{4{,}255 + 4{,}929\,j\ \ \Omega}$$

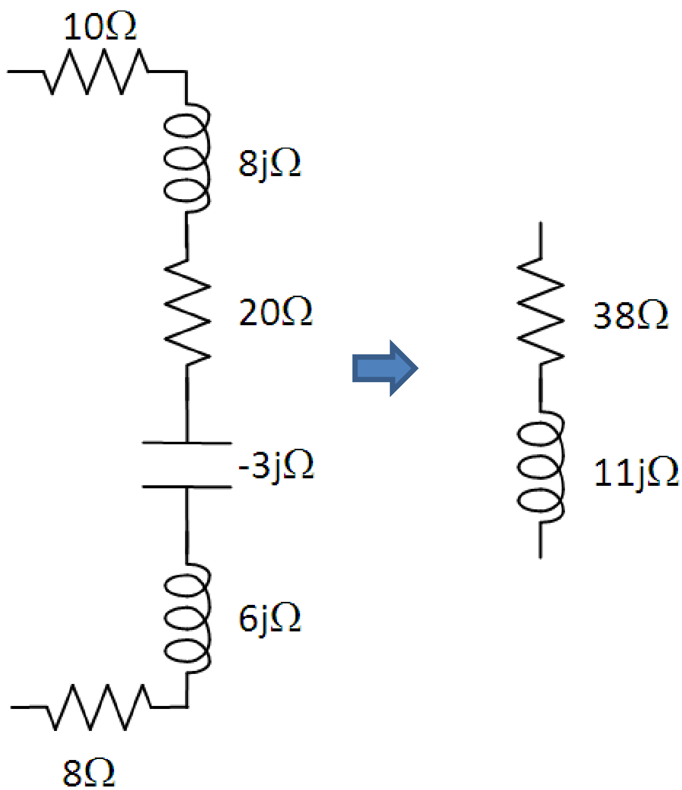
$$= 0{,}1004 - 0{,}1162\,j\ ℧$$

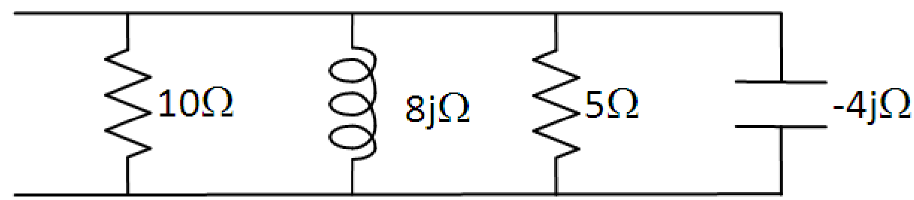
$$L = 1{,}7212\,H$$

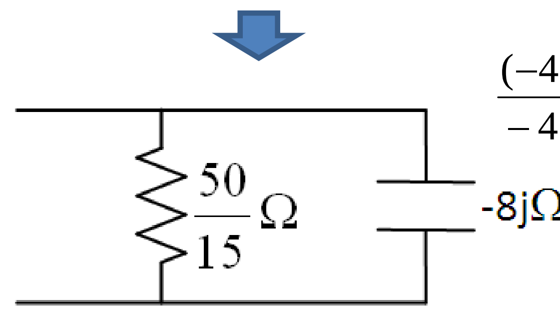


$$\omega Lj$$

Ejemplo

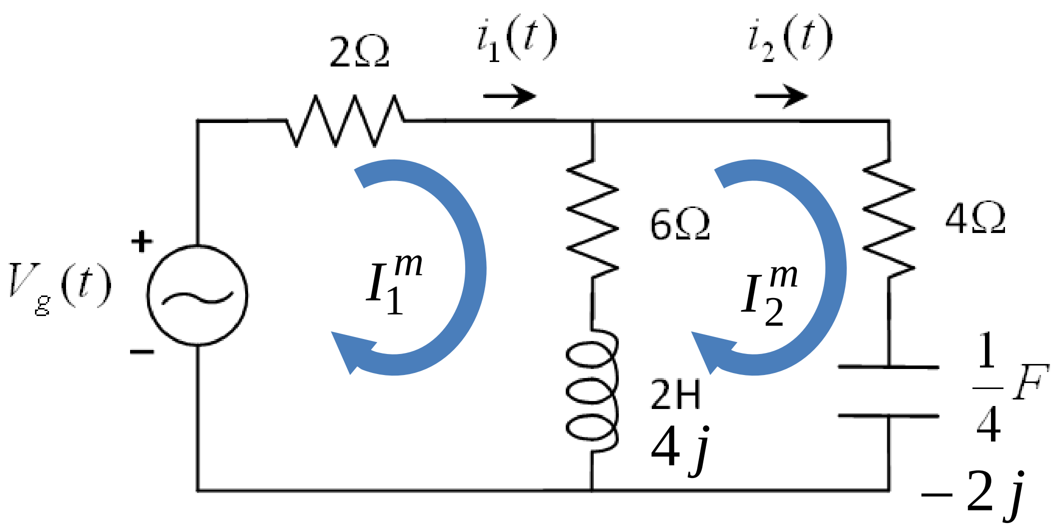






$$\frac{(-4j)(8j)}{-4j+8j}=\frac{32}{4j}$$

Ejemplos



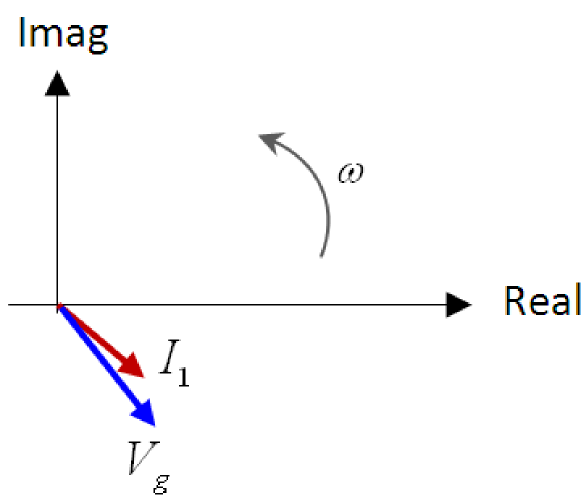
$$V_g(t) = 12Sen(2t - 70^o)$$

$$(8+4j)I_1^m - (6+4j)I_2^m = 12\angle{-70^o}$$

$$-(6+4j)I_1^m + (10+2j)I_2^m = 0$$

$$I_1^m = 2{,}326\angle{-67{,}4^o}$$

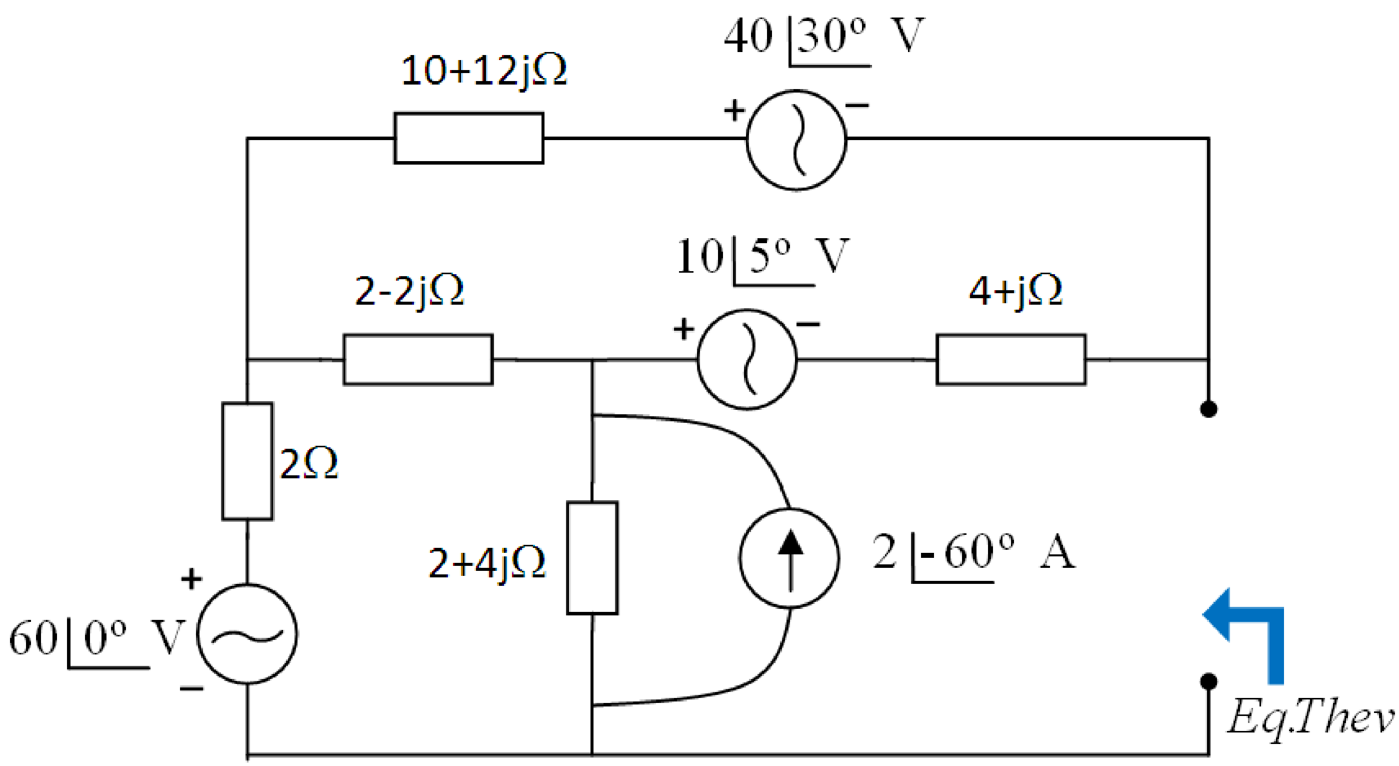
$$i_1(t) = 2{,}326Sen(2t - 67{,}4^o)\ A$$



Red predominantemente capacitiva

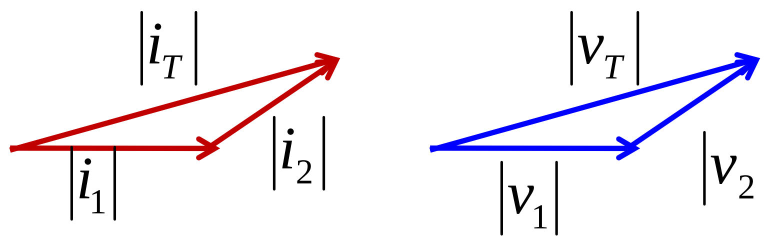
Ejercicios:

- Hallar el equivalente de Thevenin



Ejercicios:

- Problemas con módulo de fasores
- Ver la página web la solución de ejercicios de este tipo
- Teorema del coseno



- Tips: asumir 0º en el voltaje de elementos en paralelo o la corriente de elementos en serie
- Generalmente una ecuación con números complejos puede dar solución de 2 incógnitas
  - Modulo=módulo, ángulo=ángulo
  - Parte real=parte real, parte imag=parte imag