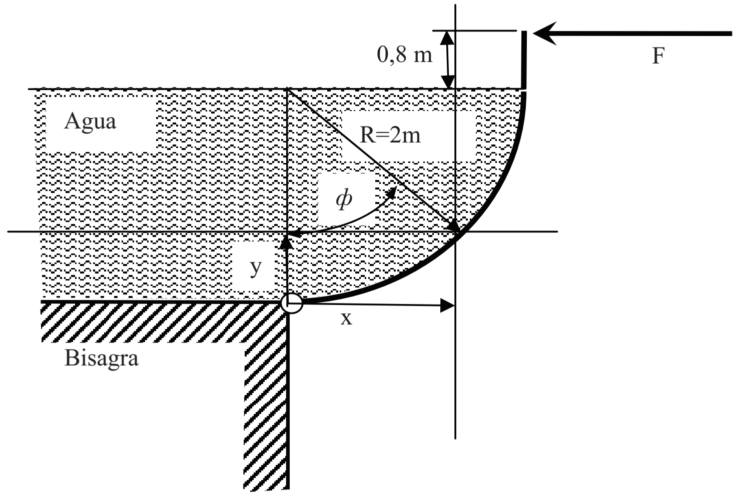
6. Calcular la fuerza F necesaria para sostener la puerta en la posición que se muestra en la figura.

(4 puntos)

$W = 1m$



$$F_{rx} = \rho g\bar{h}A_x = \frac{\rho g R}{2}RW = \frac{\rho g R^2 W}{2} = \frac{1000(9.81)(2^2)(1)}{2} = 19620N$$

$$F_{ry} = \rho g V_{ol} = \frac{\rho g \pi R^2 W}{4} = \frac{1000(9.81)\pi(2^2)(1)}{4} = 30819N$$

$$y' = \frac{1}{F_{rx}}\int_{A_x} yPdA_x = \frac{2}{\rho g R^2 W}\int_0^R y\rho g h W dy = \frac{2\rho g W}{\rho g R^2 W}\int_0^R y(R-y)dy = \frac{2}{R^2}\int_0^R (Ry - y^2)dy$$

$$y' = \frac{2}{R^2}\left(\frac{Ry^2}{2} - \frac{y^3}{3}\right)_0^R = \frac{2}{R^2}\left(\frac{R^3}{2} - \frac{R^3}{3}\right) = 2R\left(\frac{1}{2} - \frac{1}{3}\right) = 2R\left(\frac{1}{6}\right) = \frac{1}{3}R = 0.66667m$$

$$x' = \frac{1}{F_{ry}}\int_{A_y} xPdA_y = \frac{4}{\rho g \pi R^2 W}\int_0^R x\rho g h W dx = \frac{4\rho g W}{\rho g \pi R^2 W}\int_0^R x(R-y)dx$$

Expresamos las distancias en función del ángulo: $y = R - Rcos\phi; x = Rsin\phi; dx = Rcos\phi d\phi$

$$x' = \frac{4}{\pi R^2}\int_0^{\frac{\pi}{2}} Rsin\phi(R - R + Rcos\phi)Rcos\phi d\phi = \frac{4}{\pi R^2}\int_0^{\frac{\pi}{2}} R^3 sin\phi cos^2\phi d\phi = \frac{4R}{\pi}\int_0^{\frac{\pi}{2}} sin\phi cos^2\phi d\phi$$

$$x' = -\frac{4R}{\pi}\frac{cos^3\phi}{3}\bigg|_0^{\frac{\pi}{2}} = -\frac{4R}{\pi}\frac{cos^3\left(\frac{\pi}{2}\right)}{3} + \frac{4R}{\pi}\frac{cos^3(0)}{3} = \frac{4R}{3\pi} = \frac{4(2)}{3\pi} = 0.84883m$$

Para calcular la fuerza se hace sumatoria de momentos en la articulación:

$$F_{rx}y' + F_{ry}x' - F(R + 0.8) = 0$$

$$F = \frac{(F_{rx}y' + F_{ry}x')}{(R + 0.8)} = \frac{19620\left(\frac{2}{3}\right) + 30819\left(\frac{8}{3\pi}\right)}{(2 + 0.8)} = 14014N$$