# Modelando la Curva de Luz Secular de los Cometas

# Modeling the Secular Light Curves of Comets

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#### Resumen

Ferrín (2010) ha presentado la Curva de Luz Secular (SLC) de 27 cometas periódicos y no periódicos, entre ellos la del 1P/Halley. Estas SLCs nunca han sido modeladas, y en este trabajo vamos a modelar la curva secular del cometa 1P/Halley. Hemos modelado la curva de producción de agua, usando valores para el albedo visual y el infrarrojo y usando una correlación entre la producción de agua y la magnitud reducida (Jorda et al., 2008). Obtenemos orientaciones probables del eje de rotación (ángulos I,  $\Phi$ ). Estas orientaciones son comparadas con las soluciones de otros autores. Calculamos la temperatura superficial para varias orientaciones del eje de rotación. Para valores cerca del perihelio la temperatura superficial es constante. Este trabajo es parte de la tesis de maestria del segundo autor.

*PACS*: 96.30C, 96.25H, 96.25F *Palabras Claves*: comets, composition, atmospheres

#### Abstract

Ferrín (2010) has presented the Secular Light Curve (SLC) for 27 periodic and not periodic comets, among them 1P/Halley. These SLCs have never been modeled, and in this work we will be modeling the envelope of the secular light curve of comet 1P/Halley. We have modeled the water production rate of the comet, using values for the visual albedo and infrared albedo and using the correlation between the water production rate and the reduced magnitude (Jorda et al., 2008). We obtain probable orientation of the rotation axis  $(I,\Phi)$ . These orientations are compared with several solutions by other authors. We have calculated the surface temperature for several orientation of the rotation axis. For values near perihelion the surface temperature is constant. This work is part of the master's thesis of the second author.

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# 1. Introduction

Comet 1P/Halley has a large numbers of observed apparitions. Many of these visual observation have been published in the literature. (Ferrín, 2010) has published the brightness observation of 27 comets, considering that the envelope curve of these data is the best description of the brightness vs time. The reason is that there are many effects that wash out the outer regions of a comet, decreasing the measured brightness, like twilight, moon light, cirrus clouds, dirty optics, insufficient CCD aperture, and lack of dark adaptation, among others. However there is no physical effect that could increase the measured brightness of a comet. Thus the envelope of the data is the correct interpretation of the SLC. The orientation of the rotation axis can explain the shape of the secular light curve (Rondón and Ferrín, 2010; Rondón, 2007). The importance of studying the secular light curves is that they give a large amount of physical informations of the comet. Modeling the secular



Figura 1. Water production rate vs log(r), a) with an active region of  $80km^2$ , b) with an active region of  $180km^2$ . Due to sensitivity and detectability problems, many measurements lie below the envelope. Thus the correct interpretation of this plot, is the envelope.

light curve provides theoretical insight on the behavior of the brightness, and can give physical information on the location of the rotational axis.

## 2. Model Calculations

The first step for modeling the light curve is to calculate the sublimation rate of water. The equation that describe the vaporization rate of a comet is the energy conservation equation, given by:

$$\frac{Fo(1-A_v)}{r_H^2}\cos(\Theta) = (1-A_i)\sigma T^4 + Z(T)L(T) + K\frac{\partial T}{\partial z}$$
(1)

where  $A_i$  is the infrared albedo,  $r_H$  the Sun-Comet Distance,  $cos(\Theta)$  is the projection factor of the surface (Cowan and A'Hearn, 1979),  $\sigma$  is the Stefan Boltzmann constant, T is the temperature, Z(T) is the sublimation function, L(T) the latent heat function, K the thermal conductivity constant, z the layer depth.

$$Z(T) = \frac{P(T)m}{2\pi kT}$$
(2)

where P(T) is the vapor pressure function, m is the molecular weight, and k is the ideal gas constant. If we know the vapor pressure function and the latent heat function, we can solve for the energy conservation equation(1).



Figura 2. Standar deviation for each of the orientations of the rotational axis for comet 1P/Halley, in the I vs  $\Phi$  plane. Note that solutions by Skanina and Grun et al., have standard deviations of  $\pm 1$  magnitude and more, while we have found solutions with standard deviation less than  $\pm 0.5$  magnitudes (black regions of the plot). There are two identical zones (blue torus) because the nucleus is symmetric under a 180 deg rotation.



Figura 3. We show the theoretical model in red, vs the SLC (Ferrín 2010), using an orientation of the rotation axis of I= 90 deg,  $\Phi = 112$  deg for an active region of  $180km^2$  (top curve, red line), and  $80km^2$  (down curve blue line). The blue line obviously does not fit the observations well. On the other hand, the red thick line fits the envelope rather well over a range of 16 magnitudes (a factor of over one million in brightness). Red lines after perihelion represent several solutions for the SLC corresponding to different values of the conductivity and pole orientation.

$$Z_t otal = Z(i) = (1/2) \int Z(i,b) cos(b) db \quad (3)$$

We have modeled the water production rate for comet 1P/Halley using the observational data showed in (Schleicher and Woodney, 1998). We found that when these authors considers an active region of only  $80km^2$  the average of the observational data (Figure 1a) is well described. However, the secular light curve considers the envelope of the data as the correct interpretation of the SLC (Ferrin, 2005). Thus if instead we take the envelope of Figure 1a, we find an active region of 180km2 (Figure 1b). In both cases we have assumed a visual al-

bedo Av = 0.009, an infrared albedo  $_i$  = 0.5 and a thermal conductivity K=0. The correlation equation between the reduce visual magnitude and the water production rate is known (Jorda et al., 2008) and is given by:

$$m = 125,051 - 4,077 \log(Ztotal) \tag{4}$$

The secular light curve of 1P/Halley is asymmetric. This can be explained through the orientation of the rotation axis of the comet or through the thermal conductivity of the nucleus (Rondón, 2007; Rondón and Ferrín, 2010). These solutions are studied in the papers by Rondón.

We have calculated the standard deviation for our model with respect to the observational data, and found that the solution with minimal standard deviation is for an obliquity, I = 90 deg and a pole orbital longitude,  $\Phi$ = 112 deg, (Figure 2). Several authors obtain solutions for the of the rotation axis (Skanina, 1986). In Figure 2 we can see that the solution by (Grun et al., 1986) has a standard deviation of 0,98 magnitudes.

In Figure 3 we show the theoretical model in red, vs the SLC (Ferrín 2010), using an orientation of the rotation axis of I= 90 deg,  $\Phi = 112$  deg for an active region of  $180km^2$  (top curve, red line), and  $80km^2$  (down curve blue line). The blue line obviously does not fit the observations well. On the other hand, the red thick line fits the envelope rather well over a range of 16 magnitudes (a factor of over one million in brightness).

Red lines after perihelion represent several solutions for the SLC corresponding to different values of the conductivity and pole orientation studied by Rondón (2007, 2009).

### 3. Summary and conclusions

We have developed a simple theoretical model capable of reproducing the observational data of the secular SUMMARY AND CONCLUSIONS

with minimal standard deviation  $\sigma = \pm 0.3$  mag, for an orientation of the rotation axis I = 90 deg,  $\Phi = 112$  deg. The solution for the pole orientation found by (Grun et al., 1986) does no fit the SLC, and has a standard deviation of 0.98 magnitudes. The same happens with the solution by Skanina, with a standard deviation of  $\pm 1.07$  magnitudes. In conclusion, our simple model, if further developed, seems capable of explaining the envelopes of the SLCs over a brightness range of over one million in intensity and can constrain the orientation of the nuclear poles. This work is part of the master's thesis of E.Rondón (Rondón, 2007; Rondón, 2009; Rondón and Ferrin, 2010).

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3

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