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# A Simple Model for the Secular Light Curve of Comet C/1996 B2 Hyakutake

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**Abstract.** The secular light curves of comets (Ferrín, 2005) give a large amount of physical information on the cometary nucleus. We have developed a model that allows the prediction of a secular light curve, from which we derive parameters like the orientation of the rotational axis (I, $\phi$) and optical thickness of the cometary coma. The model is based on the paper published by (Cowan & A'Hearn, 1979). To do the calculation we found a correlation between the water production rate and the reduced magnitude. We obtain probable orientations of the nucleus pole for several combinations of parameters for comet C/1996 B2 Hyakutake.

**Keywords.** Hyakutake, Secular Light Curve, Water Production rate, Sublimation

## 1. Introduction

The secular light curve of a comet is a plot of the visual reduced magnitude vs logarithm of Sun-Comet distance. These light curves have a large amount of information on the cometary coma and nucleus: composition, age, size, structure, activity and cometary opacity (Ferrín 2005, Rondón 2009). These parameters are:

1)$R_{ON}$ (AU): The turn on distance of the coma. Physically $R_{ON}$ corresponds to the onset of steady activity. 2)$R_{OFF}$ (AU): The turn off distance of the coma. It measures the end of activity of the nucleus. 3)$V_{ON}$: The magnitude at which the nucleus turns on. 4)$V_{OFF}$: The magnitude at which the nucleus turns off. 5)$R_{OFF}/R_{ON}$: An asymmetry parameter for the secular light curve. 6) m(1, 1): The absolute magnitude of the coma. For the nucleus the following parameters are listed:

8)V (1, 1, 0) = $V_{NUC}$: Absolute nuclear magnitude. 9)$A_{SEC} = V_{NUC} - m(1,1)$: Amplitude of the secular light curve. 10)$D_{EFFE}$: The effective diameter of the comet. 11)P-AGE = Photometric Age, it is related to the loss of volatiles. P-AGE$=\frac{1440}{A_{SEC}}*(R_{ON}+R_{OFF})$ comet years (cy).

## 2. Factors Affecting the Sublimation of a Comet

It is known that the main factor that affects the sublimation of a comet is the orientation of the axis of rotation (Cowan & A'Hearn, 1979). In our model we consider this parameter of the first order. We have assumed a chemical composition of water to study the vapor pressure and the heat latent function. The equation that describes the vaporization rate of a comet is the energy conservation equation, given by:

$$F_0(1-A_v)r_H^{-2}\overline{cos(\theta)} = (1-A_{ir})\sigma T^4 + Z(T)L(T) + K\frac{\partial(T)}{\partial(z)} \tag{2.1}$$

$A_{ir}$ is the Infrared albedo, $r_H$ Sun-Comet Distance, $\overline{cos(\theta)}$ is the projection factor, $\sigma$ is the Steffan Bolztmann constant, T is the temperature, Z(T) is the sublimation function, L(T) latent heat function, K thermal conductivity constant, z layer depth.
The projection factor is given by the piecewise function:

Where i is the angle between the sun-comet direction and the rotation axis, and b is the latitude of the sublimation point. The projection factor is equal to 1 when the angle between the normal to the surface and the incident solar radiation is equal to 0 degree, and the projection factor is equal to 0 when the angle between the normal to the surface and the incident radiation is equal to 90 degree.
For pure ice the vaporization of the comet is given by:

$$Z(T) = \frac{P(T)m}{2(\pi)kT} \tag{2.2}$$

where : $P(T)$ is the vapor pressure function, m is the molecular weight, k is the ideal gas constant. The vapor pressure function is given by:

$$\log(P) = \frac{-2445.5646}{T} + 82312\log(T) - 0.01677006T + 120514x10^{-5}T^2 - 6.757169 \tag{2.3}$$

The latent heat function is given by:

$$L(T) = 12420 - 4.8T \tag{2.4}$$

If we know the vapor pressure function and the latent heat function we can solve $\underline{Z(i,b)}$ at a variaty of values of i and b using the energy conservation equation(1) through $\overline{cos\theta}$. Then, for estimate the total sublimation we evaluate the integral using (Eq. 2.5)

$$Z_{total} = \overline{Z(i)} = \frac{1}{2}\int (Z(i,b)cos(b)db) \tag{2.5}$$

## 3. Modeling the Light Curve of Comet C/1996B2 (Hyakutake)

The first step for modeling the light curve is to calculate the sublimation rate of water, using our model. Then, we can find the correlation equation between the reduce visual magnitude (Fig. 1a) and the water production rate, (Fig. 1b), plotting the reduce visual magnitude vs Log(r/q), (Fig. 2), where r is the heliocentric distance and q is the perihelion distance.

The correlation law between the reduce visual magnitude and the water production rate for comet Hyakutake is (Fig. 2) .

$$m = 139.9 - 4.59logQ \tag{3.1}$$

If we solve this equation for Q we find the same relation obtained by Jorda (2008) (Eq. 3.2) but with slightly different coefficients:

$$logQ = 30.48 - 0.22m \tag{3.2}$$

With equation (3.1) we calculate the reduce visual magnitude as a function of the heliocentric distance, then we can plot the light curve considering an obliquity of $I = 90^o$ and varying the pole orbital longitude (Fig. 3) as a first approximation. In these plot we can see the effects produced by changing the orientation of the axis of rotation, and is able to explain the asimmetry of the light curve. We have calculated the standard deviation for our model with the observer data, and found that the (Schleicher 2003)

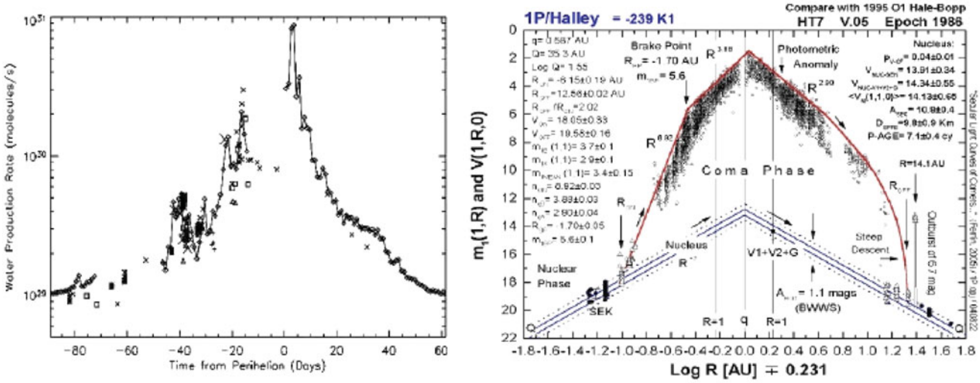


**Figure 1.** Figure 1a. Water production rate vs time from perihelion using observations by SOHO, SWAN and other authors (Combi, M., Makinen, T., *et al.*) (http://www-personal.umich.edu/∼mcombi/SWANhya/index.html). Figure 1b. Secular light curve of Hyakutake Comet (Ferrín. 2009).

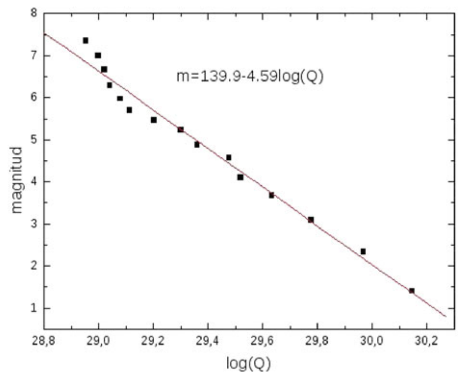


**Figure 2.** Calibration curve of the light curve, showing the equation of the magnitude as a function of the logarithm of the sublimation rate for comet C/1996 B2 Hyakutake, ajusted by least squares.

solution by the orientation of the axis of rotation $I = 108^o, \phi = 228^o$ has the smallest standar deviation (Fig. 4). We can also see the simmetry in the solution.

For small heliocentric distance, the light curve is affected for the increment of the dust rate produced for the comet, we are mesurement the effect produced by the dust and we have found the correlation beetwen the optical thickness and the heliocentric distance (Eq.3.3)(Fig. 5). Once found the correlation equation we can correct the reduce magnitude through equation. (Eq. 3.4).

$$\tau = 1.92 \mp 3.31 log(r/q) \tag{3.3}$$

$$m_{\Delta} = m_{model} + 2.5 log(e^{\tau}) \tag{3.4}$$

We can see (Fig. 6) that including the dust producion rate in the calculation of the optical thikness gives a better aproximation to the envelope of the observations.

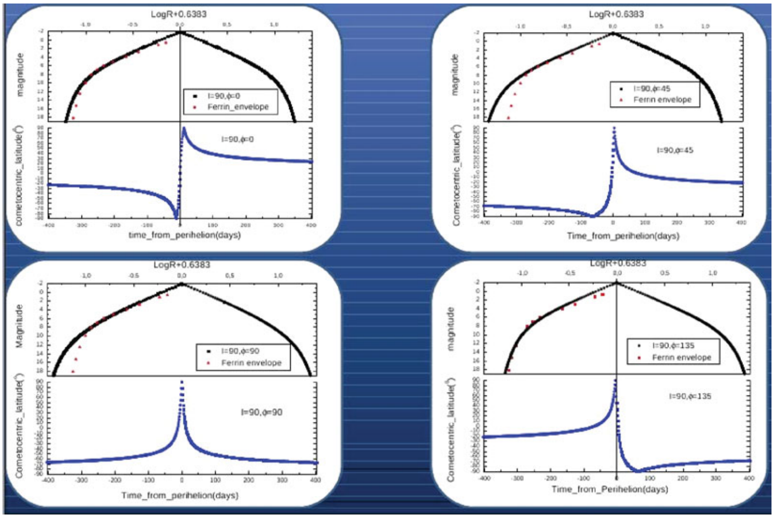


**Figure 3.** Seculars light curves calculate for the Hyakutake Comet considering $I = 90^o$ and varying the longitude of the orbital pole.

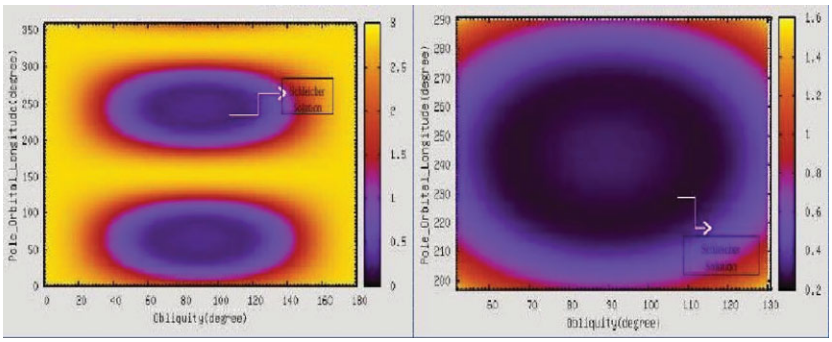


**Figure 4.** Contorn map made for our model for comet C/1996 B2 Hyakutake. The plot show the obliquity, I, versus the pole orbital longitude, $\phi$. In this plot see the standard deviation for each of the orientations of the axis of rotation with a color profile of one degree.

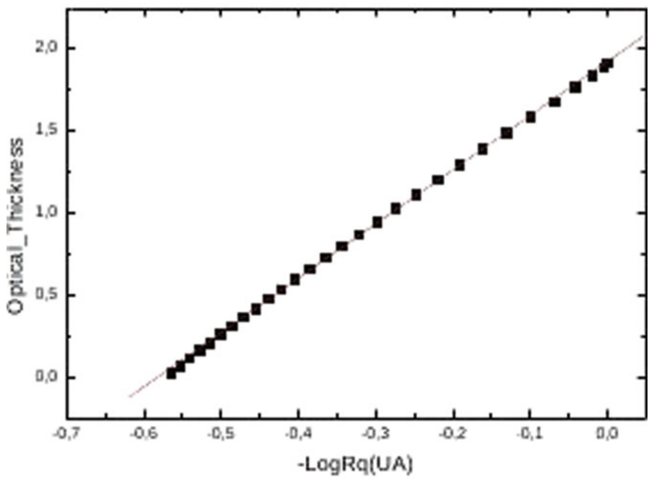


**Figure 5.** Optical thickness of the coma vs heliocentric distance.

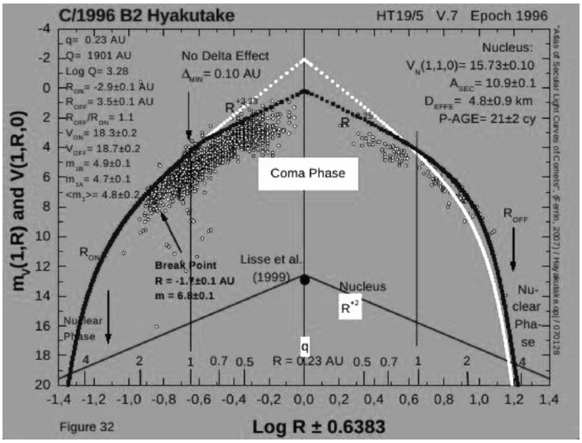


**Figure 6.** Modeling of the secular light curve. The white dots is our model without considering the effect produced by dust, and the blue dots is our model considering the effect produced by dust.

## 4. Conclusions

We have developed a theoretical model capable of reproducing the observational data of the secular light curve of comet C/1996 B2 Hyakutake. Since our model calculates the sublimation rate of water, we have found a correlation equation between the reduce visual magnitude and the sublimation rate, as a result yielding the same expression found by (Jorda 2008). The modeling of the envelope of the secular light curve is dependent on the orientation of the rotation axis and the dust production rate for small heliocentric distances. If we have the pole orientation we can calculate the water production rate as a function of time and the envelope of the secular light curve. We found a envelope that reproduces the same results obtained by (Schleicher 2003) with minimal standard deviation.

## References

Cowan, J. & A'Hearn. 1979, *Icarus*, 178, 493
Ferrín, I. 2005, *Icarus*, 178, 493
Ferrín, I. 2009, *Planetary and Space Science*, in press
Jorda, L., Crovisier, J., & Green, D. W. E. 2003, *ACM 2008*, 8046
Rondón, E. 2009, *Master Thesis, University of Los Andes*
Schleicher, D. & Woodney, L. 2003, *Icarus*, 162, 190