

## FORMULARIO DE TRANSFERENCIA DE CALOR - TEMA 2

### ECUACION DE DIFUSION DE CALOR EN COORDENADAS CARTESIANAS

$$\frac{\partial}{\partial x} \left( K \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

### ECUACION DE DIFUSION DE CALOR EN COORDENADAS CILINDRICAS

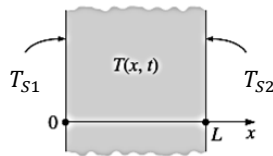
$$\frac{1}{r} \frac{\partial}{\partial r} \left( Kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( K \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

### ECUACION DE DIFUSION DE CALOR EN COORDENADAS ESFERICAS

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( Kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( K \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \left( K \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

### CONDICIONES DE FRONTERA

#### 1. TEMPERATURA SUPERFICIAL CONSTANTE (DIRICHLET)

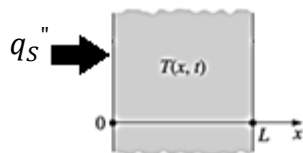


$$T_{x=0} = T_{S1}$$

$$T_{x=L} = T_{S2}$$

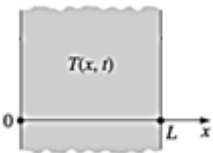
#### 2. FLUJO DE CALOR SUPERFICIAL CONSTANTE

##### a) FLUJO FINITO DE CALOR



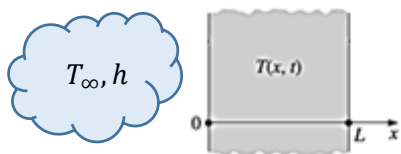
$$-K \frac{\partial T}{\partial x} \Big|_{x=0} = q_s''$$

**B) SUPERFICIE ADIABATICA O AISLADA (NEUMANN)**



$$\frac{\partial T}{\partial x} = 0 \quad \frac{\partial T}{\partial x} \Big|_{x=0} = 0$$

**3. CONDICION DE CONVECCION SUPERFICIAL**



$$h(T_{\infty} - T_{x=0}) = -K \frac{\partial T}{\partial x} \Big|_{x=0}$$

**SOLUCION GENERAL DE LA ECUACION DE DIFUSION DE CALOR: UNIDIMENSIONAL Y EN ESTADO ESTABLE, SIN GENERACION**

**PARED PLANA**

$$T_{(x)} = C_1 x + C_2$$

**CILINDRO**

$$T_{(r)} = C_1 \ln(r) + C_2$$

**ESFERA**

$$T_{(r)} = C_1 (-r^{-1}) + C_2$$

**SOLUCION GENERAL DE LA ECUACION DE DIFUSION DE CALOR: UNIDIMENSIONAL Y EN ESTADO ESTABLE, CON GENERACION**

**PARED PLANA**

$$T_{(x)} = -\frac{\dot{q}}{2K} x^2 + C_1 x + C_2$$

**CILINDRO**

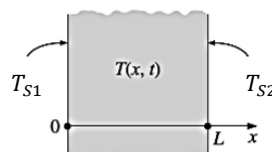
$$T_{(r)} = -\frac{\dot{q}}{4K} r^2 + C_1 \ln r + C_2$$

**ESFERA**

$$T_{(r)} = -\frac{\dot{q}}{6K} r^2 + C_1 \left(-\frac{1}{r}\right) + C_2$$

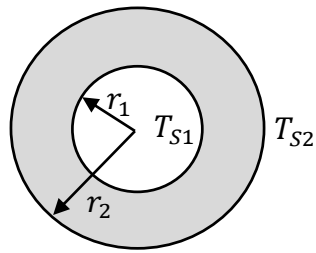
**SOLUCIONES PARTICULARES**

**DISTRIBUCION DE TEMPERATURA DE UNA PARED PLANA SIN GENERACION**



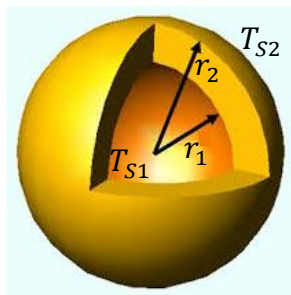
$$T_{(x)} = T_{S1} - \Delta T \frac{x}{L}$$

DISTRIBUCION DE TEMPERATURA DE UN CILINDRO HUECO SIN GENERACION



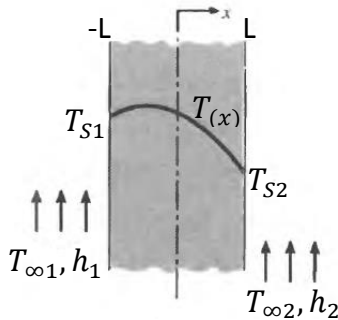
$$T_{(r)} = T_{S2} + \Delta T \frac{\ln(r/r_2)}{\ln(r_2/r_1)}$$

DISTRIBUCION DE TEMPERATURA DE UNA ESFERA HUECA SIN GENERACION



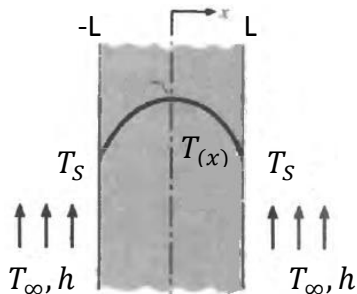
$$T_{(r)} = T_{S1} - \Delta T \left[ \frac{1 - (r_1/r)}{1 - (r_1/r_2)} \right]$$

DISTRIBUCION DE TEMPERATURA DE UNA PARED PLANA CON GENERACION Y CONDICIONES DE FRONTERA ASIMETRICAS



$$T_{(x)} = \frac{\dot{q} L^2}{2K} \left( 1 - \frac{x^2}{L^2} \right) + \frac{T_{S2} - T_{S1}}{2} \frac{x}{L} + \frac{T_{S1} + T_{S2}}{2}$$

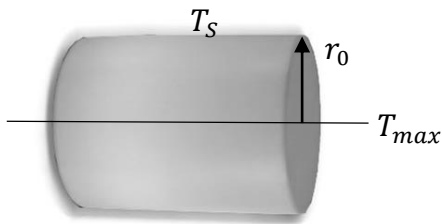
DISTRIBUCION DE TEMPERATURA DE UNA PARED PLANA CON GENERACION Y CONDICIONES DE FRONTERA SIMETRICAS



$$T_{(x)} = \frac{\dot{q} L^2}{2K} \left( 1 - \frac{x^2}{L^2} \right) + T_S$$

$$T_{\max} = \frac{\dot{q} L^2}{2K} + T_s$$

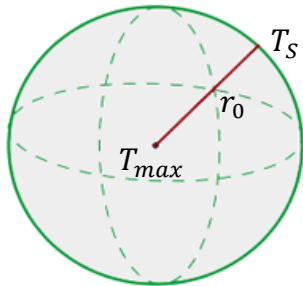
DISTRIBUCION DE TEMPERATURA DE UN CILINDRO MACIZO CON GENERACION



$$T_{(r)} = \frac{\dot{q} r_0^2}{4K} \left( 1 - \frac{r^2}{r_0^2} \right) + T_s$$

$$T_{\max} = \frac{\dot{q} r_0^2}{4K} + T_s$$

DISTRIBUCION DE TEMPERATURA DE UNA ESFERA MACIZA CON GENERACION



$$T_{(r)} = \frac{\dot{q} r_0^2}{6K} \left( 1 - \frac{r^2}{r_0^2} \right) + T_s$$

$$T_{\max} = \frac{\dot{q} r_0^2}{6K} + T_s$$

RADIO CRITICO DE AISLAMIENTO

$$R_{cr,cilindro} = \frac{K}{h}$$

$$R_{cr,esfera} = \frac{2K}{h}$$

## RESISTENCIA DE CONTACTO

**TABLA 3.2** Resistencia térmica de interfaces sólido/sólido representativas

Interfaz	$R_{t,c}'' \times 10^4 \text{ (m}^2 \cdot \text{K/W)}$	Fuente
Chip de silicio/aluminio recubierto en aire (27–500 kN/m <sup>2</sup> )	0.3–0.6	[2]
Aluminio/aluminio con relleno de hoja de indio (~100 kN/m <sup>2</sup> )	~0.07	[1, 3]
Acero inoxidable/acero inoxidable con relleno de hoja de indio (~3500 kN/m <sup>2</sup> )	~0.04	[1, 3]
Aluminio/aluminio con recubrimiento metálico (Pb)	0.01–0.1	[4]
Aluminio/aluminio con grasa Dow Corning 340 (~100 kN/m <sup>2</sup> )	~0.07	[1, 3]
Acero inoxidable/acero inoxidable con grasa Dow Corning 340 (~3500 kN/m <sup>2</sup> )	~0.04	[1, 3]
Chip de silicio/aluminio con resina epóxica de 0.02 mm	0.2–0.9	[5]
Bronce/bronce con soldadura de estaño de 15 μm	0.025–0.14	[6]

### RESISTENCIAS TERMICAS Y SUPERFICIES EXTENDIDAS

#### PARED PLANA, CILINDRO Y ESFERA

$$R_{conv} = \frac{1}{hA}$$

$$R_{rad} = \frac{1}{h_r A}$$

$$h_r = \varepsilon \sigma (T_s + T_{alr}) (T_s^2 + T_{alr}^2)$$

#### PARED PLANA SIN GENERACION

$$R_K = \frac{L}{KA}$$

#### PARED PLANA CON GENERACION

$$R_{KG} = \frac{L}{2KA}$$

#### CILINDRO SIN GENERACION

$$R_K = \frac{\ln(r_2 / r_1)}{2\pi KL}$$

#### CILINDRO CON GENERACION

$$R_{KG} = \frac{1}{4\pi KL}$$

#### ESFERA SIN GENERACION

$$R_K = \frac{(1/r_1) - (1/r_2)}{4\pi K}$$

#### ESFERA CON GENERACION

$$R_{KG} = \frac{1}{8\pi Kr}$$

#### EFFECTIVIDAD DE UNA ALETA

$$\varepsilon = \frac{q_f}{hA_{c,b}\theta_b}$$

#### EFFECTIVIDAD DE UNA ALETA INFINITA

$$\varepsilon = \sqrt{\frac{KP}{hA_c}}$$

EFICIENCIA DE UNA ALETA

$$\eta = \frac{q_f}{hA_f\theta_b}$$

$$m = \sqrt{\frac{hP}{KA_c}} \quad \theta_b = T_b - T_\infty$$

EFICIENCIA DE UNA ALETA CON EXTREMO ADIABATICO

$$\eta = \frac{\tanh(mL)}{mL}$$

$$L_{c,rec} = L + t/2 \quad L_{c,circ} = L + d/4$$

ARREGLOS DE ALETAS

$$R_{eq} = \frac{1}{h'(\eta A_{EXP,al} + A_{S.A})}$$

EFICIENCIA DE UNA ALETA CON EXTREMO ACTIVO

$$\eta = \frac{\tanh(mL_c)}{mL_c}$$

$$h' = h_{S.A} \left( \frac{A_{original}}{A_{total}} \right)^{0.375}$$

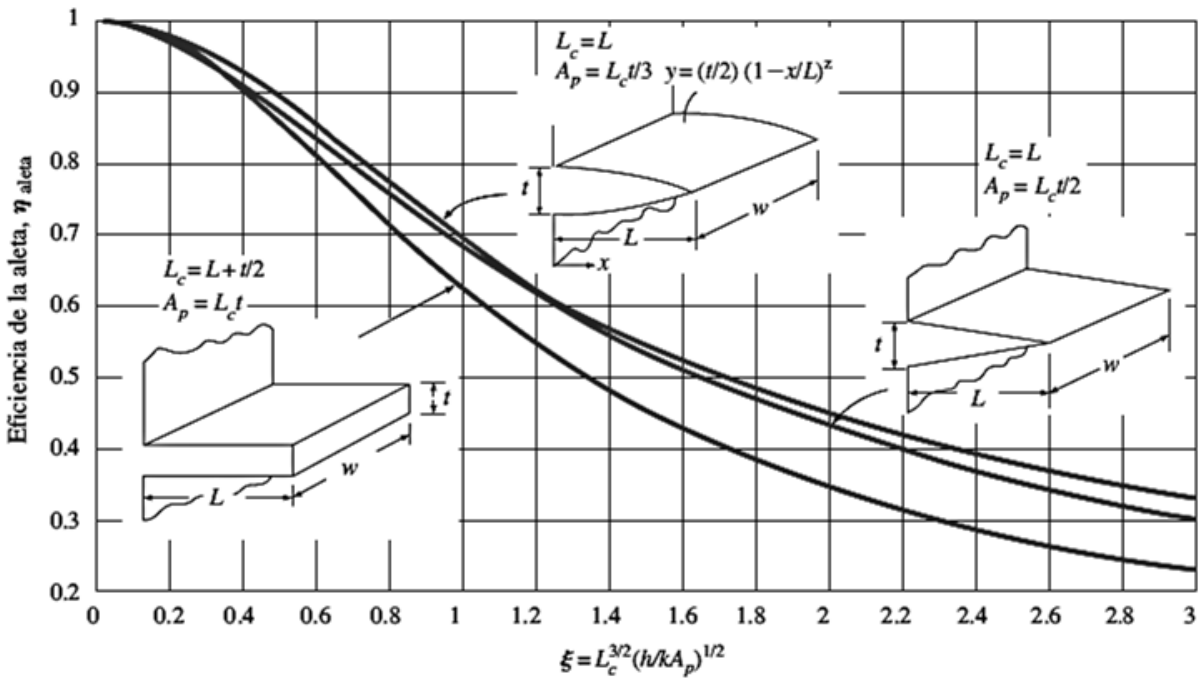
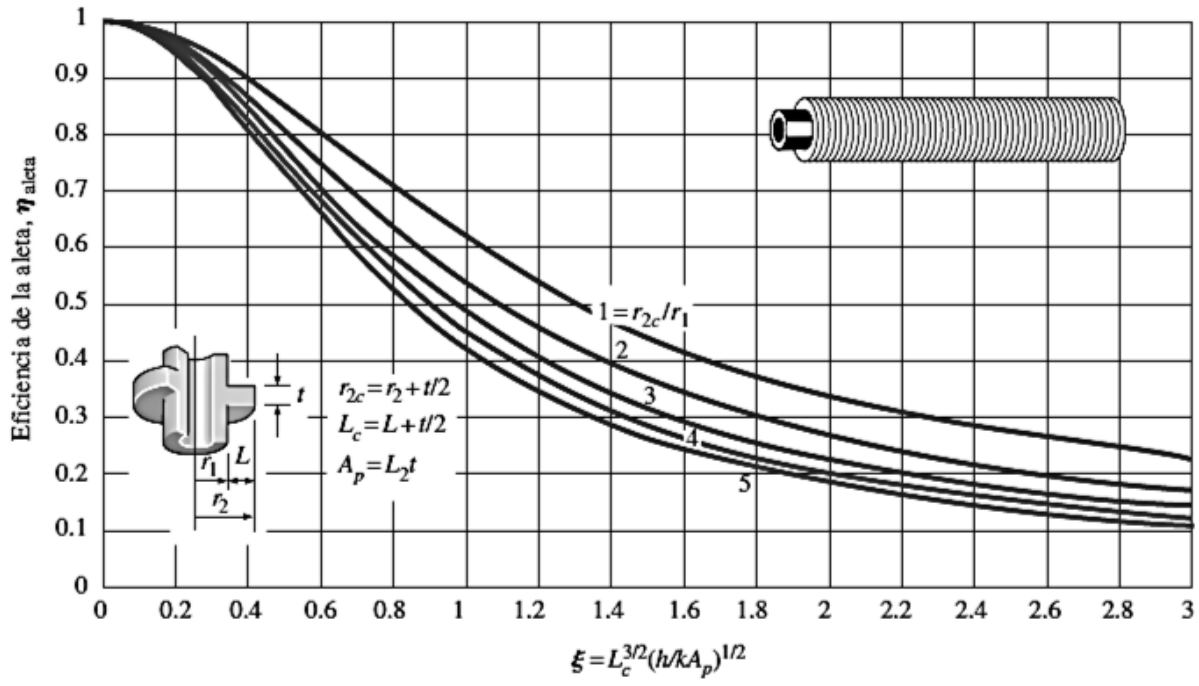


FIGURA 3-43

Eficiencia de aletas rectas de perfiles rectangular, triangular y parabólico.



**FIGURA 3-44**  
Eficiencia de aletas circulares de espesor constante  $t$ .

**TABLA 3-3**

Eficiencia y áreas de superficie de configuraciones comunes de aletas

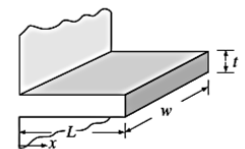
**Aletas rectangulares rectas**

$$m = \sqrt{2h/kt}$$

$$L_c = L + t/2$$

$$A_{\text{aleta}} = 2wL_c$$

$$\eta_{\text{aleta}} = \frac{\tanh mL_c}{mL_c}$$

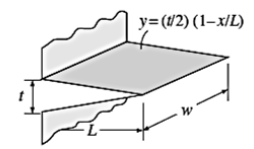


**Aletas triangulares rectas**

$$m = \sqrt{2h/kt}$$

$$A_{\text{aleta}} = 2w\sqrt{L^2 + (t/2)^2}$$

$$\eta_{\text{aleta}} = \frac{1}{mL} \frac{l_1(2mL)}{l_0(2mL)}$$



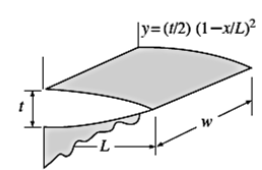
**Aletas parabólicas rectas**

$$m = \sqrt{2h/kt}$$

$$A_{\text{aleta}} = wL[C_1 + (L/t)\ln(t/L + C_1)]$$

$$C_1 = \sqrt{1 + (t/L)^2}$$

$$\eta_{\text{aleta}} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$



**Aletas circulares de perfil rectangular**

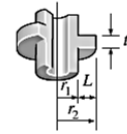
$$m = \sqrt{2h/kt}$$

$$r_{2c} = r_2 + t/2$$

$$A_{\text{aleta}} = 2\pi(r_{2c}^2 - r_1^2)$$

$$\eta_{\text{aleta}} = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



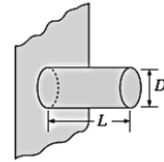
**Aletas de espiga de perfil rectangular**

$$m = \sqrt{4h/kD}$$

$$L_c = L + D/4$$

$$A_{\text{aleta}} = \pi D L_c$$

$$\eta_{\text{aleta}} = \frac{\tanh mL_c}{mL_c}$$



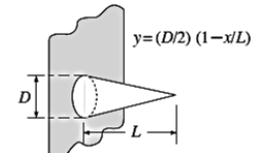
**Aletas de espiga de perfil triangular**

$$m = \sqrt{4h/kD}$$

$$A_{\text{aleta}} = \frac{\pi D}{2} \sqrt{L^2 + (D/2)^2}$$

$$\eta_{\text{aleta}} = \frac{2 l_2(2mL)}{mL l_1(2mL)}$$

$$I_2(x) = I_0(x) - (2/x)I_1(x) \text{ donde } x = 2mL$$



**Aletas de espiga de perfil parabólico**

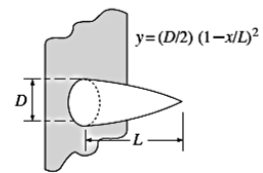
$$m = \sqrt{4h/kD}$$

$$A_{\text{aleta}} = \frac{\pi L^2}{8D} [C_3 C_4 - \frac{L}{2D} \ln(2DC_4/L + C_3)]$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = \sqrt{1 + (D/L)^2}$$

$$\eta_{\text{aleta}} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$



**Aletas de espiga de perfil parabólico (punta truncada)**

$$m = \sqrt{4h/kD}$$

$$A_{\text{aleta}} = \frac{\pi D^4}{96 L^2} \left\{ [16(L/D)^2 + 1]^{3/2} - 1 \right\}$$

$$\eta_{\text{aleta}} = \frac{3 l_1(4mL/3)}{2mL l_0(4mL/3)}$$

