

Integral indefinida. Cálculo integral

§ 1. Métodos más simples de integración

En los ejercicios 1676—1702 hallar las integrales, usando la tabla de integrales y aplicando las reglas elementales para la integración.

- | | | |
|---|--|------------------------------------|
| 1676. $\int \sqrt{x} dx.$ | 1677. $\int \sqrt[m]{x^n} dx.$ | 1678. $\int \frac{dx}{x^2}.$ |
| 1679. $\int 10^x dx.$ | 1680. $\int a^x e^x dx.$ | 1681. $\int \frac{dx}{2\sqrt{x}}.$ |
| 1682. $\int \frac{dh}{\sqrt{2gh}}.$ | 1683. $\int 3,4x^{-0,17} dx.$ | 1684. $\int (1-2u) du.$ |
| 1685. $\int (\sqrt{x+1})(x-\sqrt{x+1}) dx.$ | 1686. $\int \frac{\sqrt{x-x^3e^x+x^2}}{x^3} dx.$ | |
| 1687. $\int (2x^{-1,2} + 3x^{-0,8} - 5^{0,88}) dx.$ | | |
| 1688. $\int \left(\frac{1-z}{z}\right)^2 dz.$ | 1689. $\int \frac{(1-x)^2}{-x\sqrt{x}} dx.$ | |
| 1690. $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx.$ | 1691. $\int \frac{\sqrt[3]{x^2}-\sqrt{x}}{\sqrt{x}} dx.$ | |
| 1692. $\int \frac{dx}{\sqrt{3-3x^2}}.$ | 1693. $\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} dx.$ | |
| 1694. $\int \frac{1+\cos^2 x}{1+\cos 2x} dx.$ | 1695. $\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx.$ | |
| 1696. $\int \operatorname{tg}^2 x dx.$ | 1697. $\int \operatorname{ctg}^2 x dx.$ | |
| 1698. $\int 2 \operatorname{sen}^2 \frac{x}{2} dx.$ | 1699. $\int \frac{(1+2x^2) dx}{x^2(1+x^2)}.$ | |

1700. $\int \frac{(1+x)^2 dx}{x(1+x^2)}$

1701. $\int \frac{dx}{\cos 2x + \operatorname{sen}^2 x}$

1702. $\int (\operatorname{arcsen} x + \operatorname{arccos} x) dx$

En los ejercicios 1703—1780 hallar las integrales, aplicando el teorema sobre la invariancia de las fórmulas de integración.

1703. $\int \operatorname{sen}' x d(\operatorname{sen} x)$

1704. $\int \operatorname{tg}^3 x d(\operatorname{tg} x)$

1705. $\int \frac{d(1+x^2)}{\sqrt{1+x^2}}$

1706. $\int (x+1)^{15} dx$

1707. $\int \frac{dx}{(2x-3)^5}$

1708. $\int \frac{dx}{(a+bx)^c} (c \neq 1)$

1709. $\int \sqrt[5]{(8-3x)^6} dx$

1710. $\int \sqrt{8-2x} dx$

1711. $\int \frac{m}{\sqrt[3]{(a+bx)^2}} dx$

1712. $\int 2x \sqrt{x^2+1} dx$

1713. $\int x \sqrt{1-x^2} dx$

1714. $\int x^2 \sqrt[5]{x^3+2} dx$

1715. $\int \frac{x dx}{\sqrt{x^2+1}}$

1716. $\int \frac{x^4 dx}{\sqrt{4+x^5}}$

1717. $\int \frac{x^3 dx}{\sqrt[3]{x^4+1}}$

1718. $\int \frac{(6x-5) dx}{2 \sqrt{3x^2-5x+6}}$

1719. $\int \operatorname{sen}^3 x \cos x dx$

1720. $\int \frac{\operatorname{sen} x dx}{\cos^2 x}$

1721. $\int \frac{\cos x dx}{\sqrt[3]{\operatorname{sen}^2 x}}$

1722. $\int \cos^3 x \operatorname{sen} 2x dx$

1723. $\int \frac{\sqrt{\ln x}}{x} dx$

1724. $\int \frac{(\operatorname{arctg} x)^2 dx}{1+x^2}$

1725. $\int \frac{dx}{(\operatorname{arcsen} x)^3 \sqrt{1-x^2}}$

1726. $\int \frac{dx}{\cos^2 x \sqrt{1+\operatorname{tg} x}}$

1727. $\int \cos 3x d(3x)$

1728. $\int \frac{d(1+\ln x)}{\cos^2(1+\ln x)}$

1729. $\int \cos 3x dx$

1730. $\int (\cos \alpha - \cos 2x) dx$

1731. $\int \operatorname{sen}(2x-3) dx$

1732. $\int \cos(1-2x) dx$

1733. $\int \left[\cos \left(2x - \frac{\pi}{4} \right) \right]^{-2} dx$

1734. $\int e^x (\operatorname{sen} e^x) dx$

1735. $\int \frac{d(1+x^2)}{1+x^2}$

1736. $\int \frac{d(\operatorname{arcsen} x)}{\operatorname{arcsen} x}$

1737. $\int \frac{(2x-3) dx}{x^2-3x+8}$

$$\begin{array}{lll}
 1738. \int \frac{dx}{2x-1} & 1739. \int \frac{dx}{cx+m} & 1740. \int \frac{x dx}{x^2+1} \\
 1741. \int \frac{x^2 dx}{x^3+1} & 1742. \int \frac{e^x dx}{e^x+1} & 1743. \int \frac{e^{2x} dx}{e^{2x}+a^2} \\
 1744. \int \operatorname{tg} x dx. & 1745. \int \operatorname{ctg} x dx. & 1746. \int \operatorname{tg} 3x dx. \\
 1747. \int \operatorname{ctg}(2x+1) dx. & 1748. \int \frac{\operatorname{sen} 2x}{1+\cos^2 x} dx. & 1749. \int \frac{dx}{x \ln x} \\
 1750. \int \frac{(\ln x)^m}{x} dx, & 1751. \int e^{\operatorname{sen} x} d(\operatorname{sen} x). & \\
 1752. \int e^{\operatorname{sen} x} \cos x dx. & 1753. \int a^{3x} dx. & 1754. \int a^{-x} dx. \\
 1755. \int e^{-3x+1} dx. & 1756. \int e^{x^2} x dx. & 1757. \int e^{-x^3} x^2 dx.
 \end{array}$$

$$\begin{array}{ll}
 1758. \int \frac{d\left(\frac{x}{3}\right)}{\sqrt{1-\left(\frac{x}{3}\right)^2}} & 1759. \int \frac{dx}{\sqrt{1-25x^2}} \\
 1760. \int \frac{dx}{1+9x^2} & 1761. \int \frac{dx}{\sqrt{4-x^2}} \\
 1762. \int \frac{dx}{2x^2+9} & \\
 1763. \int \frac{dx}{\sqrt{4-9x^2}} & 1764. \int \frac{x dx}{x^4+1} \\
 1765. \int \frac{x dx}{\sqrt{a^2-x^4}} & \\
 1766. \int \frac{x^2 dx}{x^6+4} & 1767. \int \frac{x^3 dx}{\sqrt{1-x^8}} \\
 1768. \int \frac{e^x dx}{e^{2x}+4} & \\
 1769. \int \frac{2^x dx}{\sqrt{1-4^x}} & 1770. \int \frac{\cos \alpha d\alpha}{a^2+\operatorname{sen}^2 \alpha}
 \end{array}$$

$$\begin{array}{ll}
 1771. \int \frac{e^{2x}-1}{e^x} dx. & 1772. \int (e^x+1)^3 dx. \\
 1773. \int \frac{1+x}{\sqrt{1-x^2}} dx. & 1774. \int \frac{3x-1}{x^2+9} dx. \\
 1775. \int \sqrt{\frac{1-x}{1+x}} dx. & 1776. \int \frac{x(1-x^2)}{1+x^4} dx. \\
 1777. \int \frac{1+x-x^2}{\sqrt{(1-x^2)^3}} dx. & 1778. \int \frac{dx}{(x+\sqrt{x^2-1})^2} \\
 1779. \int \frac{2x-\sqrt{\operatorname{arcsen} x}}{\sqrt{1-x^2}} dx. & 1780. \int \frac{x+(\operatorname{arccos} 3x)^2}{\sqrt{1-9x^2}} dx.
 \end{array}$$

En los ejercicios 1781—1790 hallar las integrales, despejando la parte entera de la fracción bajo el signo de integral.

$$1781. \int \frac{x}{x+4} dx. \quad 1782. \int \frac{x}{2x+1} dx. \quad 1783. \int \frac{Ax}{a+bx} dx.$$

$$1784. \int \frac{3+x}{3-x} dx. \quad 1785. \int \frac{(2x-1) dx}{x-2}. \quad 1786. \int \frac{x+2}{2x-1} dx.$$

$$1787. \int \frac{(1+x)^2}{x^2+1} dx. \quad 1788. \int \frac{x^2-1}{x^2+1} dx. \quad 1789. \int \frac{x^4}{1-x} dx.$$

$$1790. \int \frac{x^4 dx}{x^2+1}.$$

En los ejercicios 1791—1807 hallar las integrales aplicando el método de descomposición de la expresión integrando y el método para despejar el cuadrado perfecto.

$$1791. \int \frac{dx}{x(x-1)}. \quad 1792. \int \frac{dx}{x(x+1)}. \quad 1793. \int \frac{dx}{(x+1)(2x-3)}.$$

$$1794. \int \frac{dx}{(a-x)(b-x)}. \quad 1795. \int \frac{x^2+1}{x^2-1} dx. \quad 1796. \int \frac{dx}{x^2-7x+10}.$$

$$1797. \int \frac{dx}{x^2+3x-10}. \quad 1798. \int \frac{dx}{4x^2-9}. \quad 1799. \int \frac{dx}{2-3x^2}.$$

$$1800. \int \frac{dx}{(x-1)^2+4}. \quad 1801. \int \frac{dx}{x^2+2x+3}. \quad 1802. \int \frac{dx}{x-x^2-2,5}.$$

$$1803. \int \frac{dx}{4x^2+4x+5}. \quad 1804. \int \frac{dx}{\sqrt{1-(2x+3)^2}}.$$

$$1805. \int \frac{dx}{\sqrt{4x-3-x^2}}. \quad 1806. \int \frac{dx}{\sqrt{8+6x-9x^2}}.$$

$$1807. \int \frac{dx}{\sqrt{2-6x-9x^2}}.$$

En los ejercicios 1808—1831 hallar las integrales aplicando fórmulas trigonométricas para transformar la expresión integrando.

$$1808. \int \cos^2 x dx. \quad 1809. \int \sin^2 x dx. \quad 1810. \int \frac{dx}{1-\cos x}.$$

$$1811. \int \frac{dx}{1+\sin x}. \quad 1812. \int \frac{1-\cos x}{1+\cos x} dx. \quad 1813. \int \frac{1+\sin x}{1-\sin x} dx.$$

$$1814. \int (\operatorname{tg}^2 x + \operatorname{tg}^4 x) dx. \quad 1815. \int \frac{\cos 2x dx}{1+\sin x \cos x}.$$

$$1816. \int \cos x \sin 3x dx. \quad 1817. \int \cos^2 x \cos 3x dx.$$

$$1818. \int \sin 2x \sin 5x dx. \quad 1819. \int \cos x \cos 2x \cos 3x dx.$$

$$1820. \int \frac{dx}{\cos x}. \quad 1821. \int \frac{1-\sin x}{\cos x} dx. \quad 1822. \int \frac{\sin^3 x}{\cos x} dx.$$

$$\begin{array}{lll}
 1823. \int \frac{\cos^3 x dx}{\operatorname{sen}^4 x} & 1824. \int \frac{\operatorname{sen}^3 \alpha}{\sqrt{\cos \alpha}} d\alpha & 1825. \int \frac{dx}{\cos^4 x} \\
 1826. \int \cos^3 x dx & 1827. \int \operatorname{tg}^4 x dx & 1828. \int \operatorname{sen}^5 x dx \\
 1829. \int \operatorname{sen}^4 x dx & 1830. \int \operatorname{tg}^3 x dx & 1831. \int \frac{dx}{\operatorname{sen}^6 x}
 \end{array}$$

§ 2. Métodos principales de integración

Integración por partes

En los ejercicios 1832—1868 hallar las integrales.

$$\begin{array}{lll}
 1832. \int x \operatorname{sen} 2x dx & 1833. \int x \cos x dx & 1834. \int x e^{-x} dx \\
 1835. \int x 3^x dx & 1836. \int x^n \ln x dx \quad (n \neq -1) & \\
 1837. \int x \operatorname{arctg} x dx & 1838. \int \arccos x dx & 1839. \int \operatorname{arctg} \sqrt{x} dx \\
 1840. \int \frac{\operatorname{arcsen} x}{\sqrt{x+1}} dx & 1841. \int x \operatorname{tg}^2 x dx & 1842. \int x \cos^2 x dx \\
 1843. \int \frac{\lg x}{x^3} dx & 1844. \int \frac{x \operatorname{arctg} x}{\sqrt{1+x^2}} dx & 1845. \int \frac{\operatorname{arcsen} \sqrt{x}}{\sqrt{1-x}} dx \\
 1846. \int \ln(x^2 + 1) dx & 1847. \int \frac{x^2 dx}{(1+x^2)^2} & \\
 1848. \int \frac{x^3 dx}{\sqrt{1+x^2}} & 1849. \int x^2 \ln(1+x) dx & \\
 1850. \int x^2 e^{-x} dx & 1851. \int x^3 e^x dx & 1852. \int x^2 a^x dx \\
 1853. \int x^3 \operatorname{sen} x dx & 1854. \int x^2 \cos^2 x dx & 1855. \int \ln^2 x dx \\
 1856. \int \frac{\ln^3 x}{x^2} dx & 1857. \int \frac{\ln^2 x}{\sqrt{x^5}} dx & 1858. \int (\operatorname{arcsen} x)^2 dx \\
 1859. \int (\operatorname{arctg} x)^2 x dx & 1860. \int e^x \operatorname{sen} x dx & \\
 1861. \int e^{3x} (\operatorname{sen} 2x - \cos 2x) dx & 1862. \int e^{ax} \cos nx dx &
 \end{array}$$

$$1863. \int \operatorname{sen} \ln x \, dx. \quad 1864. \int \operatorname{cos} \ln x \, dx. \quad 1865^*. \int \frac{x^2 \, dx}{\sqrt{1-x^2}}$$

$$1866^*. \int \sqrt{a^2+x^2} \, dx. \quad 1867. \int \frac{x^2 e^x \, dx}{(x+2)^2}. \quad 1868. \int x^2 e^x \operatorname{sen} x \, dx.$$

Cambio de variable

En los ejercicios 1869—1904 hallar las integrales.

$$1869. \int \frac{dx}{1+\sqrt{x+1}} \quad (\text{sustituyendo } x+1=z^2).$$

$$1870. \int \frac{x^3 \, dx}{\sqrt{x-1}}. \quad 1871. \int \frac{4x+3}{(x-2)^3} \, dx. \quad 1872. \int \frac{dx}{x\sqrt{x+1}}.$$

$$1873. \int \frac{x+1}{x\sqrt{x-2}} \, dx. \quad 1874. \int \frac{dx}{1+\sqrt{x}}. \quad 1875. \int \frac{\sqrt{x}}{x(x+1)} \, dx.$$

$$1876. \int \frac{\sqrt{x}}{x+1} \, dx. \quad 1877. \int \frac{dx}{1+\sqrt[3]{x+1}}. \quad 1878. \int \frac{dx}{\sqrt{ax+b+m}}.$$

$$1879. \int \frac{\sqrt{x} \, dx}{\sqrt{x}-\sqrt[3]{x}} \quad (\text{sustituyendo } x=z^6).$$

$$1880. \int \frac{dx}{\sqrt[3]{x}(\sqrt[3]{x}-1)}. \quad 1881. \int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}. \quad 1882. \int \frac{\sqrt{x}}{\sqrt{x^2-\sqrt{x}}} \, dx.$$

$$1883. \int \frac{e^{2x} \, dx}{\sqrt[4]{e^x+1}} \quad (\text{sustituyendo } e^x+1=z^4).$$

$$1884. \int \frac{dx}{\sqrt{1+e^x}}. \quad 1885. \int \frac{\sqrt{1+\ln x}}{x \ln x} \, dx.$$

$$1886. \int \sqrt{1+\cos^2 x} \cdot \operatorname{sen} 2x \cdot \operatorname{cos} 2x \, dx.$$

$$1887. \int \frac{\ln \operatorname{tg} x}{\operatorname{sen} x \cdot \operatorname{cos} x} \, dx. \quad 1888. \int \frac{x^5 \, dx}{\sqrt{a^3-x^3}}. \quad 1889. \int \frac{x^5 \, dx}{(x^2-4)^2}.$$

$$1890. \int \frac{dx}{x^2 \sqrt{x^2+a^2}}$$

(sustituyendo $x = \frac{1}{z}$, o $x = a \operatorname{tg} z$, o $x = a \operatorname{sh} z$).

$$1891. \int \frac{x^2 \, dx}{\sqrt{a^2-x^2}} \quad (\text{sustituyendo } x = a \operatorname{sen} z).$$

$$1892. \int \frac{dx}{x \sqrt{x^2-a^2}}$$

(sustituyendo $x = \frac{1}{z}$, o $x = \frac{a}{\operatorname{cos} z}$, o $x = a \operatorname{ch} z$).

1893. $\int \frac{\sqrt{1+x^2}}{x^4} dx.$

1894. $\int \frac{\sqrt{1-x^2}}{x^2} dx.$

1895. $\int \frac{dx}{\sqrt{(a^2+x^2)^3}}.$

1896. $\int \frac{\sqrt{(9-x^2)^3}}{x^6} dx.$

1897. $\int \frac{dx}{x^2 \sqrt{x^2-9}}.$

1898. $\int \frac{dx}{x \sqrt{1+x^2}}.$

1899. $\int \frac{dx}{\sqrt{(x^2-a^2)^3}}.$

1900. $\int x^2 \sqrt{4-x^2} dx.$

1901. $\int \frac{dx}{(x^2+4) \sqrt{4x^2+1}}.$

1902*. $\int \sqrt{\frac{x-1}{x+1}} \frac{dx}{x^2}.$

1903*. $\int \frac{dx}{\sqrt{x-x^2}}.$

1904*. $\int \frac{(x+1) dx}{x(1+xe^x)}.$

En los ejercicios 1905—1909 hallar las integrales efectuando primero el cambio de variable y luego integrando por partes.

1905. $\int e^{\sqrt{x}} dx.$

1906. $\int \operatorname{sen} \sqrt[3]{x} dx.$

1907. $\int \frac{\operatorname{arcsen} x}{\sqrt{(1-x^2)^3}} dx.$

1908. $\int \frac{x^2 \operatorname{arctg} x}{1+x^2} dx.$

1909. $\int \frac{\operatorname{arctg} x}{x^2(1+x^2)} dx.$

Diversos problemas

En los ejercicios 1910—2011 hallar las integrales.

1910. $\int (x+1) \sqrt{x^2+2x} dx.$

1911. $\int (1+e^{3x})^2 e^{3x} dx.$

1912. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx.$

1913. $\int \frac{\operatorname{sen} x}{e^{\cos x}} dx.$

1914. $\int \sqrt{1-e^x} e^x dx.$

1915. $\int x \cos x^2 dx.$

1916. $\int (2-3x^{\frac{4}{3}})^{\frac{1}{5}} x^{\frac{1}{3}} dx.$

1917. $\int \frac{2x^5-3x^2}{1+3x^3-x^6} dx.$

1918. $\int \frac{\sqrt{x} dx}{1+x^{\frac{3}{2}}}.$

1919. $\int \frac{dx}{e^x(3+e^{-x})}.$

1920. $\int \frac{dx}{e^x \sqrt{1-e^{-2x}}}.$

1921. $\int \frac{2x+3}{\sqrt{1+x^2}} dx.$

1922. $\int \frac{2x-1}{\sqrt{9x^2-4}} dx.$

1923. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx.$

$$\begin{array}{ll}
 1924. \int \frac{dx}{x \sqrt{3 - \ln^2 x}} & 1925. \int \frac{\ln x \, dx}{x(1 - \ln^2 x)} \\
 1926. \int \frac{x^2 - x + 1}{\sqrt{(x^2 + 1)^3}} dx & 1927. \int \frac{(\operatorname{arctg} x)^n}{1 + x^2} dx \\
 1928. \int \frac{d\varphi}{\operatorname{sen}^2 \varphi \cos^2 \varphi} & 1929. \int \frac{\cos 2x}{\cos^2 x} dx \\
 1930. \int \frac{\operatorname{sen}^4 x \, dx}{\cos^6 x} & 1931. \int \sqrt{\operatorname{tg}^3 x} \sec^4 x \, dx \\
 1932. \int (1 - \operatorname{tg} 3x)^2 dx & 1933. \int \frac{x^3 \, dx}{x + 1} \\
 1934. \int \frac{x \, dx}{(x - 1)^3} & 1935. \int \frac{x \, dx}{\sqrt{2 + 4x}} \\
 1936. \int \frac{x \, dx}{\sqrt{1 + 2x}} & 1937. \int x \sqrt{a + x} \, dx \\
 1938. \int (\sqrt{\operatorname{sen} x + \cos x})^2 dx & 1939. \int a^{mx} b^{nx} \, dx
 \end{array}$$

$$\begin{array}{ll}
 1940. \int \frac{dx}{\sqrt{5 - 2x + x^2}} & 1941. \int \frac{dx}{\sqrt{9x^2 - 6x + 2}} \\
 1942. \int \frac{dx}{\sqrt{12x - 9x^2 - 2}} & 1943. \int \frac{(8x - 11) \, dx}{\sqrt{5 + 2x - x^2}} \\
 1944. \int \frac{(x + 2) \, dx}{x^2 + 2x + 2} & 1945. \int \frac{(x - 3) \, dx}{\sqrt{3 - 2x - x^2}} \\
 1946. \int \frac{(3x - 1) \, dx}{4x^2 - 4x + 17} & 1947. \int \frac{(3x - 1) \, dx}{\sqrt{x^2 + 2x + 2}} \\
 1948. \int \frac{(x - 2) \, dx}{x^2 - 7x + 12} & 1949. \int \frac{2x + 5}{\sqrt{9x^2 + 6x + 2}} dx \\
 1950. \int \frac{3 - 4x}{2x^2 - 3x + 1} dx & 1951. \int \frac{(4 - 3x) \, dx}{5x^2 + 6x + 18} \\
 1952. \int \frac{(2 - 5x) \, dx}{\sqrt{4x^2 + 9x + 1}} & 1953. \int \frac{x \, dx}{\sqrt{3x^2 - 11x + 2}} \\
 1954. \int \frac{\sqrt{x} \, dx}{\sqrt{2x + 3}} & 1955. \int \sqrt{\frac{a - x}{x - b}} dx
 \end{array}$$

$$\begin{array}{ll}
 1956. \int \operatorname{arctg} x \, dx & 1957. \int x \operatorname{sen} x \cos x \, dx \\
 1958. \int x^2 \cos \omega x \, dx & 1959. \int e^{2x} x^3 \, dx
 \end{array}$$

1960. $\int \frac{\ln \cos x}{\cos^2 x} dx.$
1961. $\int \frac{\operatorname{ctg} x}{\ln \operatorname{sen} x} dx.$
1962. $\int \frac{x^7 dx}{(1+x^4)^2}.$
1963. $\int \frac{\cos^2 3x}{\operatorname{sen} 3x} dx.$
1964. $\int \frac{dx}{1-\operatorname{sen} 3x}.$
1965. $\int \frac{\operatorname{sen} 2x dx}{4-\cos^2 2x}.$
1966. $\int \frac{dx}{e^x+1}.$
1967. $\int \frac{e^x-1}{e^x+1} dx.$
1968. $\int e^{e^x+x} dx.$
1969. $\int e^{2x^2+\ln x} dx.$
1970. $\int \frac{3+x^3}{\sqrt{2+2x^2}} dx.$
1971. $\int \frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}} dx.$
1972. $\int \frac{x \cos x}{\operatorname{sen}^3 x} dx.$
1973. $\int e^x \operatorname{sen}^2 x dx.$
1974. $\int \frac{(1+\operatorname{tg} x) dx}{\operatorname{sen} 2x}.$
1975. $\int \frac{1-\operatorname{tg} x}{1+\operatorname{tg} x} dx.$
1976. $\int \frac{d\varphi}{\sqrt{3} \cos \varphi + \operatorname{sen} \varphi}.$
1977. $\int \frac{\operatorname{sen} x dx}{1+\operatorname{sen} x}.$
1978. $\int \frac{\operatorname{sen}^2 x \cos x}{(1+\operatorname{sen}^2 x)} dx.$
1979. $\int \frac{\sqrt{1+\cos x}}{\operatorname{sen} x} dx.$
1980. $\int \frac{\ln \ln x}{x} dx.$
1981. $\int x^3 e^{x^2} dx.$
1982. $\int e^{-x^2} x^5 dx.$
1983. $\int \frac{x^3 dx}{\sqrt{1+2x^2}}.$
1984. $\int \frac{x^4 dx}{\sqrt{(1-x^2)^3}}.$
1985. $\int \frac{\sqrt{(x^2-a^2)^3}}{x} dx.$
1986. $\int \frac{dx}{x^4 \sqrt{x^2+4}}.$
1987. $\int \frac{\sqrt{x^2+8}}{x^4} dx.$
1988. $\int \frac{\sqrt{4+x^2}}{x^6} dx.$
1989. $\int \frac{dx}{x^4 \sqrt{x^2-3}}.$
1990. $\int \frac{\sqrt{x} dx}{\sqrt[4]{x^3+1}}.$
1991. $\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1} dx.$
1992. $\int \frac{dx}{(2+x) \sqrt{1+x}}.$
1993. $\int \frac{\sqrt[3]{x} dx}{x(\sqrt{x}+\sqrt[3]{x})}.$
1994. $\int \frac{\sqrt{x^3+2x}}{x} dx.$
- 1995*. $\int \frac{x^7 dx}{(1-x^2)^5}.$
1996. $\int \frac{dx}{(ax+b) \sqrt{x}}.$
1997. $\int \frac{\sqrt{1+x^2}}{x^{13}} dx.$

$$\begin{array}{ll}
 1998. \int \frac{x dx}{(1-x^4)^{\frac{3}{2}}} & 1999. \int \frac{x^5 dx}{\sqrt{x^4+4}} \\
 2000. \int \frac{dx}{\sqrt{x}(x-1)} & 2001. \int \frac{\sqrt{1-x^3}}{x^2 \sqrt{x}} dx. \\
 2002. \int \frac{x^4 dx}{(1+x^2)^3} & 2003. \int \frac{3x^2-1}{2x \sqrt{x}} \operatorname{arctg} x dx. \\
 2004. \int \frac{e^x (1+e^x) dx}{\sqrt{1-e^{2x}}} & 2005. \int \sqrt{e^x-1} dx. \\
 2006^*. \int \frac{\ln(x+1)-\ln x}{x(x+1)} dx. & 2007. \int \frac{dx}{x^6+x^4} \\
 2008. \int \arccos \sqrt{\frac{x}{x+1}} dx. & 2009. \int \ln(x + \sqrt{1+x^2}) dx. \\
 2010. \int \sqrt[3]{\frac{\sin^2 x}{\cos^{14} x}} dx. & 2011. \int \frac{dx}{\cos^3 x \sqrt{\sin 2x}}
 \end{array}$$

§ 3. Tipos principales de las funciones integrables

Funciones fraccionarias racionales

En los ejercicios 2012—2067 hallar las integrales.
1) *El denominador tiene sólo distintas raíces reales.*

$$\begin{array}{ll}
 2012. \int \frac{x dx}{(x+1)(2x+1)} & 2013. \int \frac{x dx}{2x^2-3x-2} \\
 2014. \int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)} dx. & \\
 2015. \int \frac{dx}{6x^3-7x^2-3x} & 2016. \int \frac{x^5+x^4-8}{x^3-4x} dx. \\
 2017. \int \frac{x^3-1}{4x^3-x} dx. & \\
 2018. \int \frac{32x dx}{(2x-1)(4x^2-16x+15)} & \\
 2019. \int \frac{x dx}{x^4-3x^2+2} & 2020. \int \frac{(2x^2-5) dx}{x^4-5x^2+6} \\
 2021. \int \frac{x^6-2x^4+3x^3-9x^2+4}{x^5-5x^3+4x} dx. &
 \end{array}$$

2) El denominador tiene sólo raíces reales; algunas raíces son múltiples.

$$\begin{array}{ll}
 2022. \int \frac{(x^2 - 3x + 2) dx}{x(x^2 + 2x + 1)} & 2023. \int \left(\frac{x+2}{x-1}\right)^2 \frac{dx}{x} \\
 2024. \int \frac{x^2 dx}{x^3 + 5x^2 + 8x + 4} & 2025. \int \frac{x^3 + 1}{x^3 - x^2} dx \\
 2026. \int \frac{x^3 - 6x^2 + 11x - 5}{(x-2)^4} dx & 2027. \int \frac{dx}{x^4 - x^2} \\
 2028. \int \frac{x^2 dx}{(x+2)^2 (x+4)^2} & 2029. \int \frac{x^3 - 6x^2 + 9x + 7}{(x-2)^3 (x-5)} dx \\
 2030. \int \frac{1}{8} \left(\frac{x-1}{x+1}\right)^4 dx & 2031. \int \frac{x^5 dx}{(x-1)^2 (x^2-1)} \\
 2032. \int \frac{(x^2 - 2x + 3) dx}{(x-1)(x^3 - 4x^2 + 3x)} & 2033. \int \frac{(7x^3 - 9) dx}{x^4 - 5x^3 + 6x^2} \\
 2034. \int \frac{x^3 - 2x^2 + 4}{x^3 (x-2)^2} dx & 2035. \int \frac{3x^2 + 1}{(x^2 - 1)^3} dx
 \end{array}$$

3) El denominador tiene distintas raíces complejas.

$$\begin{array}{ll}
 2036. \int \frac{dx}{x(x^2 + 1)} & 2037. \int \frac{dx}{1 + x^3} \quad 2038. \int \frac{x dx}{x^3 - 1} \\
 2039. \int \frac{(2x^2 - 3x - 3) dx}{(x-1)(x^2 - 2x + 5)} & 2040. \int \frac{(x^4 + 1) dx}{x^3 - x^2 + x - 1} \\
 2041. \int \frac{x^2 dx}{1 - x^4} & 2042. \int \frac{dx}{(x^2 + 1)(x^2 + x)} \\
 2043. \int \frac{dx}{(x+1)^2 (x^2 + 1)} & 2044. \int \frac{(3x^2 + x + 3) dx}{(x-1)^3 (x^2 + 1)} \\
 2045. \int \frac{x^5 + 2x^3 + 4x + 4}{x^4 + 2x^3 + 2x^2} dx & 2046. \int \frac{(x^3 - 6) dx}{x^4 + 6x^2 + 8} \\
 2047*. \int \frac{dx}{1 + x^4} &
 \end{array}$$

4) El denominador tiene raíces complejas múltiples.

$$\begin{array}{ll}
 2048. \int \frac{x^3 + x - 1}{(x^2 + 2)^2} dx & 2049. \int \frac{dx}{x(4 + x^2)^2 (1 + x^2)} \\
 2050. \int \frac{(5x^2 - 12) dx}{(x^2 - 6x + 13)^2} & 2051. \int \frac{(x+1)^4 dx}{(x^2 + 2x + 2)^3} \\
 2052. \int \frac{dx}{(x^2 + 9)^3} & 2053. \int \frac{2x dx}{(1+x)(1+x^2)^2} \\
 2054. \int \frac{dx}{(1+x^2)^4} & 2055. \int \frac{x^9 dx}{(x^4 - 1)^2}
 \end{array}$$

5) Método de Ostrogradski.

$$\begin{array}{ll}
 2056. \int \frac{x^2 + 2}{(x^2 + x + 1)^2} dx & 2057. \int \frac{(4x^2 - 8x) dx}{(x-1)^2 (x^2 + 1)^2}
 \end{array}$$

$$\begin{array}{ll}
 2058. & \int \frac{x^2+x+1}{x^5-2x^4+x^3} dx. \\
 2060. & \int \frac{(x^2-1)^2 dx}{(1+x)(1+x^2)^3}. \\
 2062. & \int \frac{dx}{(x^2+2x+10)^3}. \\
 2064. & \int \frac{x^5-x^4-26x^2-24x-25}{(x^2+4x+5)^2(x^2+4)^2} dx. \\
 2065. & \int \frac{3x^4+4}{x^2(x^2+1)^3} dx. \\
 2066. & \int \frac{5-3x+6x^2+5x^3-x^4}{x^5-x^4-2x^3+2x^2+x-1} dx. \\
 2067. & \int \frac{-9 dx}{5x^2(3-2x^2)^3}.
 \end{array}
 \quad
 \begin{array}{l}
 2059. \int \frac{x^6+x^4-4x^2-2}{x^3(x^2+1)^2} dx. \\
 2061. \int \frac{dx}{x^4(x^3+1)^2}. \\
 2063. \int \frac{(x+2) dx}{(x^2+2x+2)^3}.
 \end{array}$$

Algunas funciones irracionales

En los ejercicios 2068—2089 hallar las integrales.

1) Funciones de la forma $R\left(x, \sqrt[m]{\frac{ax+b}{a_1x+b_1}}, \dots\right)$.

$$\begin{array}{ll}
 2068. & \int \frac{dx}{x(\sqrt{x}+\sqrt[3]{x^2})}. \\
 2070. & \int \frac{x dx}{(x+1)^{\frac{1}{2}}+(x+1)^{\frac{1}{3}}}. \\
 2072. & \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx. \\
 2074. & \int \sqrt[3]{\frac{1-x}{1+x}} \frac{dx}{x}. \\
 2069. & \int \frac{dx}{\sqrt{x}+\sqrt[3]{x}+2\sqrt[4]{x}}. \\
 2071. & \int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x}. \\
 2073. & \int \frac{x^2+\sqrt{1+x}}{\sqrt[3]{1+x}} dx. \\
 2075*. & \int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}}.
 \end{array}$$

2) Binomios diferenciales $x^m(a+bx^n)^p dx$.

$$\begin{array}{ll}
 2076. & \int \sqrt{x}(1+\sqrt[3]{x})^4 dx. \\
 2078. & \int \frac{dx}{x\sqrt[3]{x^2+1}}. \\
 2080. & \int \frac{dx}{\sqrt[3]{1+x^3}}. \\
 2082. & \int \frac{\sqrt{1-x^4}}{x^5} dx. \\
 2077. & \int x^{-1}(1+x^{\frac{1}{3}})^{-3} dx. \\
 2079. & \int x^5 \sqrt[3]{(1+x^3)^2} dx. \\
 2081. & \int \frac{dx}{\sqrt[4]{1+x^4}}. \\
 2083. & \int \frac{\sqrt[3]{1+\sqrt{x}}}{\sqrt{x}} dx.
 \end{array}$$

$$\begin{array}{ll}
 2084. \int \frac{\sqrt[3]{1+\sqrt{x}}}{x} dx. & 2085. \int \frac{dx}{x^2 \sqrt{1+x^5}} \\
 2086. \int \frac{\sqrt[3]{1+x^3}}{x^2} dx. & 2087. \int \frac{dx}{x^{11} \sqrt{1+x^4}} \\
 2088. \int \sqrt[3]{x(1-x^2)} dx. & 2089. \int \sqrt[3]{1+\sqrt{x}} dx.
 \end{array}$$

Funciones trigonométricas

En los ejercicios 2090—2131, hallar las integrales.

$$\begin{array}{ll}
 2090. \int \sin^3 x \cos^2 x dx. & 2091. \int \frac{\sin^3 x}{\cos^4 x} dx. \\
 2092. \int \frac{dx}{\cos x \cdot \sin^3 x}. & 2093. \int \frac{\sin^4 x}{\cos^2 x} dx. \\
 2094. \int \frac{dx}{\cos^3 x \sin^3 x}. & 2095. \int \frac{dx}{\sin^4 x \cos^4 x}. \\
 2096. \int \frac{\sin x dx}{(1-\cos x)^2}. & 2097. \int \frac{\cos x dx}{(1-\cos x)^2}. \\
 2098. \int \cos^6 x dx. & 2099. \int \operatorname{ctg}^4 x dx. \\
 2100. \int \operatorname{tg}^5 x dx. & 2101. \int \frac{dx}{\operatorname{tg}^8 x}. \\
 2102. \int \frac{dx}{\sin^3 x}. & 2103. \int \frac{\cos^4 x + \sin^4 x}{\cos^2 x - \sin^2 x} dx.
 \end{array}$$

$$\begin{array}{ll}
 2104. \int \frac{dx}{(\sin x + \cos x)^2}. & 2105. \int \frac{dx}{\sin x + \cos x}. \\
 2106. \int \frac{dx}{a \cos x + b \sin x}. & 2107. \int \frac{dx}{\operatorname{tg} x \cdot \cos 2x}. \\
 2108. \int \frac{\cos^2 x dx}{\sin x \cdot \cos 3x}. & 2109. \int \frac{dx}{1 + \operatorname{tg} x}. \\
 2110. \int \frac{dx}{5 - 3 \cos x}. & 2111. \int \frac{dx}{5 + 4 \sin x}. \\
 2112. \int \frac{2 - \sin x}{2 + \cos x} dx. & 2113. \int \frac{\sin^2 x dx}{1 - \operatorname{tg} x}. \\
 2114. \int \frac{dx}{4 + \operatorname{tg} x + 4 \operatorname{ctg} x}. & 2115. \int \frac{dx}{(\sin x + 2 \operatorname{sec} x)^2}. \\
 2116. \int \frac{dx}{5 - 4 \sin x + 3 \cos x}. & 2117. \int \frac{dx}{4 - 3 \cos^2 x + 5 \sin^2 x}. \\
 2118. \int \frac{dx}{1 + \sin^2 x}. & 2119. \int \frac{dx}{1 - \sin^4 x}.
 \end{array}$$

$$\begin{array}{ll}
 2120. \int \frac{dx}{a^2 \operatorname{sen}^2 x + b^2 \cos^2 x} & 2121. \int \frac{dx}{\operatorname{sen}^2 x + \operatorname{tg}^2 x} \\
 2122. \int \frac{\cos x dx}{\operatorname{sen}^3 x - \cos^3 x} & \\
 \hline
 2123. \int \sqrt{1 + \operatorname{sen} x} dx & 2124. \int \frac{\sqrt{\operatorname{tg} x}}{\operatorname{sen} x \cos x} dx \\
 2125^*. \int \frac{\sqrt{\operatorname{sen}^3 2x}}{\operatorname{sen}^5 x} dx & 2126. \int \frac{dx}{\sqrt[4]{\operatorname{sen}^3 x \cos^5 x}} \\
 2127. \int \frac{dx}{\sqrt{1 - \operatorname{sen}^4 x}} & 2128. \int \sqrt{1 + \operatorname{cosec} x} dx \\
 2129. \int \frac{(\cos 2x - 3) dx}{\cos^4 x \sqrt{4 - \operatorname{ctg}^2 x}} & 2130. \int \frac{dx}{\operatorname{sen} \frac{x}{2} \sqrt{\cos^3 \frac{x}{2}}} \\
 2131. \int \sqrt{\operatorname{tg} x} dx &
 \end{array}$$

Funciones hiperbólicas

En los ejercicios 2132—2150 hallar las integrales.

$$\begin{array}{ll}
 2132. \int \operatorname{ch} x dx & 2133. \int \operatorname{sh} x dx \\
 2134. \int \frac{dx}{\operatorname{ch}^2 x} & 2135. \int \frac{e^x dx}{\operatorname{ch} x + \operatorname{sh} x} \\
 2136. \int (\operatorname{ch}^2 ax + \operatorname{sh}^2 ax) dx & 2137. \int \operatorname{sh}^2 x dx \\
 2138. \int \operatorname{th}^2 x dx & 2139. \int \operatorname{cth}^2 x dx \\
 2140. \int \operatorname{sh}^3 x dx & 2141. \int \operatorname{ch}^3 x dx \\
 2142. \int \operatorname{th}^4 x dx & 2143. \int \operatorname{sh}^2 x \operatorname{ch}^3 x dx \\
 2144. \int \operatorname{cth}^5 x dx & 2145. \int \frac{dx}{\operatorname{sh} x \operatorname{ch} x} \\
 2146. \int \frac{dx}{\operatorname{sh} x} & 2147. \int \frac{dx}{(1 + \operatorname{ch} x)^2} \\
 2148. \int \sqrt{\operatorname{th} x} dx & 2149. \int \frac{x dx}{\operatorname{ch}^2 x} \\
 2150. \int \frac{e^{2x} dx}{\operatorname{sh}^4 x} &
 \end{array}$$

Funciones racionales de x y $\sqrt{ax^2 + bx + c}$

En los ejercicios 2151—2174 hallar las integrales.

2151*.
$$\int \frac{dx}{x\sqrt{x^2+x+1}}$$

2152.
$$\int \frac{dx}{x\sqrt{x^2+4x-4}}$$

2153.
$$\int \frac{dx}{x\sqrt{x^2+2x-1}}$$

2154.
$$\int \frac{dx}{x\sqrt{2+x-x^2}}$$

2155.
$$\int \frac{\sqrt{2x+x^2}}{x^2} dx,$$

2156.
$$\int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$$

2157.
$$\int \frac{dx}{(2x-3)\sqrt{4x-x^2}}$$

2158.
$$\int \sqrt{x^2-2x-1} dx.$$

2159.
$$\int \sqrt{3x^2-3x+1} dx.$$

2160.
$$\int \sqrt{1-4x-x^2} dx.$$

2161.
$$\int \frac{dx}{x-\sqrt{x^2-x+1}}$$

2162.
$$\int \frac{dx}{x^2(x+\sqrt{1+x^2})}$$

2163.
$$\int \frac{dx}{1+\sqrt{x^2+2x+2}}$$

2164.
$$\int \frac{x^2 dx}{\sqrt{1-2x-x^2}}$$

2165.
$$\int \frac{(2x^2-3x) dx}{\sqrt{x^2-2x+5}}$$

2166.
$$\int \frac{3x^2-5x}{\sqrt{3-2x-x^2}} dx$$

2167.
$$\int \frac{3x^3 dx}{\sqrt{x^2+4x+5}}$$

2168.
$$\int \frac{x^3-x+1}{\sqrt{x^2+2x+2}} dx$$

2169.
$$\int \frac{3x^3-8x+5}{\sqrt{x^2-4x-7}} dx.$$

2170.
$$\int \frac{x^4 dx}{\sqrt{x^2+4x+5}}$$

2171.
$$\int \frac{dx}{(x^3+3x^2+3x+1)\sqrt{x^2+2x-3}}$$

2172.
$$\int \frac{\sqrt{1+x^2}}{2+x^2} dx.$$

2173.
$$\int \frac{(x-1) dx}{x^2\sqrt{2x^2-2x+1}}$$

2174.
$$\int \frac{(2x+3) dx}{(x^2+2x+3)\sqrt{x^2+2x+4}}$$

Diversas funciones

En los ejercicios 2175—2230 hallar las integrales.

2175.
$$\int \frac{x^3 dx}{(x-1)^{13}}$$

2176.
$$\int \frac{x dx}{x-\sqrt{x^2-1}}$$

2177.
$$\int x^3\sqrt{a+x} dx.$$

2178.
$$\int \frac{dx}{ae^{mx}+be^{-mx}}$$