

Final exam, ODE and Linear Algebra, Section 001. Fall 2013

1) Determine the order of the given ode and state whether it is linear or nonlinear.

$$4\frac{d^3y}{dx^3} + \cos x \frac{d^2y}{dx^2} + e^x \frac{dy}{dx} + y = x^9.$$

a)3,linear. b)4,nonlinear. c)3,nonlinear d)1,linear. e) 6,nonlinear.

2) Determine all values of the constant r such that the given function solves the given ode,

$$y(x) = e^{rx}, y'' - 8y' + 16y = 0.$$

a)4 b)3 c)2 d)1. e) 0

3) Solve $y'(t) = y$, $y(0) = 2$.

a) $y = 2$ b) $y = 2t$ c) $y = t+2$ d) $y = e^{2t}$ e) $y = 2e^t$

4) Solve $(1 - x^2)y' + xy = ax$, $y(0) = 2a$.

a) $y = 2ae^{-x^2}$ b) $y = a + ae^{-x^2}$

c) $y = a(1 + \frac{1}{\sqrt{1-x^2}})$ d) $y = a(1 + \sqrt{1-x^2})$ e)) $y = 2a$

5) Solve $y' + y = e^x$, $y(0) = 1$.

a) $y = e^x$ b) $y = e^{2x}$ c) $(e^x - e^{-x})/2$ d) $y = \frac{e^x + e^{-x}}{2}$ e) $y = e^x + e^{-x}$

6) Solve $y' = 1$, $y(0) = 0$.

a) $y = 0$ b) $y = x$ c) $y = x + 1$ d) $y = 1$ e) $y = xe^x$

7) Find the set of all the solutions of the linear system,

$$3x_1 + 2x_2 + x_3 = 0$$

$$6x_1 - x_2 + 2x_3 = 0$$

$$12x_1 + 6x_2 + 4x_3 = 0.$$

a) $(0, 1, -1/2)$ b) $(t, 0, -6t)$ c) $(t, 0, -3s)$ d) $(t, 0, -3t)$ e) $(0,0,0)$

8) Find the set of all the solutions of $x + y + z = 1$

a) $(0, 0, 1)$ b) $(1, 0, 0)$ c) $(1/3, 1/3, 1/3)$ d) $(1 - t - s, t, s)$ e) Empty

9) Decide whether p_1, p_2 is a spanning set for P_3 , if $p_1 = 1 + 3x$, $p_2 = x + x^2$

a) No b) Yes

10) If V is the vector space of **all real-valued functions defined on the interval $[a,b]$** , and S is the subset of V consisting of all functions satisfying $f(a)=f(b)$. Determine whether S is a subspace of the given vector space V .

a) Yes b) No.

11) Determine the matrix of the transformation $T(x_1, x_2, x_3) = \langle x_1 + 5x_2 - 3x_3, x_2 \rangle$.

12) If $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear transformation with matrix A . Find $\text{Ker}(T)$ and $\text{Rng}(T)$ and give a geometrical description of each. Also, find $\dim[\text{Ker}(T)]$ and $\dim[\text{Rng}(T)]$.

$A =$

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 6 & 5 \end{pmatrix}$$

13) Check if $T(x, y) = \langle x + y, 1 \rangle$ is a linear transformation.

14) **Problem 1. Find the general solution to the differential equation.**

$$y''(x) + 16y(x) = 4\cos x.$$

15) **Solve. Interpret what happens with the solution after a long time ($t \rightarrow \infty$). Sketch.**

$$\begin{aligned}\frac{dx_1}{dt} &= -2x_1 - 7x_2 \\ \frac{dx_2}{dt} &= -x_1 + 4x_2\end{aligned}$$

(1)

16) Sketch the **slope field** and **3 representative solution curves** for the given ODE.

$$y' = x^2 \cos y. \text{ (20pts)}$$