ROBUST FILTERS FOR FAULT DETECTION AND DIAGNOSIS: AN $\mathcal{H}_\infty$ OPTIMIZATION APPROACH

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Keywords: Fault diagnosis. Robust fault detection and diagnosis filter . $\mathcal{H}_\infty$ optimal control. Filtering $\mathcal{H}_\infty$. Robust estimation.

Abstract

In this paper, the design of filters for fault detection and diagnosis using a post filter and an $\mathcal{H}_\infty$ optimization approach is investigated. The problem is translated into an $\mathcal{H}_\infty$ output feedback control and some classical methods can be used to solve it. The design of the detection filter is done in two steps, and the filter allows the detection and the isolation of multiple faults, in presence of perturbation and uncertainties.

1 Introduction

The control of complex systems needs the use of sophisticated apparatus which guarantee acceptable security and productivity levels. Among those, the process monitoring system in which we can find elements based on diagnosis and fault detection mechanism, has an important role.

One of these elements is the fault detection and isolation filter (FDI) which has to be designed to operate in an uncertain environnement, (perturbations, model uncertainties, etc.). In this context, there are important connections between robust estimation or filtering and FDI filter design. In general, the problem consists in designing an asymptotic stable dynamical system (filter) able to cope with perturbations (rejection, for example).

In the robust estimation problem, the goal is to derive an optimal estimate of the system state vector or a linear combination of the states taking into account the presence of perturbation and uncertainties (model). For the robust $\mathcal{H}_\infty$ estimation problem, the $\mathcal{H}_\infty$ norm of the transfer matrix from the perturbations to the error estimation is lower than a prespecified level $\gamma > 0$: [Nagpal *et al.*, 1991]; [Khargonekar *et al.*, 1992].

For robust FDI filter design, the first step consists in the generation of residuals, used in a second step for the diagnosis process. The residuals are generated by a dynamical system (filter) and are only significative when the system is affected by a fault. The input of this dynamical system is a measured output and it has to be able to distinguish if the qualitative changes on the system behavior are due to perturbations, uncertainties or to faults. [Edelmayer *et al.*, 1994], [Patton *et al.*, 1997], [Ríos-Bolívar *et al.* 1999].

The relations between the $\mathcal{H}_\infty$ robust estimation problem and FDI filter design has been investigated in [Edelmayer *et al.*, 1994]. In this case the addressed problem is the fault detection in presence of perturbations; the diagnosis problem is considered in a second level and the fault separation is obtained through a multiple filtering, [Niemann and Stoustrup, 1998].

A way to consider robustness in the FDI filter design can be to define a sensitivity measure which characterizes the filter sensitivity with respect to the possible faults in comparison to the filter sensitivity with respect to perturbations.

Let us introduce

$$\mathcal{S}_i = \frac{||H_{e_z\nu_i}||_\infty}{||H_{e_z\omega}||_\infty};$$

where ν_i are the signals characterizing the faults, ω is the perturbation and e_z the error estimation. It is clear that if for a given i, $\mathcal{S}_i$ is significative, this means that for fault i, the filter is more sensitive to ν_i than to ω. The problem consists in designing a filter which in some sense maximizes $\mathcal{S}_i$: [Edelmayer *et al.*, 1994], [Niemann and Stoustrup, 1998], [Ríos-Bolívar *et al.* 1999]. It is also clear that the problem can be formulated as a multi objective design problem and multiple filters are necessary in this case to solve the fault separation problem.

In this paper, a method is proposed for the design of a robust FDI filter. The particularity of the approach is the use of a post filter which in connection with the robust fault detection filter, obtained by solving an $\mathcal{H}_\infty$ design problem, allows to solve simultaneously the fault detection and diagnosis problems.

The paper is organized as follows. Section 2 adresses the $\mathcal{H}_\infty$

filtering problem and introduces the architecture based on the use of a post filter. Section 3 introduces the robust filter design based on the proposed architecture with the post filter. A numerical example illustrating the method is developed in Section 4. We end the paper by a conclusion.

2 On $\mathcal{H}_\infty$ optimal filtering

Consider the following system:

$$\Sigma_1 \left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B_1\omega(t) \\ z(t) & = & C_1 x(t) \\ y(t) & = & C_2 x(t) + D\omega(t), \end{array} \right. \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $z \in \mathbb{R}^m$ is the signal to be estimated from the measured signal $y \in \mathbb{R}^p$; $\omega \in \mathcal{L}_2$ is a perturbation. The matrices A, B_1, C_1, C_2 and D are matrices of appropriate dimensions. The pair (A, B_1) is stabilizable and pair (A, C_2) is detectable.

The problem considered here is to find an estimate $\hat{z}$ of z, where $\hat{z}$ is the output of a dynamical system (filter) satisfying the following conditions:

1. The filter is asymptotically stable.
2. The effect of the perturbation on the estimation error is as small as possible.

More explicitly, if $\mathcal{F}$ denotes the filter, we have:

- $\hat{z}(t) = \mathcal{F}y(t)$.
- If we define
$$e_z(t) = z(t) - \hat{z}(t)$$
and if $\omega(t) = 0$, then
$$\lim_{t\to\infty} e_z(t) = 0.$$
- If the transfer matrix from ω to e_z is denoted by $H_{e_z\omega}$ we want:
$$||H_{e_z\omega}||_\infty = \sup_{0\neq\omega\in\mathcal{L}_2} \frac{||e_z||_2^2}{||\omega||_2^2} < \gamma, \quad \gamma > 0.$$

Under the considered assumptions, an admissible filter can be expressed as, [9, 13]:

$$\mathcal{F}_L \left\{ \begin{array}{rcl} \dot{\hat{x}}(t) & = & A\hat{x}(t) + L(y(t) - C_2\hat{x}(t)) \\ \hat{z}(t) & = & C_1\hat{x}(t), \end{array} \right. \quad (2)$$

where L is a gain matrix to be designed. Defining $e_x(t) = x(t) - \hat{x}(t)$, we have:

$$\begin{array}{rcl} \dot{e}_x(t) & = & (A - LC_2)e_x(t) + (B_1 - LD)\omega(t) \\ e_z(t) & = & C_1 e_x, \end{array}$$

in which $(A - LC_2)$ has to be asymptotically stable. It is well known [8, 9], that $||H_{e_z\omega}||_\infty < \gamma$ if only if there exists a positive definite solution $\mathcal{X}$ to

$$(A - LC_2)^T\mathcal{X} + \mathcal{X}(A - LC_2) + \frac{1}{\gamma^2}\mathcal{X}(B_1 - LD)(B_1 - LD)^T\mathcal{X} + C_1^T C_1 = 0$$

and

$$(A - LC_2) + \frac{1}{\gamma^2}(B_1 - LD)(B_1 - LD)^T$$

is asymptotically stable.

We can note that an optimal solution for the problem consists in finding L such that $(A - LC_2)$ is asymptotically stable and $B_1 = LD$. We have the following theorem, [13, 20], considering $DD^T = I$, $DB_1^T = 0$:

Theorem 2.1 *There exists a filter satisfying all the previous conditions if and only if there exists a symmetrical matrix $\mathcal{X} > 0$, solution of the algebraic Riccati equation*

$$A\mathcal{X} + \mathcal{X}A^T - \mathcal{X}\left(C_2^T C_2 - \frac{1}{\gamma^2}C_1^T C_1\right)\mathcal{X} + B_1 B_1^T = 0. \quad (3)$$

A filter gain is given by

$$L = \mathcal{X}C_2^T. \quad (4)$$

With the considered filter, the transfer matrix $H_{e_z\omega}(s)$ is given by

$$\begin{array}{rcl} H_{e_z\omega}(s) & = & \left[\begin{array}{c|c} A - LC_2 & B_1 - LD \\ \hline C_1 & 0 \end{array}\right] \\ & = & C_1\left(sI - A + LC_2\right)^{-1}(B_1 - LD). \end{array}$$

In this way, under the previous assumptions, a dynamical system can be designed to estimate the signal z and ensuring a certain perturbation rejection level.

Another important solved problem, in the context of $\mathcal{H}_\infty$ optimization, is the design of dynamic output controllers ensuring a certain level of rejection of a perturbation, [8, 7].

3 The post filter design

In connection with the FDI filter design problem, in [12] two techniques based on the multi objective design method are presented. The major problem is related to faults separability which is solved with the use of multiple filters. Roughly speaking, one can say that only the fault detection problem is considered. Another drawback is that the filter design is not systematic.

In the FDI filter design, if the fault separability conditions are met, and if one filter is used, it is difficult to guarantee asymptotic stability and fault isolation simultaneously. This is why in some works a solution consists in using multiple filters. The idea in this section, is to use in conjunction with the filter, a post filter.

The post filter is a dynamical system whose input is the innovation signal defined as the difference between the output and its estimate. We denote the post filter by $\mathcal{F}_p$. We consider the filter $\mathcal{F}_L$ defined by:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + L(y(t) - C_2\hat{x}(t)) - B_e u_e(t)\\ \hat{z}(t) &= C_1\hat{x}(t);\end{aligned} \tag{5}$$

where $u_e(t)$ is the output of the post filter $\mathcal{F}_p$, and B_e is its input matrix of appropriate dimension. The error dynamic is obtained manipuling straightforwardly the filter equation (5). It is given by (see Figure 1):

$$\begin{aligned}\dot{e}_x(t) &= (A - LC_2)e_x(t) + (B_1 - LD)\omega(t) + B_e u_e(t)\\ e_z(t) &= C_1 e_x(t).\end{aligned} \tag{6}$$

First, we can note that if $B_e = -(B_1 - LD)$ and $u_e = \omega$, we can isolate completely the error from perturbation ω. The reconstruction of ω can be obtained from (1), considering $V = D^T D$ non singular, by the inverse system

$$\begin{aligned}\dot{\zeta}(t) &= (A + B_1 V^{-1} D^T C_2)\zeta(t) - B_1 V^{-1} D^T y(t)\\ u_e(t) &= V^{-1} D^T C_2 \zeta(t) - V^{-1} D^T y(t);\end{aligned}$$

where $u_e(t)$ can be considered as an estimate of $\omega(t)$. In this way, a good rejection level can be attained and this is the pursued idea in the introduction of a post filter. Introduce the innovation signal

$$\begin{aligned}e_y(t) &= y(t) - C_2\hat{x}(t)\\ &= C_2 e_x(t) + D\omega(t);\end{aligned}$$

and define the dynamical post filter equation

$$\mathcal{F}_p \left\{ \begin{aligned}\dot{\zeta}(t) &= F\zeta(t) + G e_y(t)\\ u_e(t) &= H\zeta(t) + J e_y(t),\end{aligned}\right. \tag{7}$$

where F, G, H and J are matrices of appropriate dimensions to be designed. In closed loop we obtain

$$\left\{ \begin{aligned}\dot{e}_x(t) &= (A - LC_2 + B_e J C_2)e_x(t) +\\ &\quad B_e H\zeta(t) + (B_1 - LD + B_e J D)\omega(t)\\ \dot{\zeta}(t) &= G C_2 e_x(t) + F\zeta(t) + G D\omega(t)\\ e_z(t) &= C_1 e_x(t).\end{aligned}\right.$$

As one can see, it is possible to select now B_e and L, and after determine F, G, H and J such that

$$\begin{pmatrix} A - LC_2 + B_e J C_2 & B_e H\\ G C_2 & F \end{pmatrix}$$

be asymptotically stable and $||H_{e_z\omega}||_\infty < \gamma$. If $B_e = 0$ we recover the previous case. We can note that in some cases $L = 0$ leads to a solution.

In summary, the problem to be solved is the design of a post filter $\mathcal{F}_p$, whose output is u_e. Introduce the error equations:

$$\left\{ \begin{aligned}\dot{e}_x(t) &= (A - LC_2)e_x(t) + (B_1 - LD)\omega(t) + B_e u_e(t)\\ e_z(t) &= C_1 e_x(t)\\ e_y(t) &= C_2 e_x(t) + D\omega(t).\end{aligned}\right. \tag{8}$$

The problem is to design a control u_e obtained, from the output e_y, in such a way the $\mathcal{H}_\infty$ norm of the transfer matrix from the perturbation ω to the controlled output e_z be minimum. This is typically a well known $\mathcal{H}_\infty$ optimal control design for which a solution is given in [2, 7, 10, 17]. Then, we have translated the FDI filter design to an $\mathcal{H}_\infty$ optimal control design.

With this formulation, the transfer matrix $H_{e_z\omega}(s)$, is given by

$$H_{e_z\omega}(s) = \left[\begin{array}{c|c} \mathfrak{A} & \mathfrak{B}\\ \hline (C_1 \quad 0) & 0\end{array}\right];$$

where

$$\begin{aligned}\mathfrak{A} &= \begin{pmatrix} A - LC_2 + B_e J C_2 & B_e H\\ G C_2 & F\end{pmatrix}\\ \mathfrak{B} &= \begin{pmatrix} B_1 - LD + B_e J D\\ G D\end{pmatrix}\end{aligned}$$

To design $\mathcal{F}_p$, the two steps procedure is the following

- Choose L.
- Solve the $\mathcal{H}_\infty$ optimal control problem, selecting B_e in order to satisfy the main assumptions guaranteeing that the problem has a solution. For example, we can select B_e as a linear combination of columns of $B_1 - LD$. We obtain solving this problem $\mathcal{F}_p$.

The Figure 1 gives the architecture of the post filter.

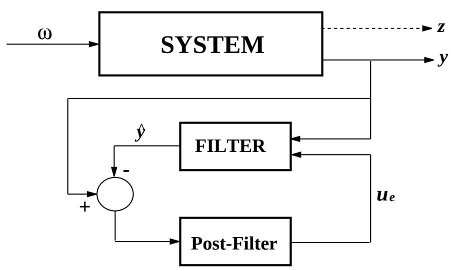

Figure 1: Post filter scheme.

3.1 Robust FDI design

In this paragraph, we move to the case where some one bad operation affects temporary the behavior of the system. Let us introduce the system model:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1\omega(t) + B_2 u(t) + F_i\\ y(t) &= C_2 x(t) + D\omega(t),\end{aligned} \tag{9}$$

where $x(t)$, $u(t)$, and $y(t)$ are previously defined and F_i are the faults, which can be represented by

$$F_i = \sum_{i=1}^{f} L_i \nu_i(t). \tag{10}$$

L_i are *fault directions* and $\nu_i(t)$, is a signal characterizing the *fault mode*.

To detect the faults, it is necessary to generate residuals obtained from the estimation of the following signal:

$$z(t) = C_1 x(t),$$

using a state estimator. As stated in the introduction, we want to minimize $||H_{e_z\omega}||_\infty$ and to maximize $||H_{e_z\nu_i}||_\infty$, for $i = 1,\dots,f$, where $H_{e_z\nu_i}$ are transfer matrices from ν_i to e_z. It is difficult to take simultaneously these requirements, [14].

The conditions ensuring that faults are detectable and separable are given by, [15, 19]:

$$\ker(C_1 L_i) = 0, \qquad i = 1,\dots,f$$
$$Im(C_1L_i) \bigcap \left(\sum_{i,j=1_{j\neq i}}^{f} Im(C_1 L_j)\right) = 0,$$

Using the results of the previous section, we can write:

$$\left\{ \begin{array}{rcl} \dot{e}_x(t) & = & (A - LC_2)e_x(t) + (B_1 - LD)\omega(t) + \\ & & B_e u_e(t) + \sum_{i=1}^{f} L_i \nu_i(t) \\ e_z(t) & = & C_1 e_x(t) \\ e_y(t) & = & C_2 e_x(t) + D\omega(t). \end{array} \right. \tag{11}$$

The B_e matrix has to be selected in a way guaranteeing that the $\mathcal{H}_\infty$ optimal control consisting in minimizing the $\mathcal{H}_\infty$ norm of the transfer matrix function between ω and e_z problem has a solution.

Remark 3.1

- *The first step consists in selecting L in order to associate to each fault a particular direction and ensure faults separability and detectability. This can be done, if it is possible, diagonalising the matrix $A - LC_2$, which can be unstable.*
- *The matrix B_e has to be chosen to guarantee the solvability of the $\mathcal{H}_\infty$ optimal control problem.*
- *When $D = 0$, the results obtained using a post filter and the ones based on Theorem 2.1 are similar. In this case the results presented in [15] can be applied.*
- *The case which leads to the best results corresponds to the situation where the directions of perturbations are independent of the faults ones.*
- *Results can be extended to systems represented by*

$$\begin{array}{rcl} \dot{x}(t) & = & A(t)x(t) + B_1\omega(t) + B_2 u(t) + F_i \\ y(t) & = & C_2 x(t), \end{array}$$

where $A(t) = A_0 + \sum_{i=1}^{g} a_i(t) A_i$; A_0 stable matrix, A_i are non destabilizing terms, and $a_i(t) \in \mathcal{L}_2$, [6, 18].

- *It is clear that the $\mathcal{H}_\infty$ optimal control problem can be solved by Riccati equation approach or by LMI machinery.*

4 Numerical example

Consider the following state equation

$$\begin{array}{rcl} \dot{x} & = & \begin{pmatrix} 0 & -102 & 0 \\ 181 & -171 & 0 \\ 0 & -1.12\times 10^{-2} & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 4.44 \\ 0 \end{pmatrix} \omega + \\ & & \begin{pmatrix} 102 \\ 163 \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ 181 \\ 0 \end{pmatrix} \nu_1 \\ y & = & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0.978 \end{pmatrix} \nu_2; \end{array}$$

which is the model of a diesel engine actuator, [1]. Two faults are considered: fault on the actuator or fault on the sensor. We suppose that:

$$z = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} x.$$

The first step consists in selecting L, we choose, in order to obtain $A - LC_2$ decoupled in relation to the faults:

$$L = \begin{pmatrix} -102 & 0 \\ 0 & 0 \\ 1.12\times 10^{-2} & 10 \end{pmatrix};$$

and the dynamic of the error estimation is described by:

$$\begin{array}{rcl} \dot{e}_x & = & \begin{pmatrix} 0 & 0 & 0 \\ 181 & -171 & 0 \\ 0 & 0 & -9.78 \end{pmatrix} e_x + \begin{pmatrix} 0 \\ 4.44 \\ 0 \end{pmatrix} \omega + \\ & & \begin{pmatrix} 0 \\ 181 \\ 0 \end{pmatrix} \nu_1 + \begin{pmatrix} 0 \\ 0 \\ -0.978 \end{pmatrix} \nu_2 + B_e u_e \\ e_z & = & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} e_x \\ e_y & = & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} e_x + \begin{pmatrix} 0 \\ 0.978 \end{pmatrix} \nu_2 \end{array}$$

We can note that in order to ensure the fault separability, one fault is associated to each output error. To solve the $\mathcal{H}_\infty$ optimization problem presented in the previous section, we select B_e as

$$B_e = \begin{pmatrix} 0.1 \\ 1 \\ 0 \end{pmatrix}.$$

Using the LMI approach to solve the $\mathcal{H}_\infty$ optimization problem, [7], we obtain

$$F = \begin{pmatrix} -160.03 & 256.28 & 0 \\ -92.72 & -90.59 & 0 \\ 0 & 0 & -175.07 \end{pmatrix}$$

$$G = \begin{pmatrix} 54.44 & 0 \\ 165.66 & 0 \\ 0 & 134.82 \end{pmatrix}$$

$$H = \begin{pmatrix} -21.21 & -13.70 & 0 \end{pmatrix} \quad J = 0.$$

Thus, the Figure 2 represents the frequency responses of the transfer matrices from the fault to the estimation error ($H_{e_z \nu_i}$) and from the perturbation to the estimation error ($H_{e_z \omega}$). We can note that $\mathcal{S}_i = \frac{||H_{e_z \nu_i}||_\infty}{||H_{e_z \omega}||_\infty}$ is significative, the perturbation is rejected.

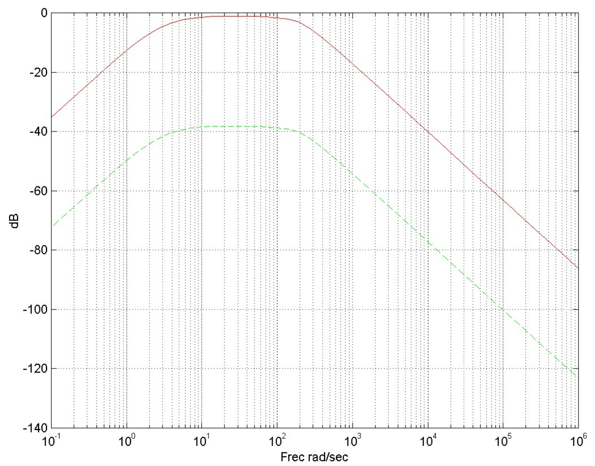

Figure 2: Singular values diagram: $H_{e_z \nu_1}$ (solid line), $H_{e_z \omega}$ (dotted line).

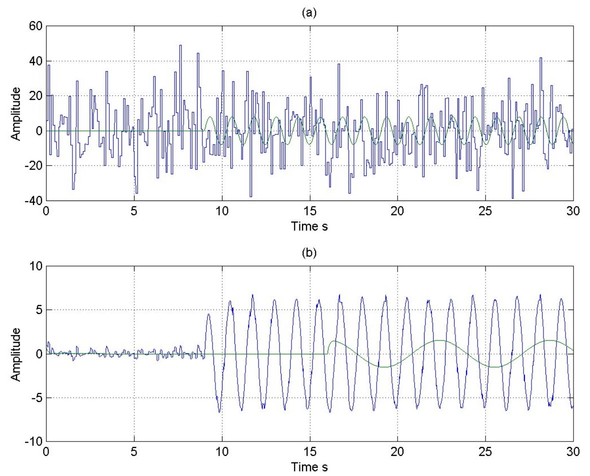

Figure 3: Filter responses in presence of faults and perturbation.

Figure 3 represents the estimation errors with the perturbation and in presence of faults. Figure 3 (a) shows the perturbation and a fault affecting the actuator at $t = 9s$. The perturbation magnitude is important.

Figure 3 (b) shows the presence of a fault in the sensor at $t = 16s$. In the same figure we can see that residuals are significatives when the faults occur at $t = 9s$ and at $t = 16s$, and that each residual is associated with a fault guaranteeing faults separability. We can also note that the perturbation is rejected.

5 Conclusion

In this paper, we have presented a method to design robust filters for fault detection and isolation. This method is based on two steps. The first step consists in designing a full state observer in a way ensuring faults separability. In a second step, a post filter is designed to ensure asymptotic stability of the error dynamic and rejection of the perturbation. The problem can be translated in an $\mathcal{H}_\infty$ optimal control problem which can be solved using standard methods.

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