SYSTEM INVERSION AND FAULT DETECTION:
THE FAILURE AFFINE NONLINEAR CASE

F. Szigeti†, A. Ríos-Bolívar†
† Universidad de Los Andes
Facultad de Ingeniería
Departamento de Control
Av. Tulio Febres Cordero
Mérida 5101 - Venezuela
fax: +58 274 240 2846
e-mail: szigeti@ula.ve, ilich@ula.ve

Abstract

In this paper, nonlinear system inversion is revisited in order to apply it to fault detection and isolation problem. The considered concept of invertibility is less conservative than ones known in the literature, and the invertibility condition, in spite that a state space algorithm, is proposed, does not depend on the concrete state space realization. Fault detectability concepts based on that inversion algorithm hence neither will depend on the state space realization. The main difficulty is in the residual generation combined by inversion.

Keywords: Fault detection and Isolation, Nonlinear System, System Inversion, State Elimination.

1 Introduction

The main purpose of this paper is to present a design procedure to FDI filters for failure affine nonlinear systems based on system inversion. System inversion or dynamic inverse is very intrinsic concept of the control theory and practice. Instead of list exhaustively examples for our opinion, some examples will be recounted with respect to the basic concepts of FDI.

1. The input observability, see [7] of LTI systems is “almost” equivalent to their left invertibility. Indeed, the input observability of

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t),
\end{align*}
\]

is equivalent to the following property: The outputs \( y_1, y_2 \), corresponding to the initial states \( x_1, x_2 \) and inputs \( u_1, u_2 \), respectively, are equal, if and only if \( x_1 \) and \( x_2 \) are undistinguishable and \( u_1 = u_2 \) [13].

More direct relationship can be established between these concepts, if the input set will be restricted to the smooth inputs

\[
U = \{ u; u(0) = 0, \ldots, u^{(n-1)}(0) = 0 \}.
\]

Indeed in this case input observability and system invertibility are equivalent. In applications to FDI the failure modes belong to \( U \) by a natural way.

2. Consider a detectable diagnostic LTI system

\[
\begin{align*}
\dot{x} &= Ax + Bu + \sum_{i=1}^{m} L_i \nu_i \\
y &= Cx + Du + \sum_{i=1}^{m} M_i \nu_i.
\end{align*}
\]

Failures \( \nu_1, \nu_2, \ldots, \nu_m \) are detectable if and only if there exists a gain matrix \( G \), such that

\[
\begin{align*}
\dot{e} &= (A - GC)e + (L_i - GM_i) \nu_i \\
\eta &= Ce + M_i \nu_i,
\end{align*}
\]

are invertible.
Failures $\nu_1, \nu_2, \ldots, \nu_m$ are detectable and isolable if and only if there exists a gain matrix $G$ such that

$$
\dot{e} = (A - GC)e + \sum_{i=1}^{m} (L_i - GM_i)\nu_i,
$$

$$
\eta = Ce + \sum_{i=1}^{m} M_i\nu_i
$$
is invertible from $\nu = (\nu_1, \nu_2, \ldots, \nu_m)$ to $\eta$, [13].

The first results on system inversion are dated in the 60’s, [14]. The inversion of input affine systems is due to Hirschorn, [6]. In 1986, invertibility of nonlinear input-output differential systems is considered by M. Fliess, [4]. The transformation in canonical form of an input affine system (Isidori [8]) is also related to nonlinear system inversion.

Instead of mention more and more recent papers on dynamic inversion, just two remark will be mentioned.

1. The Hirschorn’s paper [6], because of his invertibility concept formulated in state space terms: the exceptional states, where invertibility fails must be a closed nowhere dense subset of the state space. However, invertibility may also fail for certain input values, as it is shown in one of the Hirschorn’s example, [6].

The very inconvenience is in the fact that, the invertibility of the input-output map is the most natural, and the algorithms and the conditions for invertibility depends on the particularity of a state space realization.

2. The invertibility condition in Isidori [8] is very restrictive, admitting only a one-step algorithm in our framework.

In the last years many papers pay attention to extend the linear FDI filter design to nonlinear diagnostic systems. However, in contrast to the linear case, there exist few results on the fault detection and isolation (FDI) problem for the nonlinear systems, [1, 3, 10, 5, 19]. In general, the well-known linear methods are looked for extend to nonlinear systems [9, 18].

With respect to system inversion applied to the FDI filters design, some results have been obtained in [16, 15, 12, 11, 17], for linear and nonlinear diagnostic models with applications. In this paper an inversion based FDI filter scheme is given, with the conditions for invertibility in terms of the inputs $u, \nu_1, \ldots, \nu_m$ and output $y$.

Examples show the power of system inversion in FDI.

## 2 FDI and system inversion

FDI based on analytic redundancy requires the knowledge of an appropriate diagnostic model, the failure signatures, or failure directions in order to design residual generators. The diagnostic model considered in this paper will be affine in the failure modes:

$$
\dot{x}(t) = f(x, u) + \sum_{i=1}^{m} g_i(x, u)\nu_i(t), \quad x(0) = x_0
$$

$$
y(t) = h(x);
$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^r$ is the control input, $y \in \mathbb{R}^m$ is the output vector. The functions $f, g_i, h$ are of appropriate dimensions. $f, g_i, h$ are algebraic (analytic) functions, respectively.

The varying failure signatures of (2) are $g_i(x, u)$ and the unknown failure modes are $\nu_i$. In this context, the detectability of one fault, $\nu_i$, can be defined by:

**Definition 2.1** The fault $\nu_i$, $i = 1, \ldots, m$, is said to be non detectable if for $\nu_i \neq 0$ the relation

$$
y(x_0, x, u, 0) = y(x_0, x, u, 0, \ldots, \nu_i, \ldots, 0)
$$

is satisfied; otherwise the fault $\nu_i$ is said detectable.

Consequently, if the fault $\nu_i$ is detectable, the residues can be obtained from the difference between the output $y(x_0, x, u, \nu_i)$ and the output $y(x_0, x, u, 0)$, i.e.

$$
\eta(t) = y(x_0, x, u, 0, \ldots, \nu_i, \ldots, 0) - y(x_0, x, u, 0).
$$

The residual generator is an other dynamic system, with inputs $u, y$,

$$
\dot{e}(t) = F(e, u, \dot{u}, \ldots, y, \dot{y}, \ldots)
$$

$$
\eta(t) = H(e, u, \dot{u}, \ldots, y, \dot{y}, \ldots)
$$

(3)
Therefore, a residual $\eta(t)$ is a time function, which is ideally zero in absence of faults and different to zero in the presence of any fault.

In this moment the method to design a residual generator is irrelevant. If $\nu(t) = (\nu_1 \cdots \nu_m)^T = 0$, then $H(0, u, \dot{u}, y, \dot{y}, \ldots) = 0$ must hold, and the corresponding error $e(t) \to 0$ if $t \to \infty$. Then, FDI is based on the reconstruction of the unknown inputs $\nu(t) = (\nu_1 \cdots \nu_m)^T = 0$, from the coupled system (2) and (3), using the knowledge of $u, y$ and $\eta$. The reconstruction of $\nu$ may take place in different information granulation, for example, $\nu$ and $\eta$ of system (2) and (3), using the knowledge of $u, y$ and $\eta$. The reconstruction of $\nu$ may take place in different information granulation, for example, $\nu$, or ones of its components: 1) are zero or not (binary granulation); 2.a) are less than lower threshold, 2.b) are between the lower and critical upper threshold (tolerable fault), and 2.c) are higher than the upper threshold (fault state), (ternary granulation); 3) are reconstructed as functions. Nevertheless, each of these tasks can be solved by system inversion.

Thus, the fault detectability notion can be formulated based on the coupling of systems (2), (3):

**Definition 2.2** Fault $\nu_i$ is said detectable if there exists a residual generator and a differential polynomial $P(u, \dot{u}, \ldots, y, \dot{y}, \ldots, \nu_i, \dot{\nu}_i, \ldots)$ such that coupled system (2), (3), is left invertible from $\nu_i \mapsto \eta$, $\nu_j = 0$, $j \neq i$, and supposing that $(u, y, \nu_i)$ satisfies the non equality

$$P(u, \dot{u}, \ldots, y, \dot{y}, \ldots, \nu_i, \dot{\nu}_i, \ldots) \neq 0.$$  

**Remark 2.1** If system (2) has an asymptotically stable observer, then its error equation is a residual generator. However, the observer design and system inversion procedure can also be interchanged. First system (2) from $\nu \mapsto y$ is inverted and the states of the inverse system are estimated by an observer.

**Remark 2.2** The role of the differential polynomial $P(u, \dot{u}, \ldots, y, \dot{y}, \ldots, \nu_i, \dot{\nu}_i, \ldots)$ is the invertibility condition: certain determinant of a functional matrix is not zero. Later, the inversion algorithm and an example will clarify the real meaning of it.

Detectability and isolability of the faults can also be expressed, similarly to the linear case, [16, 11], in terms of system inversion. If there exists an asymptotically stable residual generator such that the coupled system (2), (3) is invertible from $\nu \mapsto \eta$, then faults are detectable and isolable.

### 2.1 Invertibility of nonlinear systems

Invertibility of the control systems, affine in the input can be solved by algorithms within the framework of output differentiating and elementary linear algebra. Those algorithms will be reconsidered and structured in order to obtain more systems which are invertible and to point out the real nature of the invertibility conditions.

As a first approach, an input-output system $\Sigma$ from inputs $U_\Sigma$ to outputs $Y_\Sigma$ is left invertible if there exists an input-output system $\Sigma^{-1}$ from inputs $Y_\Sigma$ into outputs $U_\Sigma$, such that the cascade system $\Sigma^{-1} \Sigma : U_\Sigma \to Y_\Sigma \to U_\Sigma$ is the identity. In our mainly algebraic setting, it is supposed that $U_\Sigma, Y_\Sigma$ are good class of functions equipped with an algebraic structure; for example, those are differential vector space.

Now, the invertibility will be refined, or extended by the following way. The exceptions when inversion algorithm fails, will be expressed in a differential algebraic condition, which will be natural explaining in the description of the algorithm.

**Definition 2.3** A system $\Sigma$ from inputs $U_\Sigma$ into outputs $Y_\Sigma$ is left invertible if there exists an input-output system $\Sigma^{-1}$ from inputs $Y_\Sigma$ into outputs $U_\Sigma$, and a differential polynomial $P(u, \dot{u}, \ldots, y, \dot{y}, \ldots)$, such that if $y = \Sigma u$, then $\Sigma^{-1}(y) = u$, for all pairs $(u, y) \in U_\Sigma \times Y_\Sigma$, if

$$P(u, \dot{u}, \ldots, y, \dot{y}, \ldots) \neq 0.$$  

Our objective is to give the inversion algorithm for the diagnostic system with respect to the inputs $\nu = (\nu_1, \ldots, \nu_m)$, and output $y$. This system is affine with respect to a sub-family of inputs and the inversion is achieved to this sub-family.

Consider the diagnostic system (2). The $i$-th 0-partial relative degree with respect to the system (2) is $r_{0i}$ is the first derivative of $y_i$, where the failure
modes \( \nu_i \) appear, that is, considering the \( i \)-th output, \( y_i \), then

\[
\dot{y}_i = \frac{\partial h_i(x)}{\partial x} \dot{x} = \frac{\partial h_i(x)}{\partial x} f(x, u)
\]

Let \( \mathfrak{A} \) be invertible, that is, the maximal rank of \( \mathfrak{A} \) is \( m \). Thus, \( \det \mathfrak{A}(x, u, \dot{u}, \ldots) = 0 \) is the non invertibility condition.

For domain of \( \mathfrak{A} \), the subset of the \( (x, u, \dot{u}, \ldots) \) where \( \mathfrak{A} \) is invertible is a dense and open subset (generic property). The non invertibility condition is an algebraic condition (analytic).

Applying the Diop’s state elimination, [2], for the system (2), with output map

\[
z = \det \mathfrak{A}(x, u, \dot{u}, \ldots)
\]
a differential analytic algebraic equation and non equation

\[
P(u, \dot{u}, \ldots, \nu, \dot{\nu}, \ldots, z, \dot{z}, \ldots) = 0,
\]
\[
Q(u, \dot{u}, \ldots, \nu, \dot{\nu}, \ldots, z, \dot{z}, \ldots) \neq 0
\]
are obtained. This input-output differential system is equivalent to system (2),(4). Hence, the non invertibility condition \( z = 0 \) is

\[
P(u, \dot{u}, \ldots, \nu, \dot{\nu}, \ldots, 0, \ldots, 0) = 0,
\]
\[
Q(u, \dot{u}, \ldots, \nu, \dot{\nu}, \ldots, 0, \ldots, 0) \neq 0.
\]

**Remark 2.3** The output \( y \) does not appear in this representation because the invertibility of matrix \( \mathfrak{A} \) is not the general invertibility concept of system.

Introduce the notations

\[
Y_i = y_i - h_i(x) = 0,
\]
\[
\dot{Y}_i = \dot{y}_i - L_f(x,u)h_i(x) = 0,
\]
\[
\vdots
\]
\[
Y_i^{(r_{\nu}-1)} = y_i^{(r_{\nu}-1)} - L_f(x,u)h_i(x) = 0;
\]
\[
i = 1, \ldots, m.
\]

Let \( r = \sum_{i=1}^{m} r_{\nu_i} \). Then, the maximal rank of the Jacobian of the mapping (5) in \( x \), must be \( r \) if \( \mathfrak{A} \) is invertible. In another case, if maximal rank is \( \tilde{r} < r \), then there exists (implicit function theorem),

\[
F(\ldots, Y^{(k)}_j, \ldots) = 0
\]

such that \( Y_1, \ldots, Y_{\tilde{r}} \) depend only of \( x \). Thus,

\[
F_i(\ldots, Y^{(k)}_j, \ldots) = \mathfrak{A}_i(u, \dot{u}, \ldots, y, \dot{y}, \ldots) = 0,
\]

The following matrix is defined, [8]:

\[
\mathfrak{A}(x, u, \dot{u}, \ldots) = \begin{pmatrix}
L_{g_1(x,u)}f^{(r_{\nu_1}-1)}_f h_1(x) & \cdots & L_{g_m(x,u)}f^{(r_{\nu_1}-1)}_f h_1(x) \\
L_{g_1(x,u)}f^{(r_{\nu_2}-1)}_f h_2(x) & \cdots & L_{g_m(x,u)}f^{(r_{\nu_2}-1)}_f h_2(x) \\
\vdots & \ddots & \vdots \\
L_{g_1(x,u)}f^{(r_{\nu_m}-1)}_f h_m(x) & \cdots & L_{g_m(x,u)}f^{(r_{\nu_m}-1)}_f h_m(x)
\end{pmatrix}^{m,m}
\]

\[
= \left( L_{g_j(x,u)}f^{(r_{\nu_i}-1)}_f h_i(x) \right)_{i=1, j=1}^{m,m}
\]
\( i = r + 1, \ldots, r \), are differential equations between \( u \) and \( y \) without \( \nu \), which contradicts the invertibility.

If the inversion is not finished in one step, that is, if \( \mathcal{A}(x, u, \dot{u}, \ldots) \) is not invertible, then let \( m_1 \) the maximal rank of \( \mathcal{A} \), \( m_1 < m \). There exists an invertible matrix \( \Phi(x, u, \ldots) \) which is polynomial of \( L_{g_j(x,u)}L_{f(x,u)}^{r_0-1}x \), such that

\[
\sum_{i=1}^{m} \Phi_{li}(x, u, \dot{u}, \ldots) \left( y_i^{(r_0)} \right) = \sum_{j=1}^{m} L_{g_j(x,u)}L_{f(x,u)}^{r_0-1}h_i(x)\nu_j = 0;
\]

\( l = 1, \ldots, m \), where the matrix \( \tilde{\mathcal{A}}_l \),

\[
\tilde{\mathcal{A}}_l = \left( \sum_{i=1}^{m} \Phi_{li}(x, u, \dot{u}, \ldots) L_{g_j(x,u)}L_{f(x,u)}^{r_0-1}h_i(x) \right)_{l=1,j=1}^{m_1,m}
\]

has maximal rank and

\[
\sum_{i=1}^{m} \Phi_{li}(x, u, \dot{u}, \ldots) L_{g_j(x,u)}L_{f(x,u)}^{r_0-1}h_i(x) = 0,
\]

for all \( l = m_1 + 1, \ldots, m \).

Defining

\[
Y_{ii}(x, u, \dot{u}, \ldots, y_i^{(r_0)}, \ldots, y_m^{(r_0)}) = \sum_{i=1}^{m} \Phi_{ii}(x, u, \dot{u}, \ldots) \left( y_i^{(r_0)} \right) = 0;
\]

\( l = m_1 + 1, \ldots, m \), which don’t involve the failure modes by (6).

Let \( r_{ii}, l = m_1 + 1, \ldots, m \), the 1-partial relative degree, that is, the order of the first derivative when the failure mode appears. Thus,

\[
Y_{ii} = 0, \quad L_{f(x,u)}^{r_{ii}}Y_{ii} = 0; \quad i = 1, \ldots, r_{ii} - 1,
\]

\[
L_{f(x,u)}^{r_{ii}}Y_{ii} + \sum_{j=1}^{m} \left( L_{g_j(x,u)}L_{f(x,u)}^{r_{ii}-1}Y_{ii} \right) \nu_j = 0
\]

\( l = m_1 + 1, \ldots, m \).

Denote \( \tilde{\mathcal{A}}_1 \) the matrix

\[
\tilde{\mathcal{A}}_1 = \left( L_{g_j(x,u)}L_{f(x,u)}^{r_{ii}-1}Y_{ii} \right)_{l=m_1+1,j=1}^{m,m}
\]

and

\[
\mathcal{A}_1(x, u, \dot{u}, \ldots, y, \dot{y}, \ldots) = \left( \tilde{\mathcal{A}}_1(x, u, \dot{u}, \ldots, y, \dot{y}, \ldots) \right).
\]

Let \( \max \text{ rank } \mathcal{A}_1(x, u, \dot{u}, \ldots, y, \dot{y}, \ldots) = m_2, m_2 \geq m_1 \), if \( m_2 = m \), the inversion is finished and the partial relative degrees are \( r_1 = r_{01}, \ldots, r_{m_1} = r_{0m_1} \), \( r_{m_1+1} = r_{0m_1+1}+r_{1m_1+1}+\ldots, r_m = r_{0m}+r_{1m} \). Then, similarly to the case considered above, if \( p = m \), then equations (5), (7), and (8) define an implicit function with respect to \( x \) with maximal rank \( r = r_1 + \cdots + r_m \). If \( m_2 < m \), then the algorithm will be continued, with the construction of the invertible matrix \( \Phi_2 \), the new outputs \( Y_{2m_2+1} = 0, \ldots, Y_{2m} = 0 \), and so on.

System (2) is invertible if there exists a \( k \), sequence of \( m_1 < m_2 < \cdots < m_k \) such that \( Y_{k-1,m_k-1} = 0 \), \( \ldots, Y_{k-1,m} = 0 \) such that \( Y_{k-1,i} = 0, L_{g(x,u)}Y_{k-1,i} = 0, \ldots, L_{f(x,u)}^{r_{k-1,i}-1}Y_{k-1,i} = 0 \), and

\[
L_{f(x,u)}^{r_{k-1,i}}Y_{k-1,i} + \sum_{j=1}^{m} \left( L_{g_j(x,u)}L_{f(x,u)}^{r_{k-1,i}-1}Y_{k-1,i} \right) \nu_j = 0.
\]

Then the implicit function

\[
L_{f(x,u)}^{r_{i,j}}h_i(x) = 0, \quad \forall j = 0, 1, \ldots, r_{0j} - 1, \quad i = 1, \ldots, m
\]

\[
L_{f(x,u)}^{r_{i,j}}Y_{i} = 0, \quad \forall j = 0, 1, \ldots, r_{ii} - 1, \quad i = m_l + 1, \ldots, m; \quad l = 1, \ldots, k - 1;
\]

for \( x \) has maximal rank,

\[
r = \sum_{l=0}^{k-1} \sum_{i=m_l+1}^{m} r_{ii}
\]

We notice, using the implicit function theorem, that the dimension of zero dynamics and the inverse dynamic is \( n - r \).
Corollary 2.1 The dimension of zero dynamics and the inverse dynamic is \( n - r \). This can be proven by the inverse function theorem for (9).

Corollary 2.2 If the zero dynamics is asymptotically stable, then the system (2) is detectable.

Using the inverse function theorem for (9) the states different from the zero dynamics are expressed by \( u, u, \ldots, y, \hat{y}, \ldots \), hence the desired dynamics for the “observable” states can be assigned arbitrarily, such that the states of the zero dynamics are the same, thus substituting 0 in the observer for the states of the zero dynamics the obtained observer has an asymptotically stable error.

Corollary 2.3 Let us suppose that the inverse dynamics is asymptotically stable over the state of the zero dynamics, then the inverse system is detectable.

3 Example

Consider a diagnostic model

\[
\begin{align*}
\dot{x}_1 &= x_1v_1 + x_3v_2 \\
\dot{x}_2 &= -x_2v_1 + x_2u \\
\dot{x}_3 &= -x_1x_2x_3v_2 + x_1 + u \\
y_1 &= x_1x_2, \quad y_2 = x_3.
\end{align*}
\]

Differentiating the outputs, then:

\[
\begin{align*}
\dot{y}_1 &= x_2x_3v_2 + x_1x_2u \\
\dot{y}_2 &= -x_1x_2x_3v_2 + x_1 + u.
\end{align*}
\]

Thus:

\[
\dot{y}_1x_1 + \dot{y}_2 = x_1y_1u + x_1 + u
\]

From the implicit function theorem:

\[
\begin{align*}
y_1 - x_1x_2 &= 0 \\
y_2 - x_3 &= 0 \\
u - \dot{y}_2 + x_1(y_1u - \dot{y}_1 + 1) &= 0.
\end{align*}
\]

In this case:

\[
\begin{align*}
x_1 &= \frac{\dot{y}_2 - u}{y_1u - \dot{y}_1 + 1}, \quad y_1u - \dot{y}_1 + 1 \neq 0; \\
x_2 &= \frac{y_1(y_1u - \dot{y}_1 + 1)}{\dot{y}_2 - u}, \quad \dot{y}_2 - u \neq 0; \\
x_3 &= y_2.
\end{align*}
\]

Evaluate the invertibility of matrix,

\[
A_0(x) = \begin{pmatrix} 0 & x_2x_3 \\ 0 & -x_1x_2x_3 \end{pmatrix}
\]

which is not invertible.

Differentiate:

\[
\begin{align*}
\frac{d}{dt}(u - \dot{y}_2 + x_1(y_1u - \dot{y}_1 + 1)) &= \dot{u} - \dot{y}_2 + x_1(y_1u - \dot{y}_1 + 1) \nu_1 + x_3(y_1u - \dot{y}_1 + 1) \nu_2 + x_1(\dot{y}_1u + y_1\dot{u} - \dot{y}_1) &= 0
\end{align*}
\]

Hence, system (10) is invertible if and only if

\[
A_1(x) = \begin{pmatrix} 0 & x_2x_3 \\ x_1(y_1u - \dot{y}_1 + 1) & x_3(y_1u - \dot{y}_1 + 1) \end{pmatrix};
\]

or equivalently, (due to \( y_1u - \dot{y}_1 + 1 \neq 0 \)),

\[
\bar{A}_1(x) = \begin{pmatrix} 0 & x_2x_3 \\ 0 & -x_1x_2x_3 \end{pmatrix}
\]

has maximal rank 2, that is \( x_1 \neq 0, x_2x_3 \neq 0, (\ll=\gg x_2 \neq 0 \text{ and } x_3 \neq 0) \).

The negation of \( x_1 \neq 0 \) and \( x_2 \neq 0 \) and \( x_3 \neq 0 \) is \( x_1 = 0 \) or \( x_2 = 0 \) or \( x_3 = 0 \).

1. \( x_1 = 0 \). In this case,

\[
\begin{align*}
0 &= x_3\nu_2, \\
\dot{x}_2 &= -x_2v_1 + x_2u, \\
\dot{x}_3 &= u, \\
y_1 &= 0, \quad y_2 = x_3.
\end{align*}
\]

That is, \( y_2\nu_2 = 0, y_1 = 0, \) and \( \dot{y}_2 = u \). Hence

\[
0 = \dot{y}_2\nu_2 + y_2\dot{\nu}_2 = uv_2 + y_2\dot{\nu}_2
\]

thus, \( \nu_2 = \nu_2(0) \exp \left(-\frac{u}{y_2}\right) \).

2. \( x_2 = 0 \). In this case,

\[
\begin{align*}
\dot{x}_1 &= x_1v_1 + x_3\nu_2 \\
\dot{x}_3 &= x_1 + u, \\
y_1 &= 0, \quad y_2 = x_3.
\end{align*}
\]
Hence, $\dot{y}_2 = x_1 + u$, and $\ddot{y}_2 = x_1 \nu_1 + x_3 \nu_2 + \dot{u}$. Thus
\[ \ddot{y}_2 = (\dot{y}_2 - u) \nu_1 + y_2 \nu_2 + \dot{u}. \]

This case is “equivalent” to
\[
\begin{align*}
y_1 &= 0, \\
\dot{y}_2 &= (\dot{y}_2 - u) \nu_1 + y_2 \nu_2 + \dot{u}.
\end{align*}
\]

3. $x_3 = 0$. In this case,
\[
\begin{align*}
\dot{x}_1 &= x_1 \nu_1, \\
\dot{x}_2 &= -x_2 \nu_1 + x_2 u, \\
0 &= x_1 + u, \\
y_1 &= x_1 x_2, \quad y_2 = 0.
\end{align*}
\]
Hence, $y_1 = -x_2 u$, $\dot{y}_1 = - \dot{x}_2 u - x_2 u$. Thus
\[
\begin{align*}
\dot{y}_1 u &= - (x_2 u - x_2 \nu_1) u^2 - x_2 u \dot{u} \\
&= y_1 u^2 - y_1 u \nu_1 - y_1 \dot{u}.
\end{align*}
\]

Then,
\[ \dot{y}_1 u - y_1 u^2 + y_1 u \nu_1 + y_1 \dot{u} = 0 \]
are the input-output relation condition to not invertibility.

The fault detectability and isolability condition was expressed in terms of invertibility of systems. Conditions to invertibility and an algorithm for system inversion were given in an algebraic and differential algebraic framework. System invertibility and fault detectability were characterized by an algebraic input-output differential equation.

The persistence with respect to an input-output pair is required to detect faults.

4 Conclusions

Applying the relation between inverse systems and FDI problem, a procedure for fault detection and isolation filter design in nonlinear systems has been presented. The approach is based on the reconstruction of the failure modes. The failure mode reconstruction is completed using a state observer. The method can be applied to a great class of nonlinear systems, when the inverse dynamics respect to $\nu$ is asymptotically stable.

When the number of outputs is greater than the number of inputs (failure modes), the inversion algorithm can be given by small modification. Having more outputs than inputs, the states of the zero dynamics can not be observable from the outputs which are used for the inversion. However, agreeing redundant outputs, the state space of the zero dynamics will decrease and the inverse may become asymptotically stable over it.

The References


