TABLE 5.2
Performance of fan in Prob. 5.22

rad/s	Q, m^3/s	SP, Pa	rad/s	Q, m^3/s	SP, Pa
157	1.42	861	126	3.30	114
	1.89	861	94	0.94	304
	2.36	796		1.27	299
	2.83	694		1.89	219
	3.02	635		2.22	134
	3.30	525		2.36	100
126	1.42	548	63	0.80	134
	1.79	530		1.04	122
	2.17	473		1.42	70
	2.36	428		1.51	55
	2.60	351			

(*b*) If the SP is to be computed as a function of Q and ω, propose a convenient form of the equation (just use symbols for the coefficients; do not evaluate them numerically).

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CHAPTER 6

SYSTEM SIMULATION

6.1 DESCRIPTION OF SYSTEM SIMULATION

System simulation, as practiced in this chapter, is the calculation of operating variables (such as pressures, temperatures, and flow rates of energy and fluids) in a thermal system operating in a steady state. System simulation presumes knowledge of the performance characteristics of all components as well as equations for thermodynamic properties of the working substances. The equations for performance characteristics of the components and thermodynamic properties, along with energy and mass balances, form a set of simultaneous equations relating the operating variables. The mathematical description of system simulation is that of solving these simultaneous equations, many of which may be nonlinear.

A *system* is a collection of components whose performance parameters are interrelated. *System simulation* means observing a synthetic system that imitates the performance of a real system. The type of simulation studied in this chapter can be accomplished by calculation procedures, in contrast to simulating one physical system by observing the performance of another physical system. An example of two corresponding physical systems is when an electrical system of resistors and capacitors represents the heat-flow system in a solid wall.

6.2 SOME USES OF SIMULATION

System simulation may be used in the design stage to help achieve an improved design, or it may be applied to an existing system to explore prospective modifications. Simulation is not needed at the *design conditions* because in the design process the engineer probably chooses reasonable values of the operating variables (pressures, temperatures, flow rates, etc.) and selects the components (pumps, compressors, heat exchangers, etc.) that correspond to the operating variables. It would be for the nondesign conditions that system simulation would be applied, e.g., as at part-load or overload conditions. The designer may wish to investigate off-design operation to be sure that pressures, temperatures, or flow rates will not be too high or too low.

The steep increase in the cost of energy has probably been responsible for the blossoming of system simulation during recent years. Thermal systems (power generation, thermal processing, heating, and refrigeration) operate most of the time at off-design conditions. To perform energy studies in the design stage the operation of the system must be simulated throughout the range of operation the system will experience.

System simulation is sometimes applied to existing systems when there is an operating problem or a possible improvement is being considered. The effect on the system of changing a component can be examined before the actual change to ensure that the operating problem will be corrected and to find the cheapest means of achieving the desired improvement.

After listing some of the classes of system simulation, this chapter concentrates on just one class for the remainder of the chapter. Next the use of information-flow diagrams and the application to sequential and simultaneous calculations are discussed. The process of simulating thermal systems operating at steady state usually simmers down to the solution of simultaneous nonlinear algebraic equations, and procedures for their solution are examined.

6.3 CLASSES OF SIMULATION

System simulation is a popular term and is used in different senses by various workers. We shall first list some of the classes of system simulation and then designate the type to which our attention will be confined.

Systems may be classified as *continuous* or *discrete*. In a continuous system, the flow through the system is that of a continuum, e.g., a fluid or even solid particles, flowing at such rates relative to particle sizes that the stream can be considered as a continuum. In discrete systems, the flow is treated as a certain number of integers. The analysis of the flow of people through a supermarket involving the time spent at various shopping areas and the checkout counter is a discrete system. Another example of a discrete-system analysis is that performed in traffic control on expressways and city streets. Our concern, since it is primarily directed toward fluid and energy systems, is continuous systems.

Another classification is *deterministic* v. *stochastic*. In the deterministic analysis the input variables are precisely specified. In stochastic analysis the input conditions are uncertain, either being completely random or (more commonly) following some probability distribution. In simulating the performance of a steam-electric generating plant that supplies both process steam and electric power to a facility, for example, a deterministic analysis starts with one specified value of the steam demand along with one specified value of the power demand. A stochastic analysis might begin with some probability description of the steam and power demands. We shall concentrate in this chapter on deterministic analysis, and reserve for Chapter 19 the study of probabilistic influences.

Finally, system simulation may be classed as *steady-state* or *dynamic*, where in a dynamic simulation there are changes of operating variables with respect to time. Dynamic analyses are used for such purposes as the study of a control system in order to achieve greater precision of control and to avoid unstable operating conditions (Chapter 15). The dynamic simulation of a given system is more difficult than the steady-state simulation, since the steady state falls out as one special case of the transient analysis. On the other hand, steady-state simulations are required much more often than dynamic simulations and are normally applied to much larger systems.

The simulation to be practiced here will be that of continuous, deterministic steady-state systems.

6.4 INFORMATION-FLOW DIAGRAMS

Fluid- and energy-flow diagrams are standard engineering tools. In system simulation, an equally useful tool is the information-flow diagram. A block diagram of a control system is an information-flow diagram in which a block signifies that an output can be calculated when the input is known. In the block diagram used in automatic-control work the blocks represent *transfer functions*, which could be considered differential equations. In steady-state system simulation the block represents an algebraic equation. A centrifugal pump might appear in a fluid-flow diagram like that shown in Fig. 6-1*a*, while in the information-flow diagram the blocks (Fig. 6-1*b*) represent

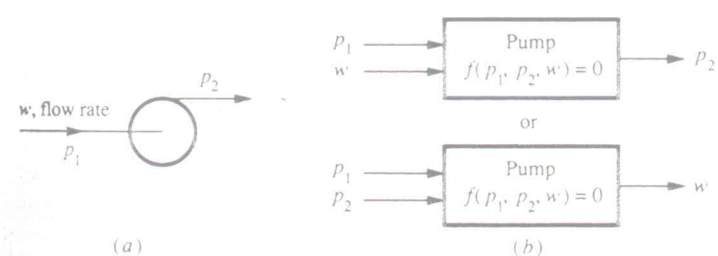

FIGURE 6-1
(a) Centrifugal pump in fluid-flow diagram (*b*) possible information-flow blocks representing pump.

functions or expressions that permit calculation of the outlet pressure for the one block and the flow rate for the other. A block, as in Fig. 6-1*b*, is usually an equation, here designated as $f(p_1, p_2, w) = 0$, or it may be tabular data to which interpolation would be applicable.

Figure 6-1 shows only one component. To illustrate how these individual blocks can build the information-flow diagram for a system, consider the fire-water facility shown in Fig. 6-2. A pump having pressure-flow characteristics shown in Fig. 6-2 draws water from an open reservoir and delivers it through a length of pipe to hydrant *A*, with some water continuing through additional pipe to hydrant *B*. The water flow rates in the pipe sections are designated w_1 and w_2, and the flow rates passing out the hydrants are w_A and w_B. The equations for the water flow rate through open hydrants are $w_A = C_A\sqrt{p_3 - p_{at}}$ and $w_B = C_B\sqrt{p_4 - p_{at}}$, where C_A and C_B are constants and p_{at} is the atmospheric pressure. The equation for the pipe section 0–1 is $p_{at} - p_1 = C_1 w_1^2 + h\rho g$, where $C_1 w_1^2$ accounts for friction and $h\rho g$ is the pressure drop due to the change in elevation h. In pipe sections 2–3 and 3–4

$$p_2 - p_3 = C_2 w_1^2 \qquad \text{and} \qquad p_3 - p_4 = C_3 w_2^2$$

These five equations can be written in functional form

$$f_1(w_A, p_3) = 0 \tag{6.1}$$

$$f_2(w_B, p_4) = 0 \tag{6.2}$$

$$f_3(w_1, p_1) = 0 \tag{6.3}$$

$$f_4(w_1, p_2, p_3) = 0 \tag{6.4}$$

$$f_5(w_2, p_3, p_4) = 0 \tag{6.5}$$

The atmospheric pressure p_{at} is not listed as a variable since it will have a

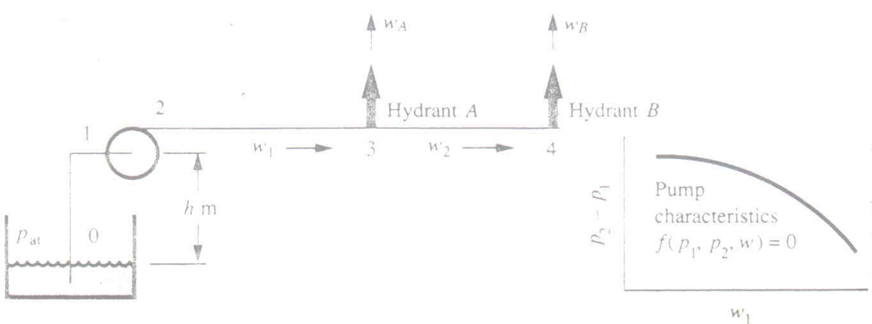

FIGURE 6-2
Fire-water system and pump characteristics.

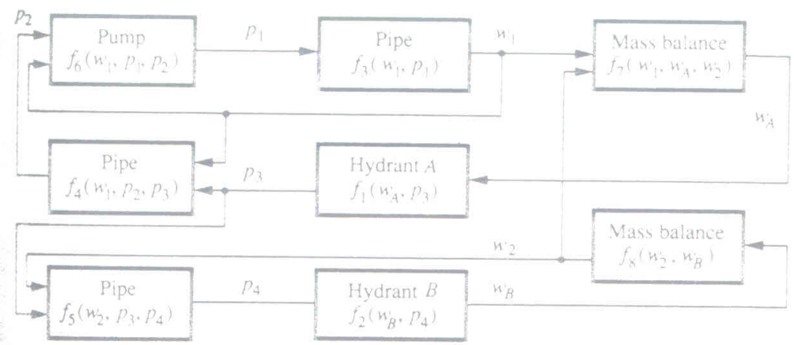

FIGURE 6-3
Information-flow diagram for fire-water system.

known value. An additional function is provided by the pump characteristics

$$f_6(w_1, p_1, p_2) = 0 \tag{6.6}$$

The preceding six equations can all be designated as *component performance characteristics*. There are eight unknown variables, $w_1, w_2, w_A, w_B, p_1, p_2, p_3$, and p_4, but only six equations so far. Mass balances provide the other two equations

$$w_1 = w_A + w_2 \qquad \text{or} \qquad f_7(w_1, w_A, w_2) = 0 \tag{6.7}$$

and

$$w_2 = w_B \qquad \text{or} \qquad f_8(w_2, w_B) = 0 \tag{6.8}$$

Several correct flow diagrams can be developed to express this system, one of which is shown in Fig. 6-3. Each block is arranged so that there is only one output, which indicates that the equation represented by that block is solved for the output variable.

6.5 SEQUENTIAL AND SIMULTANEOUS CALCULATIONS

Sometimes it is possible to start with the input information and immediately calculate the output of a component. The output information from this first component is all that is needed to calculate the output information of the next component, and so on to the final component of the system, whose output is the output information of the system. Such a system simulation consists of *sequential calculations*. An example of a sequential calculation might occur in an on-site power-generating plant using heat recovery to generate steam for heating or refrigeration, as shown schematically in Fig. 6-4. The exhaust gas from the engine flows through the boiler, which generates steam

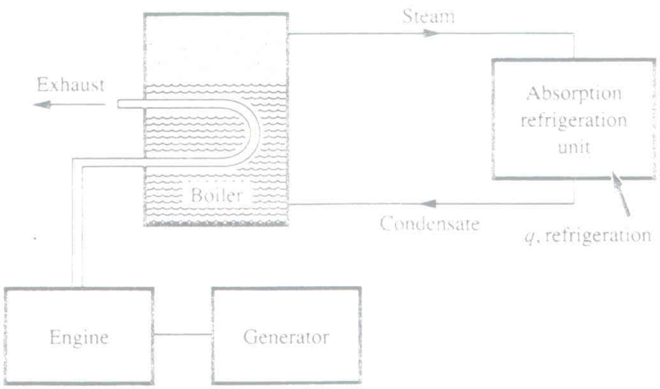

FIGURE 6-4
On-site power generation with heat recovery to develop steam for refrigeration.

to operate an absorption refrigeration unit. If the output information is the refrigeration capacity that would be available when the unit generates a given electric-power requirement, a possible information-flow diagram for this simulation is shown in Fig. 6-5.

Starting with the knowledge of the engine-generator speed and electric-power demand, we can solve the equations representing performance characteristics of the components in sequence to arrive at the output information, the refrigeration capacity.

The *sequential* simulation shown by the information-flow diagram of

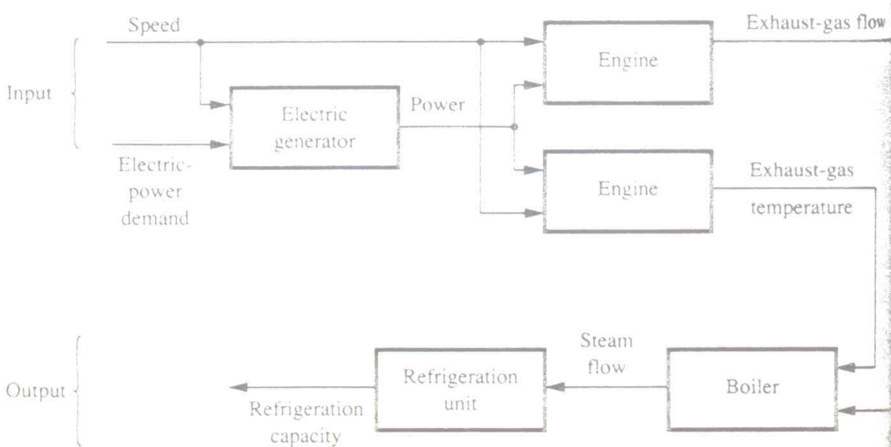

FIGURE 6-5
Information-flow diagram for an on-site power-generating plant of Fig. 6-4.

Fig. 6-5 is in contrast to the *simultaneous* simulation required for the information-flow diagram of Fig. 6-3. Sequential simulations are straightforward, but simultaneous simulations are the challenges on which the remainder of the chapter concentrates.

6.6 TWO METHODS OF SIMULATION: SUCCESSIVE SUBSTITUTION AND NEWTON-RAPHSON

The task of simulating a system, after the functional relationships and interconnections have been established, is one of solving a set of simultaneous algebraic equations, some or all of which may be nonlinear. Two of the methods available for this simultaneous solution are *successive substitution* and *Newton-Raphson*. Each method has advantages and disadvantages which will be pointed out.

6.7 SUCCESSIVE SUBSTITUTION

The method of successive substitution is closely associated with the information-flow diagram of the system (Fig. 6-3). There seems to be no way to find a toe-hold to begin the calculations. The problem is circumvented by assuming a value of one or more variables, beginning the calculation, and proceeding through the system until the originally-assumed variables have been recalculated. The recalculated values are substituted successively (which is the basis for the name of the method), and the calculation loop is repeated until satisfactory convergence is achieved.

Example 6.1 A water-pumping system consists of two parallel pumps drawing water from a lower reservoir and delivering it to another that is 40 m higher, as illustrated in Fig. 6-6. In addition to overcoming the pressure difference due to the elevation, the friction in the pipe is $7.2w^2$ kPa, where w is the combined flow rate in kilograms per second. The pressure-flow-rate characteristics of the pumps are

Pump 1: Δp, kPa $= 810 - 25w_1 - 3.75w_1^2$

Pump 2: Δp, kPa $= 900 - 65w_2 - 30w_2^2$

where w_1 and w_2 are the flow rates through pump 1 and pump 2, respectively.

Use successive substitution to simulate this system and determine the values of Δp, w_1, w_2, and w.

Solution. The system can be represented by four simultaneous equations. The pressure difference due to elevation and friction is

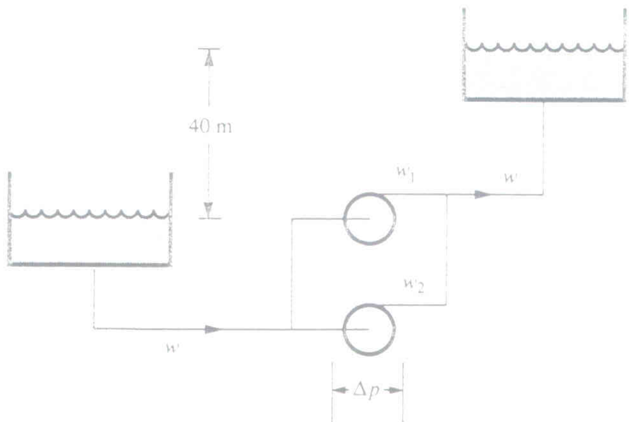

FIGURE 6-6
Water-pumping system in Example 6.1.

$$\Delta p = 7.2w^2 + \frac{(40\ \text{m})(1000\ \text{kg/m}^3)(9.807\ \text{m/s}^2)}{1000\ \text{Pa/kPa}} \tag{6.9}$$

Pump 1: $\Delta p = 810 - 25w_1 - 3.75w_1^2$ (6.10)

Pump 2: $\Delta p = 900 - 65w_2 - 30w_2^2$ (6.11)

Mass balance: $w = w_1 + w_2$ (6.12)

One possible information-flow diagram that represents this system is shown in Fig. 6-7. If a trial value of 4.2 is chosen for w_1, the value of Δp can be computed from Eq. (6.10), and so on about the loop. The values of the variables resulting from these iterations are shown in Table 6.1. The calculation appears to be converging slowly to the values $w_1 = 3.991$, $w_2 = 1.99$, $w = 5.988$, and $\Delta p = 650.5$.

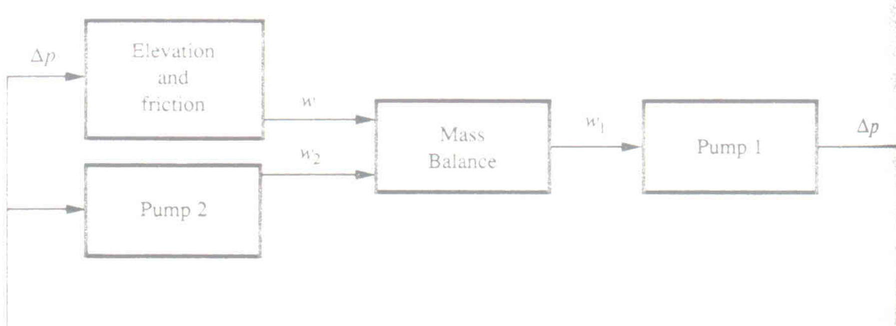

FIGURE 6-7
Information-flow diagram 1 for Example 6.1.

TABLE 6.1
Successive substitution on information-flow diagram of Fig. 6-7

Iteration	Δp	w_2	w	w_1
1	638.85	2.060	5.852	3.792
2	661.26	1.939	6.112	4.174
3	640.34	2.052	5.870	3.818
4	659.90	1.946	6.097	4.151
...	...	...	...	...
47	649.98	2.000	5.983	3.983
48	650.96	1.995	5.994	3.999
49	650.04	2.000	5.983	3.984
50	650.90	1.995	5.993	3.998

6.8 PITFALLS IN THE METHOD OF SUCCESSIVE SUBSTITUTION

Figure 6-7 is only one of the possible information-flow diagrams that can be generated from the set of equations (6.9) to (6.12). Two additional flow diagrams are shown in Figs. 6-8 and 6-9.

The trial value of $w_2 = 2.0$ was chosen for the successive substitution method on information-flow diagram 2, and the results of the iterations are shown in Table 6.2. A trial value of $w = 6.0$ was chosen for the solution of information-flow diagram 3, and the results are shown in Table 6.3.

Information-flow diagram 1 converged to the solution, while diagrams 2 and 3 diverged. This experience is typical of successive substitution. It should be observed that the divergence in diagrams 2 and 3 is attributable to the calculation sequence and not a faulty choice of the trial value. In both cases the trial value was essentially the correct solution: $w_2 = 2.0$ and $w = 6.0$.

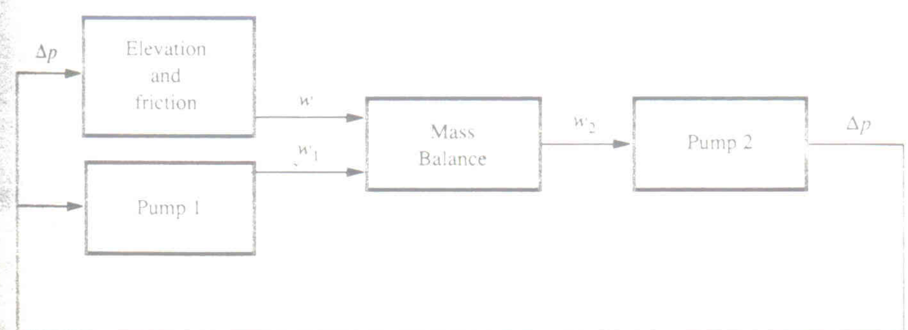

FIGURE 6-8
Information-flow diagram 2 for Example 6.1.

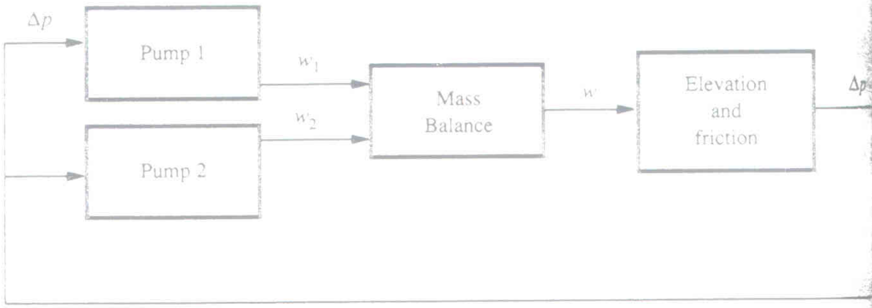

FIGURE 6-9
Information-flow diagram 3 for Example 6.1.

TABLE 6.2
Iterations of information-flow diagram 2

Iteration	Δp	w	w_1	w_2
1	650.0	5.983	4.000	1.983
2	653.2	6.019	3.942	2.077
3	635.5	5.812	4.258	1.554
4	726.5	6.814	2.443	4.371
5	42.8	†		

† Value of w became imaginary.

TABLE 6.3
Iterations of information-flow diagram 3

Iteration	Δp	w_1	w_2	w
1	651.5	3.973	1.992	5.965
2	648.5	4.028	2.008	6.036
3	654.6	3.916	1.975	5.891
4	642.1	4.142	2.042	6.184
5	667.6	3.672	1.903	5.575
6	616.1	4.593	2.178	6.771
7	722.3	2.539	1.580	4.120
8	514.5	6.149	2.662	8.811
9	951.2	†		

† Value of w_1 became imaginary.

Are there means of checking a flow diagram in advance to determine whether the calculations will converge or diverge? Yes, and a technique will be explained in Chapter 14, but the effort of such a check is probably greater than simply experimenting with various diagrams until one is found that converges.

In the method of successive substitution each equation is solved for one variable, and the equation may be nonlinear in that variable, as was true, for example, for the calculations of w and w_2 in diagram 1. No particular problem resulted in computing w and w_2 here because the equations were quadratic. An iterative technique, which may be required in some cases, is described in Sec. 6.10.

6.9 TAYLOR-SERIES EXPANSION

The second technique of system simulation, presented in Secs. 6.10 and 6.11, the Newton-Raphson method, is based on a Taylor-series expansion. It is therefore appropriate to review the Taylor-series expansion. If a function z, which is dependent upon two variables x and y, is to be expanded about the point $(x = a, y = b)$, the form of the series expansion is

$$z = \text{const} + \text{first-degree terms} + \text{second-degree terms} + \text{higher-degree terms}$$

or, more specifically,

$$z = c_o + [c_1(x-a) + c_2(y-b)] + [c_3(x-a)^2 + c_4(x-a)(y-b) + c_5(y-b)^2] + \cdots \tag{6.13}$$

Now determine the values of the constants in Eq. (6.13). If x is set equal to a and y is set equal to b, all the terms on the right side of the equation reduce to zero except c_o, so that the value of the function at (a, b) is

$$c_o = z(a, b) \tag{6.14}$$

To find c_1, partially differentiate Eq. (6.13) with respect to x; then set $x = a$ and $y = b$. The only term remaining on the right side of Eq. (6.13) is c_1, so

$$c_1 = \frac{\partial z(a, b)}{\partial x} \tag{6.15}$$

In a similar manner,

$$c_2 = \frac{\partial z(a, b)}{\partial y} \tag{6.16}$$

The constants c_3, c_4, and c_5 are found by partial differentiation twice followed by substitution of $x = a$ and $y = b$ to yield

$$c_3 = \frac{1}{2}\frac{\partial^2 z(a, b)}{\partial x^2} \qquad c_4 = \frac{\partial^2 z(a, b)}{\partial x\, \partial y} \qquad c_5 = \frac{1}{2}\frac{\partial^2 z(a, b)}{\partial y^2} \tag{6.17}$$

For the special case where y is a function of one independent variable x, the Taylor-series expansion about the point $x = a$ is

$$y = y(a) + \frac{dy(a)}{dx}(x-a) + \left[\frac{1}{2}\frac{d^2y(a)}{dx^2}\right](x-a)^2 + \cdots \tag{6.18}$$

The general expression for the Taylor-series expansion if y is a function of n variables $x_1, x_2, \ldots, x_n$ around the point $(x_1 = a_1, x_2 = a_2, \ldots, x_n = a_n)$ is

$$y(x_1, x_2, \ldots, x_n) = y(a_1, a_2, \ldots, a_n) + \sum_{j=1}^{n} \frac{\partial y(a_1, \ldots, a_n)}{\partial x_j}(x_j - a_j)$$
$$+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 y(a_1, \ldots, a_n)}{\partial x_i \, \partial x_j}(x_i - a_i)(x_j - a_j) + \cdots \tag{6.19}$$

Example 6.2. Express the function $\ln(x^2/y)$ as a Taylor-series expansion about the point $(x = 2, y = 1)$.

Solution

$$z = \ln\frac{x^2}{y} = c_o + c_1(x-2) + c_2(y-1) + c_3(x-2)^2 + c_4(x-2)(y-1) + c_5(y-1)^2 + \cdots$$

Evaluating the constants, we get

$$c_0 = \ln\frac{2^2}{1} = \ln 4 = 1.39$$

$$c_1 = \frac{\partial z(2, 1)}{\partial x} = \frac{2x/y}{x^2/y} = \frac{2}{x} = 1$$

$$c_2 = \frac{\partial z(2, 1)}{\partial y} = -\frac{x^2/y^2}{x^2/y} = -\frac{1}{y} = -1$$

$$c_3 = \frac{1}{2}\frac{\partial^2 z(2, 1)}{\partial x^2} = \frac{1}{2}\left(-\frac{2}{x^2}\right) = -\frac{1}{4}$$

$$c_4 = \frac{\partial^2 z(2, 1)}{\partial x \, \partial y} = 0$$

$$c_5 = \frac{1}{2}\frac{\partial^2 z(2, 1)}{\partial y^2} = \frac{1}{2}\frac{1}{y^2} = \frac{1}{2}$$

The first several terms of the expansion are

$$z = 1.39 + (x-2) - (y-1) - \left(\frac{1}{4}\right)(x-2)^2 + \left(\frac{1}{2}\right)(y-1)^2 + \cdots$$

6.10 NEWTON-RAPHSON WITH ONE EQUATION AND ONE UNKNOWN

In the Taylor-series expansion of Eq. (6.18) when x is close to a, the higher-order terms become negligible. The equation then reduces approximately to

$$y \approx y(a) + [y'(a)](x-a) \tag{6.20}$$

Equation (6.20) is the basis of the Newton-Raphson iterative technique for solving a nonlinear algebraic equation. Suppose that the value of x is sought that satisfies the equation

$$x + 2 = e^x \tag{6.21}$$

Define y as

$$y(x) = x + 2 - e^x \tag{6.22}$$

and denote x_c as the correct value of x that solves Eq. (6.21) and makes $y = 0$

$$y(x_c) = 0 \tag{6.23}$$

The Newton-Raphson process requires an initial assumption of the value of x. Denote as x_t this temporary value of x. Substituting x_t into Eq. (6.22) gives a value of y which almost certainly does not provide the desired value of $y = 0$. Specifically, if $x_t = 2$,

$$y(x_t) = x_t + 2 - e^{x_t} = 2 + 2 - 7.39 = -3.39$$

Our trial value of x is incorrect, but now the question is how the value of x should be changed in order to bring y closer to zero.

Returning to the Taylor expansion of Eq. (6.20), express y in terms of x by expanding about x_c

$$y(x) \approx y(x_c) + [y'(x_c)](x - x_c) \tag{6.24}$$

For $x = x_t$, Eq. (6.24) becomes

$$y(x_t) \approx y(x_c) + [y'(x_t)](x_t - x_c) \tag{6.25}$$

Equation (6.25) contains the further approximation of evaluating the derivative at x_t rather than at x_c, because the value of x_c is still unknown. From Eq. (6.23) $y(x_c) = 0$, and so Eq. (6.25) can be solved approximately for the unknown value of x_c

$$x_c \approx x_t - \frac{y(x_t)}{y'(x_t)} \tag{6.26}$$

In the numerical example

$$x_c \approx 2 - \frac{-3.39}{1 - e^2} = 1.469$$

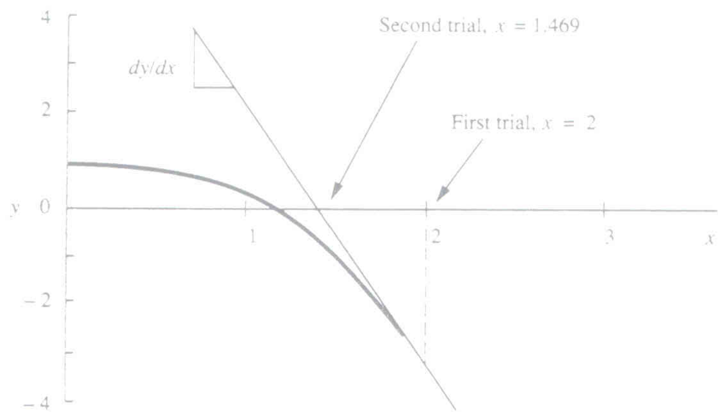

FIGURE 6-10
Newton-Raphson iteration.

The value of $x = 1.469$ is a more correct value and should be used for the next iteration. The results of the next several iterations are

x	1.469	1.208	1.152
$y(x)$	−0.876	−0.132	−0.018

The graphic visualization of the iteration is shown in Fig. 6-10, where we seek the root of the equation $y = x + 2 - e^x$. The first trial is at $x = 2$, and the deviation of y from zero divided by the slope of the curve there suggests a new trial of 1.469.

The Newton-Raphson method, while it is a powerful iteration technique, should be used carefully because if the initial trial is too far off from the correct result, the solution may not converge. Some insight into the nature of the function being solved is therefore always helpful.

6.11 NEWTON-RAPHSON METHOD WITH MULTIPLE EQUATIONS AND UNKNOWNS

The solution of a nonlinear equation for the unknown variable discussed in Sec. 6.10 is only a special case of the solution of a set of multiple nonlinear equations. Suppose that three nonlinear equations are to be solved for the three unknown variables x_1, x_2, and x_3

$$f_1(x_1, x_2, x_3) = 0 \tag{6.27}$$

$$f_2(x_1, x_2, x_3) = 0 \tag{6.28}$$

$$f_3(x_1, x_2, x_3) = 0 \tag{6.29}$$

The procedure for solving the equations is an iterative one in which the following steps are followed:

1. Rewrite the equations so that all terms are on one side of the equality sign [Eqs. (6.27) to (6.29) already exist in this form].
2. Assume temporary values for the variables, denoted $x_{1,t}$, $x_{2,t}$, and $x_{3,t}$.
3. Calculate the values of f_1, f_2, and f_3 at the temporary values of x_1, x_2, and x_3.
4. Compute the partial derivatives of all functions with respect to all variables.
5. Use the Taylor-series expansion of the form of Eq. (6.19) to establish a set of simultaneous equations. The Taylor-series expansion for Eq. (6.27), for example, is

$$\begin{aligned} f_1(x_{1,t}, x_{2,t}, x_{3,t}) \approx\ & f_1(x_{1,c}, x_{2,c}, x_{3,c}) \\ & + \frac{\partial f_1(x_{1,t}, x_{2,t}, x_{3,t})}{\partial x_1}(x_{1,t} - x_{1,c}) \\ & + \frac{\partial f_1(x_{1,t}, x_{2,t}, x_{3,t})}{\partial x_2}(x_{2,t} - x_{2,c}) \\ & + \frac{\partial f_1(x_{1,t}, x_{2,t}, x_{3,t})}{\partial x_3}(x_{3,t} - x_{3,c}) \end{aligned} \tag{6.30}$$

The set of equations is

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} x_{1,t} - x_{1,c} \\ x_{2,t} - x_{2,c} \\ x_{3,t} - x_{3,c} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \tag{6.31}$$

6. Solve the set of linear simultaneous equations (6.31) to determine $x_{i,t} - x_{i,c}$.
7. Correct the x's

$$x_{1,\text{new}} = x_{1,\text{old}} - (x_{1,t} - x_{1,c})$$
$$\cdots\cdots\cdots\cdots$$
$$x_{3,\text{new}} = x_{3,\text{old}} - (x_{3,t} - x_{3,c})$$

8. Test for convergence. If the absolute magnitudes of all the f's or all the Δx's are satisfactorily small, terminate; otherwise return to step 3.

Example 6.3. Solve Example 6.1 by the Newton-Raphson method.

Solution

Step 1.

$$f_1 = \Delta p - 7.2w^2 - 392.28 = 0$$

$$f_2 = \Delta p - 810 + 25w_1 + 3.75w_1^2 = 0$$

$$f_3 = \Delta p - 900 + 65w_2 + 30w_2^2 = 0$$

$$f_4 = w - w_1 - w_2 = 0$$

Step 2. Choose trial values of the variables, which are here selected as $\Delta p = 750$, $w_1 = 3$, $w_2 = 1.5$, and $w = 5$.

Step 3. Calculate the magnitudes of the f's at the temporary values of the variables, $f_1 = 177.7$, $f_2 = 48.75$, $f_3 = 15.0$, and $f_4 = 0.50$.

Step 4. The partial derivatives are shown in Table 6.4.

Step 5. Substituting the temporary values of the variables into the equations for the partial derivatives forms a set of linear simultaneous equations to be solved for the corrections to x:

$$\begin{bmatrix} 1.0 & 0.0 & 0.0 & -72.0 \\ 1.0 & 47.5 & 0.0 & 0.0 \\ 1.0 & 0.0 & 155.0 & 0.0 \\ 0.0 & -1.0 & -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} = \begin{bmatrix} 177.7 \\ 48.75 \\ 15.0 \\ 0.50 \end{bmatrix}$$

where $\Delta x_i = x_{i,t} - x_{i,c}$

Step 6. Solution of the simultaneous equations

$$\Delta x_1 = 98.84 \quad \Delta x_2 = -1.055 \quad \Delta x_3 = -0.541 \quad \Delta x_4 = -1.096$$

Step 7. The corrected values of the variables are

$$\Delta p = 750.0 - 98.84 = 651.16 \quad w_1 = 4.055 \quad w_2 = 2.041 \quad w = 6.096$$

These values of the variables are returned to step 3 for the next iteration.

The values of the f's and the variables resulting from continued iterations are shown in Table 6.5.

The calculations converged satisfactorily after three iterations.

TABLE 6.4

	$\partial/\partial\Delta p$	$\partial/\partial w_1$	$\partial/\partial w_2$	$\partial/\partial w$
$\partial f_1/\partial$	1	0	0	$-14.4w$
$\partial f_2/\partial$	1	$25 + 7.5w_1$	0	0
$\partial f_3/\partial$	1	0	$65 + 60w_2$	0
$\partial f_4/\partial$	0	-1	-1	1

TABLE 6.5

After iteration	f_1	f_2	f_3	f_4	Δp	w_1	w_2	w
1	−8.681	4.170	8.778	0.000	651.16	4.055	2.041	6.096
2	−0.081	0.0148	0.056	0.000	650.48	3.992	1.998	5.989
3	0.000	0.000	0.000	0.000	650.49	3.991	1.997	5.988

6.12 SIMULATION OF A GAS TURBINE SYSTEM

A simulation of a more extensive thermal system will be given for a nonregenerative gas-turbine cycle. This cycle, shown in Fig. 6-11, consists of a compressor, combustor, and turbine whose performance characteristics are known. The turbine-compressor combination operates at 120 r/s.

The objective of the simulation is to determine the power output at the shaft, E_s kW, if 8000 kW of energy is added at the combustor by burning fuel. The turbine draws air and rejects the turbine exhaust to atmospheric pressure of 101 kPa. The entering air temperature is 25°C. Certain simplifications will be introduced in the solution, but it is understood that the simulation method can be extended to more refined calculations. The simplifications are

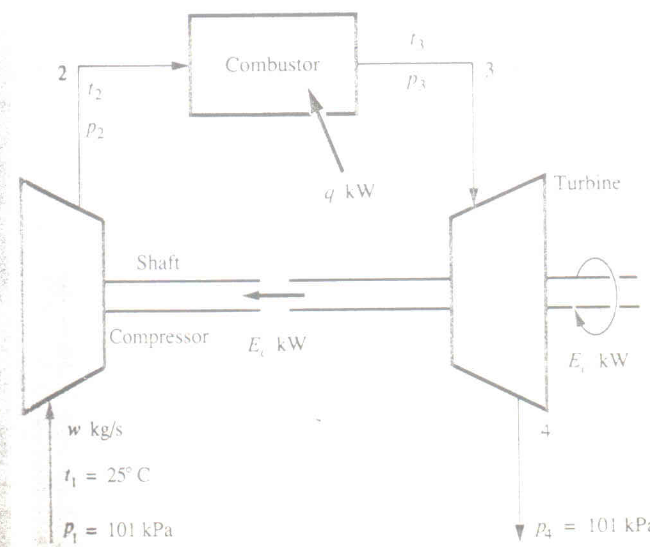

FIGURE 6-11
Gas-turbine cycle.

1. Assume perfect-gas properties throughout the cycle and a c_p constant at 1.03 kJ/(kg · K)
2. Neglect the mass added in the form of fuel in the combustor so that the mass rate of flow w is constant throughout the cycle
3. Neglect the pressure drop in the combustor so that $p_2 = p_3$ and the high pressure in the system can be designated simply as p
4. Neglect heat transfer to the environment

The performance characteristics of the axial-flow compressor and the gas turbine[1] operating at 120 r/s with an atmospheric pressure of 101 kPa that will be used in the simulation are shown in Figs. 6-12 and 6-13, respectively.

With the techniques presented in Chap. 4 equations can be developed for the curves in Fig. 6-12,

$$p = 331 + 45.6w - 4.03w^2 \tag{6.32}$$

and

$$E_c = 1020 - 0.383p + 0.00513p^2 \tag{6.33}$$

where p = discharge pressure of compressor, kPa
w = mass rate of flow, kg/s
E_c = power required by compressor, kW

When operating at a given speed and discharge pressure, the characteristics of the turbine take the form shown in Fig. 6-13. With the techniques

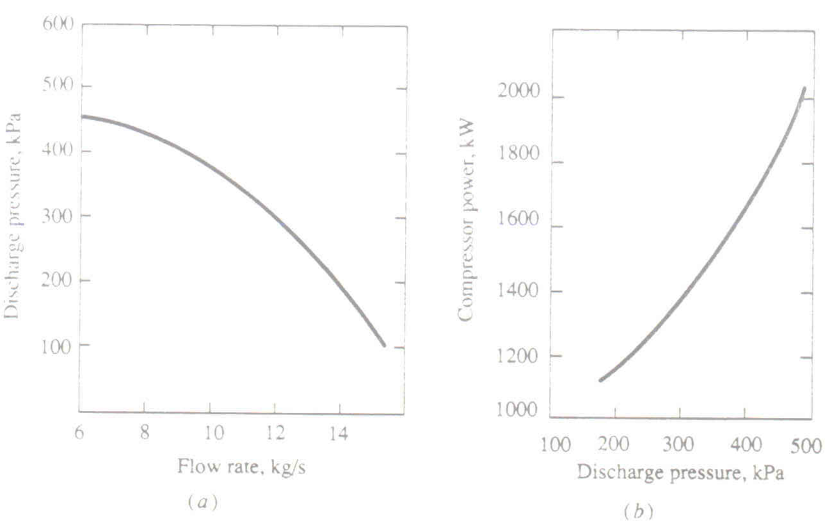

FIGURE 6-12
Performance of axial-flow compressor operating at 120 r/s with 101 kPa inlet pressure.

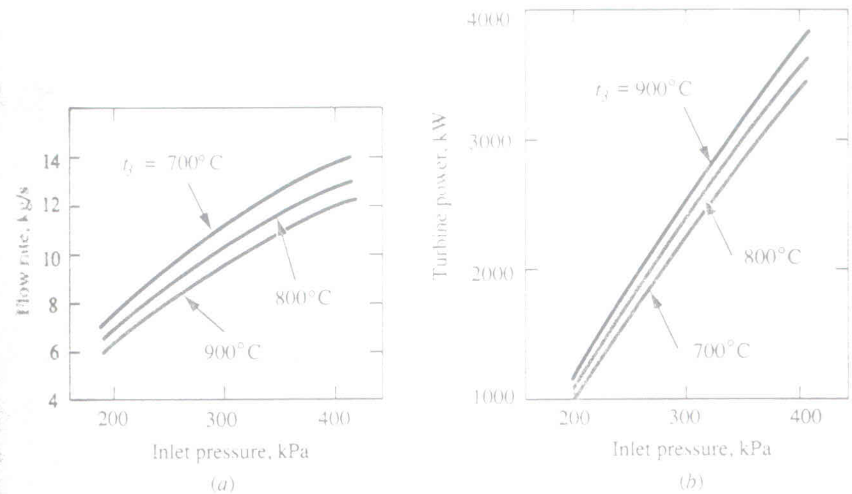

FIGURE 6-13
Performance of gas turbine operating at 120 r/s and 101 kPa discharge pressure.

of Sec. 4.8 equations can be developed for the curves in Fig. 6-13

$$\begin{aligned} w = {} & 8.5019 + 0.02332p + 0.48 \times 10^{-4}p^2 - 0.02644t \\ & + 0.1849 \times 10^{-4}t^2 + 0.000121pt - 0.2736 \times 10^{-6}p^2t \\ & - 0.1137 \times 10^{-6}pt^2 + 0.2124 \times 10^{-9}p^2t^2 \end{aligned} \tag{6.34}$$

and

$$\begin{aligned} E_t = {} & 1727.5 - 10.06p + 0.033033p^2 - 7.4709t + 0.003919t^2 \\ & + 0.050921pt - 0.8525 \times 10^{-4}p^2t - 0.2356 \times 10^{-4}pt^2 \\ & + 0.4473 \times 10^{-7}p^2t^2 \end{aligned} \tag{6.35}$$

where t = entering temperature = t_3, °C
E_t = power delivered by turbine, kW

To achieve the simulation the values of the following unknown variables must be determined: w, p, E_c, t_2, E_s, t_3, and E_t. Seven independent equations must be found to solve for this set of unknowns. Four equations are available from the performance characteristics of the compressor and turbine. The three other equations come from energy balances:

Compressor: $$E_c = wc_p(t_2 - 25) \tag{6.36}$$

Combustor: $$8000 = wc_p(t_3 - t_2) \tag{6.37}$$

Turbine power: $$E_t = E_c + E_s \tag{6.38}$$

TABLE 6.6
Newton-Raphson solution of gas-turbine simulation

	w	p	E_c	t_2	E_3	t_3	E_t
Trial value	10.00	300.0	1000.0	250.0	1500.0	800.0	2400.0
After iteration:							
1	10.91	352.2	1507.8	150.8	1569.3	877.5	3077.0
2	10.77	354.8	1530.0	162.8	1597.7	884.2	3127.7
3	10.77	354.9	1530.1	173.0	1598.5	884.5	3128.6
4	10.77	354.9	1530.1	173.0	1598.5	884.5	3128.6

The execution of the solution follows the steps outlined in Sec. 6.11. A summary of the trial values and results after the Newton-Raphson iterations is presented in Table 6.6.

The shaft power delivered by this system is 1598.5 kW.

6.13 OVERVIEW OF SYSTEM SIMULATION

Steady-state simulation of thermal systems is fast increasing in applicability. The uses of simulation include evaluation of part-load operation directed toward identifying potential operating problems and also predicting annual energy requirements of systems.[2] System simulation can also be one step in an optimization process. For example, the effect on the output of the system of making a small change in one component, e.g., the size of a heat exchanger, is essentially a partial derivative of the type that will be needed in certain of the optimization techniques to be explained in later chapters.

If the exposure to system simulation in this chapter was the reader's first experience with it, wrestling with the techniques may be the major preoccupation. After the methods have been mastered, setting up the equations becomes the major challenge. In large systems it may not be simple to choose the proper combination of equations that precisely specifies the system while avoiding combinations of dependent equations. Unfortunately, no methodical procedure has yet been developed for choosing the equations; a thorough grounding in thermal principles and a bit of intuition are still the necessary tools.

The mathematical description of steady-state system simulation is that of solving a simultaneous set of algebraic equations, some of which are nonlinear. One impulse might be first to eliminate equations and variables by substitution. This strategy is not normally recommended when using a computer to perform the successive substitution or Newton-Raphson solution. Working with the full set of equations provides the solution to a larger

number of variables directly, some of which may be of interest. Performing the substitution always presents the hazard of making an algebraic error, and the equations in combined form are more difficult to check than their simpler basic arrangement.

Two methods of system simulation have been presented in this chapter, successive substitution and Newton-Raphson. Successive substitution is a straight-forward technique and is usually easy to program. It uses computer memory sparingly. The disadvantages are that sometimes the sequence may either converge very slowly or diverge. As far as can be determined through the web of commercial secrecy, many of the large simulation programs used in the petroleum, chemical, and thermal processing industries rely heavily on the successive-substitution method.[3,4] The experienced programmer will enhance his chances of a convergent sequence by choosing the blocks in the information-flow diagram in such a way that the output is only moderately affected by large changes in the input. The Newton-Raphson technique, while a bit more complex, is powerful.

This chapter serves as an introduction to system simulation and provides the tools for solving useful engineering problems. Chapter 14 continues the study of steady-state system simulation in greater depth. In that chapter the successive-substitution method is explored further by identifying the nature of calculation sequences that result in convergence. Methods are developed to accelerate convergence or damp divergence. The Newton-Raphson technique as explained in the foregoing chapter required extracting partial derivatives by hand—a process that is tedious. Chapter 14 explains the structure of a generalized program[5] that extracts the partial derivatives numerically. A challenge also addressed by Chapter 14 is the simulation of large systems where the number of equations and unknowns, n, becomes very large. The Newton-Raphson technique requires the solution of an $n \times n$ matrix, so acceleration techniques become valuable.

PROBLEMS

6.1. The operating point of a fan-and-duct system is to be determined. The equations for the two components are

Duct: $$SP = 80 + 10.73Q^{1.8}$$

Fan: $$Q = 15 - (73.5 \times 10^{-6})SP^2$$

where SP = static pressure, Pa
Q = airflow rate, m^3/s

Use successive substitution to solve for the operating point, choosing as trial values SP = 200 Pa or $Q = 10\ m^3/s$.

Ans.: 6 m^3/s and 350 Pa.

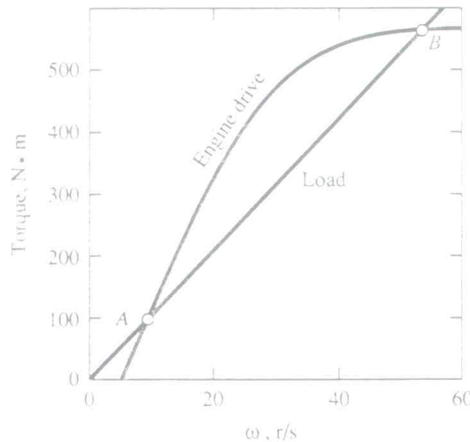

FIGURE 6-14
Torque-rotative-speed curves of engine drive and load on a truck.

6.2. The torque-rotative-speed curve of the engine-and-drive train of a truck operating at a certain transmission setting is shown in Fig. 6-14. The T vs. ω curve for the load on the truck is also shown and is appropriate for the truck moving slowly uphill. The equations for the two curves are

Engine drive: $T = -170 + 29.4\omega - 0.284\omega^2$

Load: $T = 10.5\omega$

where T = torque, N · m
ω = rotative speed, r/s

(*a*) Determination of the operating condition of the truck is a simulation of a two-component system. Perform this simulation with both flow diagrams shown in Fig. 6-15. Use an initial value for both simulations of $\omega = 40$ r/s and show the results in the form of Table 6.7.

(*b*) From a physical standpoint, explain the behavior of the system when operating in the immediate vicinity (on either side) of *A*.

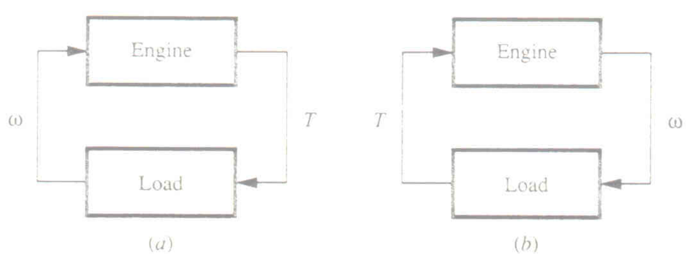

FIGURE 6-15
Flow diagrams in Prob. 6.2.

TABLE 6.7
Results of iterations in Prob. 6.2

Flow diagram Fig. 6-15*a*			Flow diagram Fig. 6-15*b*		
Iteration	T	ω	**Iteration**	T	ω
0	X	40	0	X	40
1			1		
2			2		
3			3		

☐ Converging to point ____ ☐ Converging to point ____
☐ Diverging ☐ Diverging

6.3. A seawater desalination plant operates on the cycle shown in Fig. 6-16. Seawater is pressurized, flows through a heat exchanger, where its temperature is elevated by the condensation of what becomes the desalted water, and flows next through a steam heat exchanger, where it is heated but is still in a liquid state at point 3. In passing through the float valve the pressure drops and some of the liquid flashes into vapor, which is the vapor that condenses as fresh water. The portion at point 4 that remains liquid flows out as waste at point 6. The following conditions and relationships are known:

Temperature and flow rate of entering seawater.

UA_1 and UA_2 of the heat exchangers.

Enthalpies of saturated liquid and saturated vapor of seawater and the fresh water as functions of temperature:

$$h_f = f_1(t) \qquad \text{and} \qquad h_g = f_2(t)$$

For heat exchangers with one fluid condensing, use Eq. (5.10).

The system operates so that essentially $t_4 = t_5 = t_6$.

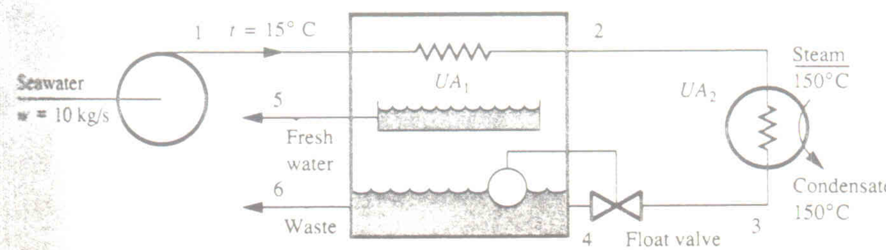

FIGURE 6-16
Desalination plant in Prob. 6.3.

Set up an information-flow diagram that would be used for a successive-substitution system simulation, indicating which equations apply to each block. For convenience in checking, use these variables: t_2, t_3, h_3, h_4, $h_{f,4}$, $h_{g,4}$, t_4, w_5, x_4, and q where x_4 is the fraction of vapor at point 4 and q is the rate of heat transfer at the fresh-water condenser.

6.4. On some high-speed aircraft an air-cycle refrigeration unit is used for cabin cooling, one concept of which is shown in Fig. 6-17. Equations available for the compressor are: power $= f_1(t_1, p_1, p_2, \dot{m})$ and $\dot{m} = f_2(t_1, p_1, \omega, p_2)$. For the turbine the equations are: power $= f_3(t_3, p_3, \dot{m}, p_4)$ and $\dot{m} = f_4(t_3, p_3, \omega, p_4)$ where $f()$ indicates a function or equation in terms of the variables in the parentheses. Assume no pressure drop through the heat exchanger. The compressor-turbine combination operates when p_1 is greater than p_4. The following data are imposed and known: p_1, t_1, p_4, the UA of the heat exchanger, and the temperature and flow rate of ambient air through the heat exchanger. Construct an information-flow diagram to simulate the system using the equations and variables previously listed as well as others that are necessary.

6.5. Steam boilers sometimes use a continuous blowdown of water to control the amount of impurities in the water. This high-temperature water is capable of heating feedwater, as shown in Fig. 6-18. A flow rate of 0.2 kg/s at a temperature of 340°C is blown down from the boiler. The flow rate of the feed water to the heater is 3 kg/s and its entering temperature is 80°C. The UA value of the feed-water heater is 10 kW/K. Equations for the enthalpy of saturated liquid and vapor are, respectively, $h_f = 4.19t$ and $h_g = 2530 + 0.4t$, where t is the temperature in °C. The system is to be simulated and the following variables computed: t_2, t_o, w_A and w_B.

(*a*) Construct an information-flow diagram.

(*b*) Using successive substitution, compute the values of the variables.

Ans.: $t_2 = 108.2$°C.

6.6. In a synthetic-ammonia plant (Fig. 6-19) a 1:3 mixture, on a molar basis, of N_2 and H_2 along with an impurity, argon, passes through a reactor where some of the nitrogen and hydrogen combine to form ammonia. The ammonia product formed leaves the system at the condenser and the remaining H_2, N_2, and Ar recycle to the reactor.

The presence of the inert gas argon is detrimental to the reaction. If no argon is present, the reactor converts 60 percent of the incoming N_2 and H_2 into ammonia, but as the flow rate of argon through the reactor increases, the

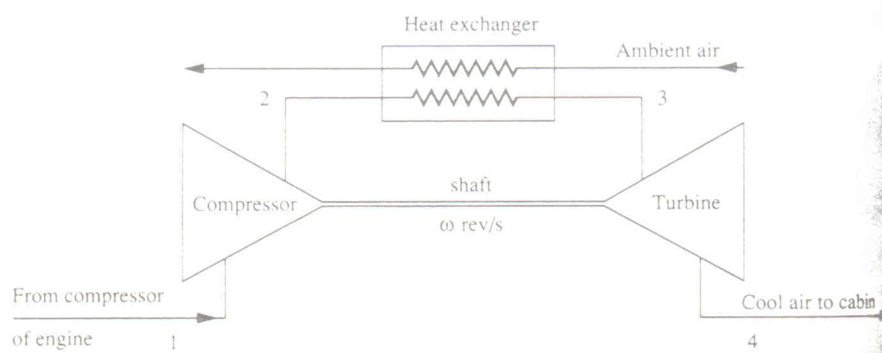

FIGURE 6-17
Air-cycle refrigeration unit in Prob. 6.4.

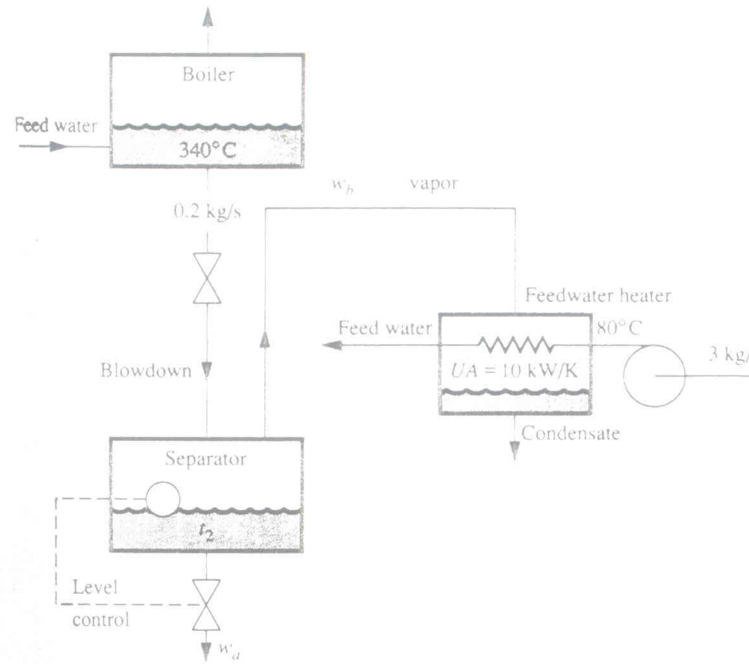

FIGURE 6-18
Blowdown from a boiler in Prob. 6.5.

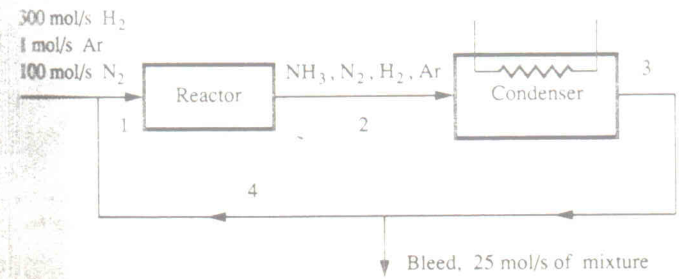

FIGURE 6-19
A synthetic-ammonia plant.

percent conversion decreases. The conversion efficiency follows the equation

$$\text{Conversion, }\% = 60e^{-0.016w}$$

where w is the flow rate of argon through the reactor in moles per second.

To prevent the reaction from coming to a standstill, a continuous bleed of 25 mol/s of mixture of N_2, H_2, and Ar is provided. If the incoming feed consists of 100 mol/s of N_2, 300 mol/s of H_2, and 1 mol/s of Ar, simulate this system by successive substitution to determine the flow rate of mixture through the reactor and the rate of liquid ammonia production in moles per second.

Ans.: 893.2 and 188 mol/s.

6.7. For $x(\tan x) = 2.0$, where x is in radians, use the Newton-Raphson method to determine the value of x.

Ans.: 1.0769.

6.8. The heat exchanger in Fig. 6-20 heats water entering at 30°C with steam entering as saturated vapor at 50°C and leaving as condensate at 50°C. The flow of water is to be chosen so that the heat exchanger transfers 50 kW. The area of the heat exchanger is 1.4 m², and the U value of the heat exchanger based on this area is given by

$$\frac{1}{U}(\text{m}^2\cdot\text{K})/\text{kW} = \frac{0.0445}{w^{0.8}} + 0.185$$

where w is the flow rate of water in kilograms per second. Use the Newton-Raphson method for one equation and one unknown to determine the value of w that results in the transfer of 50 kW.

Ans.: 0.6934 kg/s.

6.9. Solve Prob. 6.1 using the multiple-equation Newton-Raphson method with trial values of SP = 200 Pa and Q = 10 m³/s.

6.10. An oil pipeline has ten pumping stations, each station having the pressure-flow characteristic of

$$\Delta p = 2100 - 20w - 0.5w^2$$

where Δp = pressure rise in the station, kPa
w = flow rate of oil, kg/s

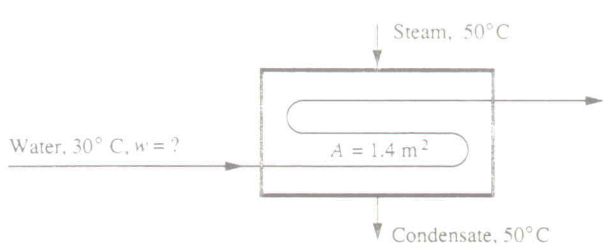

FIGURE 6-20
Heat exchanger in Prob. 6.8.

In normal operation the flow rate is 25 kg/s. The pressure drop in the pipe is proportional to the square of the flow rate. If one pumping station fails and that station bypassed, what will be the flow rate provided by the remaining nine stations?

6.11. In a closed loop a centrifugal and gear pump operate in series to deliver fluid through a long pipe. The equations relating Δp and the flow rate for the three components are:

centrifugal pump: $\Delta p = 6 + 2Q - 0.5Q^2$

gear pump: $\Delta p = 40 - 5Q$

pipe: $\Delta p = 0.1Q^2$

where Δp = pressure rise (or drop in the pipe), kPa
Q = flow rate, m³/s

(*a*) Plot on a $\Delta p - Q$ graph the performance of all components.
(*b*) If a system simulation were performed (no need to perform this simulation), what would be the approximate solution? Discuss the physical implications of the solution.

6.12. In some cryogenic liquefaction systems the temperature of a stream of liquid is reduced by flashing off some of the liquid into vapor through a throttling valve and heat exchanger as shown in Fig. 6-21. With the values shown in Fig. 6-21, use a Newton-Raphson simulation to determine the flow rate of liquid leaving the heat exchanger and its temperature.

Ans.: outlet temperature of liquid is 157.3 K.

6.13. A centrifugal pump operates with a bypass as shown in Fig. 6-22. The pressure drop through the bypass line is given by the equation

$$\Delta p = 1.2(w_{bp})^2$$

the characteristics of the pump are expressed by the equation

$$\Delta p = 50 + 5w_p - 0.1w_p^2$$

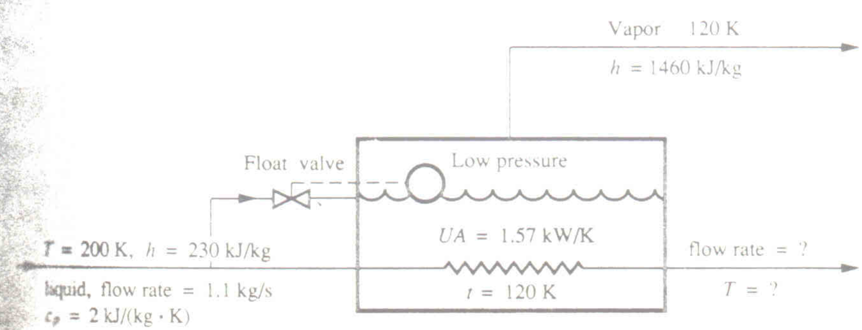

FIGURE 6-21
Cryogenic liquid cooler in Prob. 6.12.

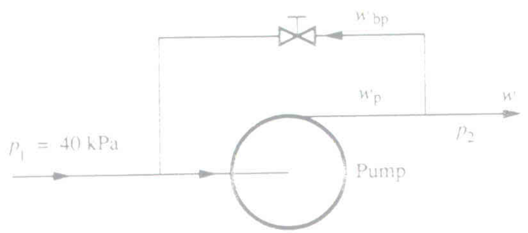

FIGURE 6-22
Centrifugal pump with bypass line in Prob. 6.13.

and the equation for the pressure drop in the piping system is

$$p_2 - p_1 = 0.018w^2$$

where the flow rates are in kg/s and the pressures in kPa. Use the Newton-Raphson technique to determine w, w_p, w_{bp} and p_2.

6.14. Air at 28°C with a flow rate of 4 kg/s flows through a cooling coil counterflow to cold water that enters at 6°C, as shown in Fig. 6-23. Air has a specific heat, c_p = 1.0 kJ/(kg · K). No dehumidification of the air occurs as it passes through the coil. The product of the area and heat-transfer coefficient for the heat exchanger is 7 kW/K. The pump just overcomes the pressure drop through the control valve and coil, such that $p_1 = p_4$. The pressure-flow characteristics of the pump are

$$p_2 - p_1, \text{Pa} = 120{,}000 - 15{,}400w_w^2$$

where w_w is the flow rate of water in kilograms per second. The specific heat of the water is c_p = 4.19 kJ/(kg · K). The pressure drop through the coil is $p_3 - p_4 = 9260w_w^2$. The outlet-air temperature regulates the control

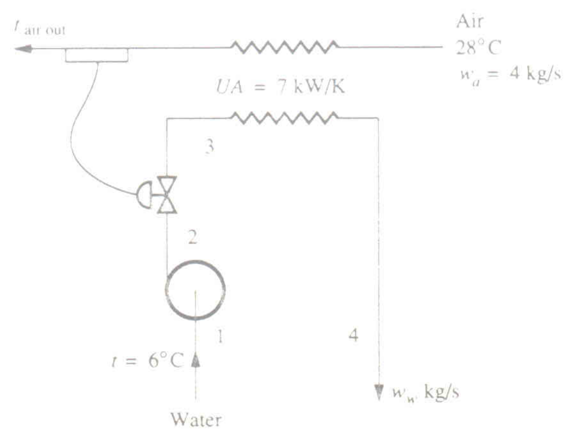

FIGURE 6-23
Cooling coil in Prob. 6.14.

valve to maintain an outlet-air temperature somewhere between 10 and 12°C. The flow-pressure-drop relation for the valve is $w_w = C_v\sqrt{p_2 - p_3}$, where C_v is a function of the degree of valve opening, a linear relation, as shown in Fig. 6-24. The fully open value of C_v is 0.012.

Use the Newton-Raphson method to simulate this system, determining at least the following variables: w_w, t_4, $t_{air\ out}$, p_2, p_3, and C_v. Use as the test for convergence that the absolute values of pressures change less than 1.0, absolute values of temperatures less than 0.001, and absolute value of C_v less than 0.000001. Limit the number of iterations to 10.

Ans.: p_2 = 64,355 (based on p_1 = 0), t_4 = 14.14, C_v = 0.0108.

6.15. A two-stage air compressor with intercooler shown in Fig. 6-25 compresses air (which is assumed dry) from 100 to 1200 kPa absolute. The following data apply to the components:

$$\text{Displacement rate} = \begin{cases} 0.2\ \text{m}^3/\text{s low-stage compressor} \\ 0.05\ \text{m}^3/\text{s high-stage compressor} \end{cases}$$

$$\text{Volumetric efficiency } \eta, \% = \frac{\text{flow rate measured at compressor suction, m}^3/\text{s}}{\text{displacement rate, m}^3/\text{s}}(100)$$

and for both compressors

$$\eta, \% = 104 - 4.0\left(\frac{p_{disch}}{p_{suction}}\right)^{1.4}$$

The polytropic exponent n in the equation $p_1v_1^n = p_2v_2^n$ is 1.2. The intercooler is a counterflow heat exchanger receiving 0.09 kg/s of water at 22°C. The

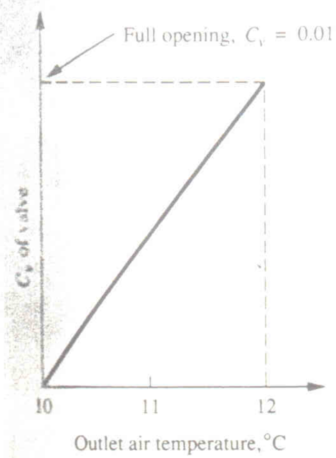

FIGURE 6-24
Characteristics of valve in Prob. 6.14.

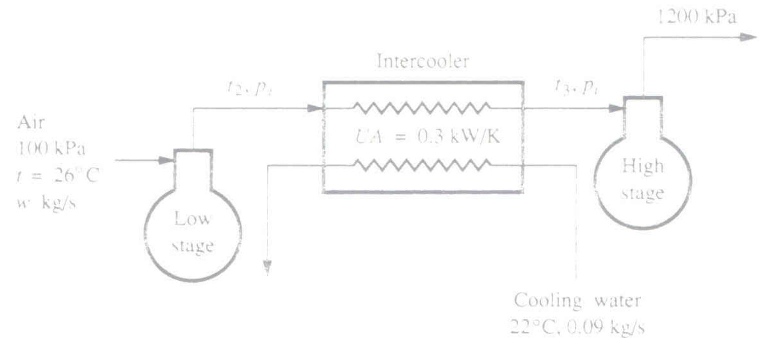

FIGURE 6-25
Two-stage air compression in Prob. 6.15.

product of the overall heat-transfer coefficient and the area of this heat exchanger is $UA = 0.3$ kW/K. Assume that the air is a perfect gas.

Use the Newton-Raphson method to simulate this system, determining at least the values of w, p_i, t_2 and t_3. Use as a test for convergence that all variables change less than 0.001 during an iteration. Limit the number of iterations to 10.

Ans.: $w = 0.18$ kg/s, $t_2 = 101.8$°C, $p_i = 387.7$ kPa, $t_3 = 43.84$°C

6.16. A helium liquefier operating according to the flow diagram shown in Fig. 6-26 receives high-pressure helium vapor, liquefies a fraction of the vapor, and returns the remainder to be recycled. The following operating conditions prevail:

Point 1 (vapor entering warm side of heat exchanger), $T = 15$ K, $h = 78.3$ kJ/kg, $w = 5$ g/s, $p = 2000$ kPa

Point 5 (vapor leaving turbine), $T = 8$ K, $h = 53$ kJ/kg, $w = 4$ g/s

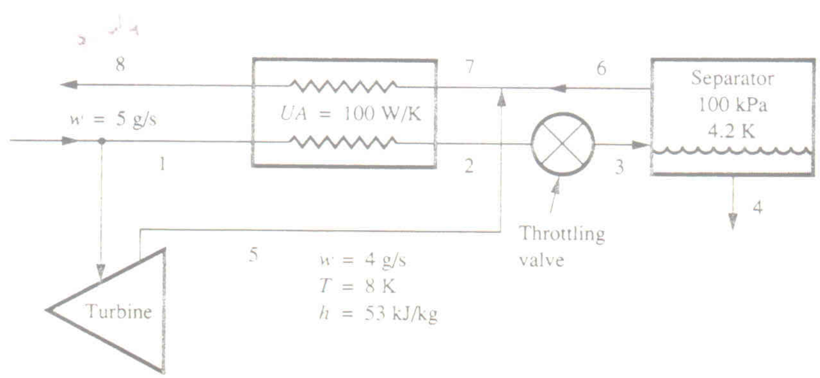

FIGURE 6-26
Helium liquefier in Prob. 6.16.

Separator, $p = 100$ kPa, saturation temperature at 100 kPa = 4.2 K, $h_f = 10$ kJ/kg, $h_g = 31$ kJ/kg

Heat exchanger, $UA = 100$ W/K

Specific heat of helium vapor:

$$c_p = \begin{cases} 6.4 \text{ kJ/(kg}\cdot\text{K)} & \text{at 2000 kPa} \\ 5.8 \text{ kJ/(kg}\cdot\text{K)} & \text{at 100 kPa} \end{cases}$$

Using the Newton-Raphson method, simulate this system, determining the values of w_4, T_2, T_7, and T_8. Use as the test for convergence that all variables change less than 0.001 during an iteration. Limit the number of iterations to 10.

Ans.: $w_4 = 0.447$ g/s, $T_2 = 7.316$, $T_7 = 5.97$, and $T_8 = 10.93$ K.

6.17. A refrigeration plant that operates on the cycle shown in Fig. 6-27 serves as a water chiller. Data on the individual components are as follows:

$$UA = \begin{cases} 30{,}600 \text{ W/K} & \text{evaporator} \\ 26{,}500 \text{ W/K} & \text{condenser} \end{cases}$$

$$\text{Rate of water flow} = \begin{cases} 6.8 \text{ kg/s} & \text{evaporator} \\ 7.6 \text{ kg/s} & \text{condenser} \end{cases}$$

The refrigeration capacity of the compressor as a function of the evaporating and condensing temperatures t_e and t_c, respectively, is given by the equation developed in Prob. 4.9.

$$q_e\text{, kW} = 239.5 + 10.073t_e - 0.109t_e^2 - 3.41t_c - 0.00250t_c^2$$
$$-0.2030t_et_c + 0.00820t_e^2t_c + 0.0013t_et_c^2 - 0.000080005t_e^2t_c^2$$

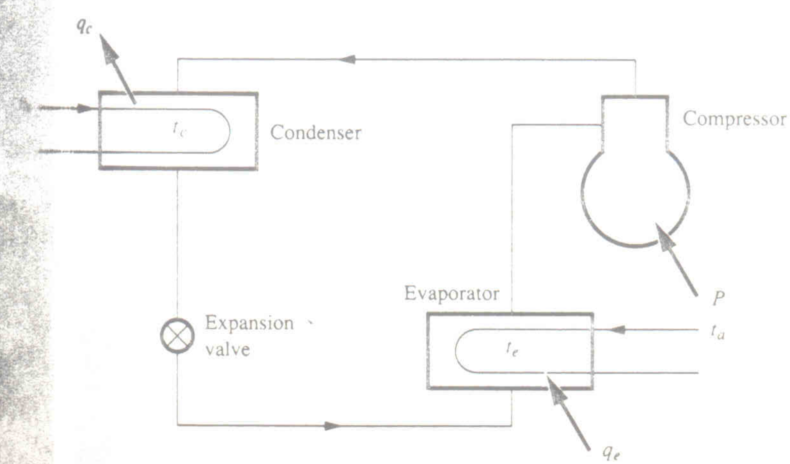

FIGURE 6-27
Refrigeration plant in Prob. 6.17.