6.11 Safe pressure vessels

Pressure vessels, from the simplest aerosol-can to the biggest boiler, are designed, for safety, to yield or leak before they break. The details of this design method vary. Small pressure vessels are usually designed to allow general yield at a pressure still too low to cause any crack the vessel may contain to propagate ("yield before break"); the distortion caused by yielding is easy to detect and the pressure can be released safely. With large pressure vessels this may not be possible. Instead, safe design is achieved by ensuring that the smallest crack that will propagate unstably has a length greater than the thickness of the vessel wall ("leak before break"); the leak is easily detected, and it releases pressure gradually and thus safely (Table 6.19). The two criteria lead to different material indices. What are they?

The model. The stress in the wall of a thin-walled spherical pressure vessel of radius R (Figure 6.19) is

$$\sigma = \frac{pR}{2t} \tag{6.38}$$

In pressure vessel design, the wall thickness, t, is chosen so that, at the working pressure p, this stress is less than the yield strength σ_f of the wall. A small

Table 6.19 Design requirements for safe pressure vessels

Function	Pressure vessel (contain pressure p safely)
Constraints	Radius R specified
Objective	• Maximize safety using yield-before-break criterion, or • Maximize safety using leak-before-break criterion
Free variables	*Choice of material*

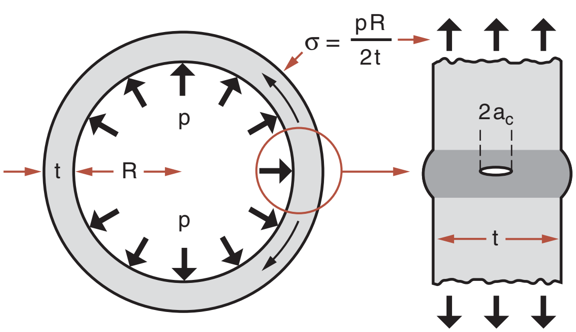

Figure 6.19 A pressure vessel containing a flaw. Safe design of small pressure vessels requires that they yield before they break; that of large pressure vessels may require, instead, that they leak before they break.

pressure vessel can be examined ultrasonically, or by X-ray methods, or proof tested, to establish that it contains no crack or flaw of diameter greater than $2a_c^*$; then the stress required to make the crack propagate[3] is

$$\sigma = \frac{CK_{1C}}{\sqrt{\pi a_c^*}}$$

where C is a constant near unity and K_{1C} is the plane-strain fracture toughness. Safety can be achieved by ensuring that the working stress is less than this, giving

$$p \leq \frac{2t}{R} \frac{K_{1C}}{\sqrt{\pi a_c^*}}$$

The largest pressure (for a given R, t and a_c^*) is carried by the material with the greatest value of

$$M_1 = K_{1C} \tag{6.39}$$

But this design is not fail-safe. If the inspection is faulty, or if, for some other reason a crack of length greater than a_c^* appears, catastrophe follows. Greater security is obtained by requiring that the crack will not propagate even if the stress reaches the general yield stress — for then the vessel will deform stably in a way that can be detected. This condition is expressed by setting σ equal to the yield stress σ_f giving

$$\pi a_c \leq C^2 \left[\frac{K_{1C}}{\sigma_f}\right]^2$$

The tolerable crack size, and thus the integrity of the vessel, is maximized by choosing a material with the largest value of

$$M_2 = \frac{K_{1C}}{\sigma_f} \tag{6.40}$$

Large pressure vessels cannot always be X-rayed or sonically tested; and proof testing them may be impractical. Further, cracks can grow slowly because of corrosion or cyclic loading, so that a single examination at the beginning of service life is not sufficient. Then safety can be ensured by arranging that a crack just large enough to penetrate both the inner and the outer surface of the vessel is still stable, because the leak caused by the crack can be detected. This is achieved if the stress is always less than or equal to

$$\sigma = \frac{CK_{1C}}{\sqrt{\pi t/2}} \tag{6.41}$$

[3] If the wall is sufficiently thin, and close to general yield, it will fail in a plane-stress mode. Then the relevant fracture toughness is that for plane stress, not the smaller value for plane strain.

The wall thickness t of the pressure vessel was, of course, designed to contain the pressure p without yielding. From equation (6.38), this means that

$$t \geq \frac{pR}{2\sigma_f} \tag{6.42}$$

Substituting this into the previous equation (with $\sigma = \sigma_f$) gives

$$p \leq \frac{4C^2}{\pi R}\left(\frac{K_{1C}^2}{\sigma_f}\right) \tag{6.43}$$

The maximum pressure is carried most safely by the material with the greatest value of

$$M_3 = \frac{K_{1C}^2}{\sigma_f} \tag{6.44}$$

Both M_1 and M_2 could be made large by making the yield strength of the wall, σ_f, very small: lead, for instance, has high values of both, but you would not choose it for a pressure vessel. That is because the vessel wall must also be as thin as possible, both for economy of material, and to keep it light. The thinnest wall, from equation (6.42), is that with the largest yield strength, σ_f. Thus we wish also to maximize

$$M_4 = \sigma_f$$

narrowing further the choice of material.

The selection. These selection criteria are explored by using the chart shown in Figure 6.20: the fracture toughness, K_{1C}, plotted against elastic limit σ_f. The indices M_1, M_2, M_3 and M_4 appear as lines of slope 0, 1, 1/2 and as lines that are vertical. Take "yield before break" as an example. A diagonal line corresponding to a constant value of $M_1 = K_{1C}/\sigma_f$ links materials with equal performance; those above the line are better. The line shown in the figure at $M_1 = 0.6\,\text{m}^{1/2}$ (corresponding to a process zone of size 100 mm) excludes everything but the toughest steels, copper, aluminum and titanium alloys, though some polymers nearly make it (pressurized lemonade and beer containers are made of these polymers). A second selection line at $M_3 = 50\,\text{MPa}$ eliminates aluminum alloys. Details are given in Table 6.20.

The leak-before-break criterion

$$M_2 = \frac{K_{1C}^2}{\sigma_f} \tag{6.45}$$

favors low alloy steel, stainless, and carbon steels more strongly, but does not greatly change the conclusions.

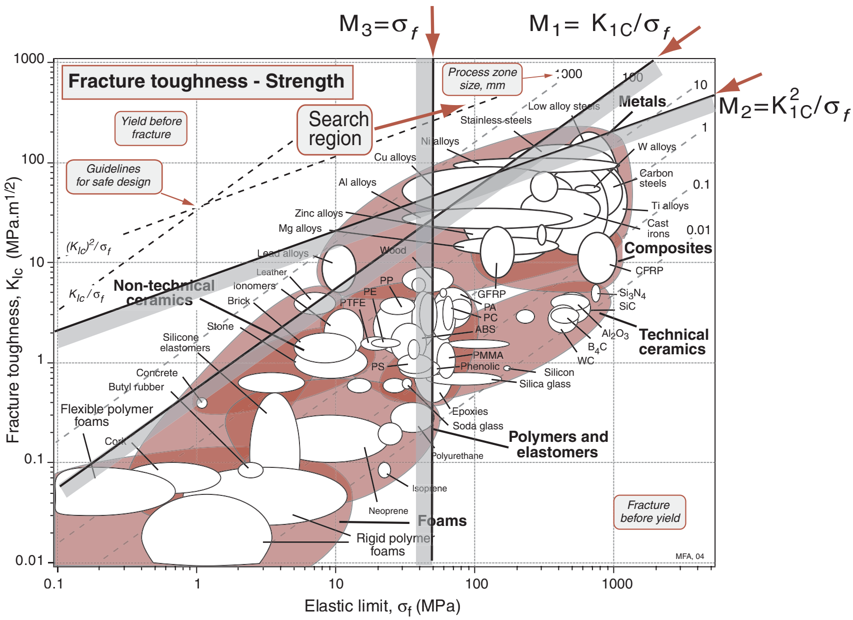

Figure 6.20 Materials for pressure vessels. Steel, copper alloys, and aluminum alloys best satisfy the "yield-before-break" criterion. In addition, a high yield strength allows a high working pressure. The materials in the "search areas" triangle are the best choice. The leak-before-break criterion leads to essentially the same selection.

Table 6.20 Materials for safe pressure vessels

Material	$M_1 = K_{IC}/\sigma_f$ ($m^{1/2}$)	$M_3 = \sigma_f$ (MPa)	Comment
Stainless steels	0.35	300	Nuclear pressure vessels are made of grade 316 stainless steel
Low alloy steels	0.2	800	These are standard in this application
Copper	0.5	200	Hard drawn copper is used for small boilers and pressure vessels
Aluminum alloys	0.15	200	Pressure tanks of rockets are aluminum
Titanium alloys	0.13	800	Good for light pressure vessels, but expensive

Postscript. Large pressure vessels are always made of steel. Those for models — a model steam engine, for instance — are made of copper. It is chosen, even though it is more expensive, because of its greater resistance to corrosion. Corrosion rates do not scale with size. The loss of 0.1 mm through corrosion is not serious in a pressure vessel that is 10 mm thick; but if it is only 1 mm thick it becomes a concern.