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Development of an Internal Model Sliding Mode Controller

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This paper shows the synthesis of a robust predictive controller from a process model that represents a good approximation for nonlinear chemical processes. The controller is designed using the internal model and sliding mode control concepts. This approach results in a fixed controller structure that depends on the characteristic parameters of the model. The controller performance is compared with internal model control and sliding mode control for different linear examples. All linear models present a controllability relationship (t_0/τ) of greater than 1. Simulation results indicate that the proposed controller can work for processes with elevated deadtime, despite modeling errors.

1. Introduction

The internal model control (IMC) structure was introduced by Garcia and Morari in 1982.[1,2] This structure uses an inverse of the process model to design the controller, and the error between the plant and model outputs is used as a feedback signal. The model inverse controller is not suitable for implementation because perfect control requires unreasonably large control moves, the controller requires derivatives of the reference, and the perfect model assumption is not practical. Thus, IMC is synthesized in two steps. In the first step, the assumption is that the model and plant are similar, which makes an easy IMC design with desired performance characteristics. The second step deals with the design of a low-pass filter such that the robustness with respect to model−plant mismatches can be guaranteed with respect to model−plant mismatches.

The sliding mode control (SMC) approach is a nonlinear control technique. SMC is a particular technique of variable structure control.[3] The SMC design is composed of two steps. At the first step, a custom-made surface is to be designed. While on the sliding surface, the plant's dynamics is restricted to the equations of the surface and is robust to match plant uncertainties and external disturbances. At the second step, a feedback control law is to be designed to provide convergence of a system's trajectory to the sliding surface; thus, the sliding surface should be reached in a finite time. The system's motion on the sliding surface is called the sliding mode.[3−5] Perfect tracking can be achieved at the price of control chattering.[6] The design of sliding mode controllers (SMCr's) for deadtime processes requires some assumptions,[6−8] which can be solved using the internal model approach.

The aim of this work is to design a new kind of a robust predictive controller based on a process model that represents a good approximation for nonlinear chemical processes. The paper shows the procedure to merge two control techniques, a robust one and a predictive one, to take benefits of both. Previous works have shown that mixing SMC and predictive structures can produce better controllers than the original ones.[6,7] Therefore, the internal model sliding mode controller (IM-SMCr) is designed in two steps: First, an invertible model of the process is chosen and, from it, the sliding surface is designed. Second, the reaching condition is satisfied, and the robustness is guaranteed.

The paper is organized as follows: section 2 shows the concept of internal model and also, briefly, the SMC theory. Section 3 shows the procedure used to design the controller. In section 4, simulations are presented to judge the performance of the proposed controllers. Finally, the conclusions are given.

2. Basic Concepts

Internal Model Structure Control. The internal model structure is shown in Figure 1. The idea behind this scheme is first to obtain a model of the process and then to decompose the model into two components: an invertible one and a noninvertible one. From the invertible model, the controller is designed.[1] Thus, the model can be represented in the following way:

$$G_m(s) = G_m^{+}(s)\, G_m^{-}(s) \tag{1}$$

where $G_m^{+}(s)$ corresponds to the noninvertible term of the model and $G_m^{-}(s)$ is the invertible part. The noninvertible part has an inverse that is not causal or is unstable, such as deadtime or unstable poles. On the other hand, the invertible component is causal and stable, which leads to a realizable controller.

Therefore, the IMC procedure eliminates all elements in the process model that can produce an unrealizable controller. Thus, the design of the controller takes into consideration only the invertible one.

SMC. The control law contains two parts: the SMC law and the reaching mode control law. The first of these is responsible for maintaining the controlled system dynamic on a sliding surface, which represents the desired closed-loop behavior. The second control law is designed in order to reach the desired surface.[3]

The first step in SMC is choosing the sliding surface that is usually formulated as a linear function of the

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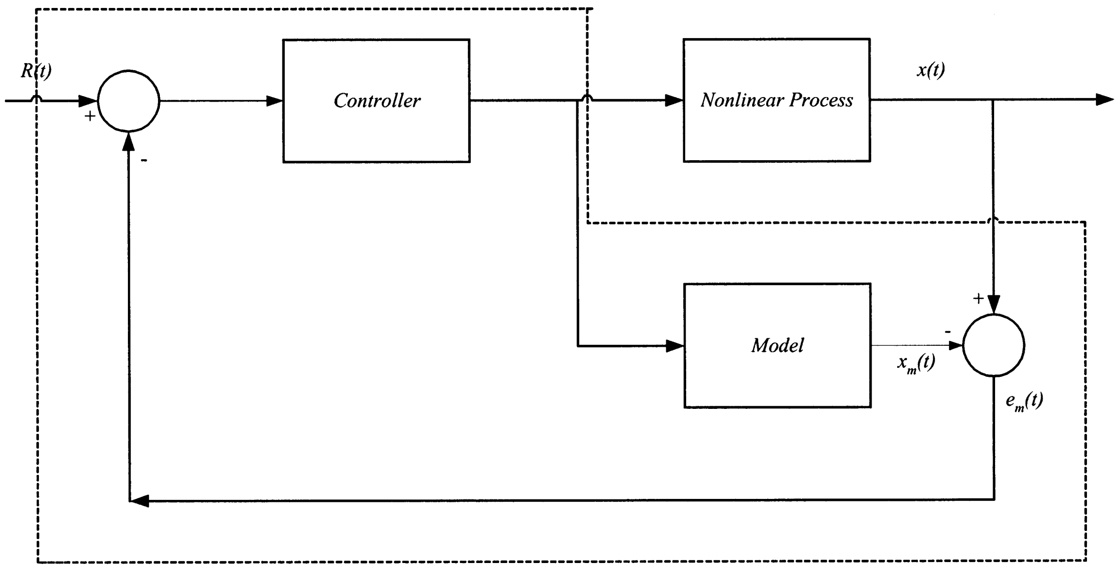

Figure 1. Internal model structure.

system states. The proposed sliding equation is composed of the reference signal, the model output, and the modeling error. Therefore, $s(t)$ can be represented as

$$s(t) = f(R(t), x_{\mathrm{m}}(t), e_{\mathrm{m}}(t), \lambda, n) \quad (2)$$

where $R(t)$ is the reference, $x_{\mathrm{m}}(t)$ is the model output, $e_{\mathrm{m}}(t)$ is the modeling error, n is the model process order, and λ is a tuning parameter.

Filippov's construction of the equivalent dynamics is the method normally used to generate the equivalent SMC law.[5] It consists of satisfying the sliding condition

$$\mathrm{d}s(t)/\mathrm{d}t = 0 \quad (3)$$

and substituting it into the system dynamic equations; the control law is thereby obtained.

To design the reaching mode control law, the signum function of $s(t)$ affected by a constant gain can be used.[5,8,9] However, this produces the undesirable effect of chattering, normally not tolerated by the actuators. A more appropriate solution is to use the sigmoid-like function, instead of the signum one, to smooth the discontinuity and to obtain a continuous approximation to the surface behavior and avoid chattering[3-5] in the control signal when the surface is (pseudo) reached. This is known in the literature as reaching a pseudo sliding mode. The expression for the reaching mode control law can then be expressed as

$$U_{\mathrm{reach}}(t) = K_{\mathrm{D}} \frac{s(t)}{|s(t)| + \delta} \quad (4)$$

where K_{D} is the tuning parameter responsible for the speed with which the sliding surface is reached and δ is used to reduce the chattering problem.

3. Synthesis of IM-SMCr

This section shows the development of a general IM-SMCr; the proposed control scheme is also shown. Nonlinear high-order models describe most processes in the industry. Some studies have shown that a simplified model of a nonlinear high-order model can be used to design a controller. A first-order plus deadtime (FOPDT) model is a recommended one[2] that can be used to get the controller because this kind of model is able to adequately represent the dynamics of many chemical processes over a range of frequencies[2] and is easily obtained from the popular reaction curve method. It can be described by

$$G(s) = \frac{K}{\tau s + 1} \mathrm{e}^{-t_0 s} \quad (5)$$

where K represents the static gain, t_0 is the deadtime, and τ is a first-order lag.

Camacho and Smith[10] showed the design of a SMCr from an FOPDT. The controller design, in that paper, requires some assumptions to deal with the deadtime term. The proposed approach in this paper has the advantage of choosing the invertible part of the model process to design the controller.

Figure 2 shows the proposed scheme. The nonlinear process has been modeled as an FOPDT. As was shown in section 2, the model can be separated into two parts:

$$G_{\mathrm{m}}^{+} = \mathrm{e}^{-t_0 s} \quad (6)$$

$$G_{\mathrm{m}}^{-} = \frac{K}{\tau s + 1} \quad (7)$$

Because $G_{\mathrm{m}}^{-}(s)$ eliminates the deadtime term from the model, this simplification facilitates the SMC design.

Let us propose the following sliding surface:

$$s(t) = e_{\mathrm{m}}^{-}(t) + \lambda \int_0^t \{[R(t) - x_{\mathrm{m}}(t)] - e_{\mathrm{m}}(t)\}\, \mathrm{d}t \quad (8)$$

$e_{\mathrm{m}}^{-}(t)$ is the error between the reference, $R(t)$, and the model output without deadtime, $x_{\mathrm{m}}^{-}(t)$, $x(t)$ is the controlled variable, and $e_{\mathrm{m}}(t)$ is the error between the process output and the complete model output, also known as modeling error. It is observed that the sliding surface is given as a function of the reference, the model output, and the modeling error. This representation is very important because the controlled variable is given as feedback indirectly through the model output response.

$$R(t) - x_{\mathrm{m}}(t) - e_{\mathrm{m}}(t) = R(t) - x_{\mathrm{m}}(t) - [x(t) - x_{\mathrm{m}}(t)] = R(t) - x(t) = e(t) \quad (9)$$

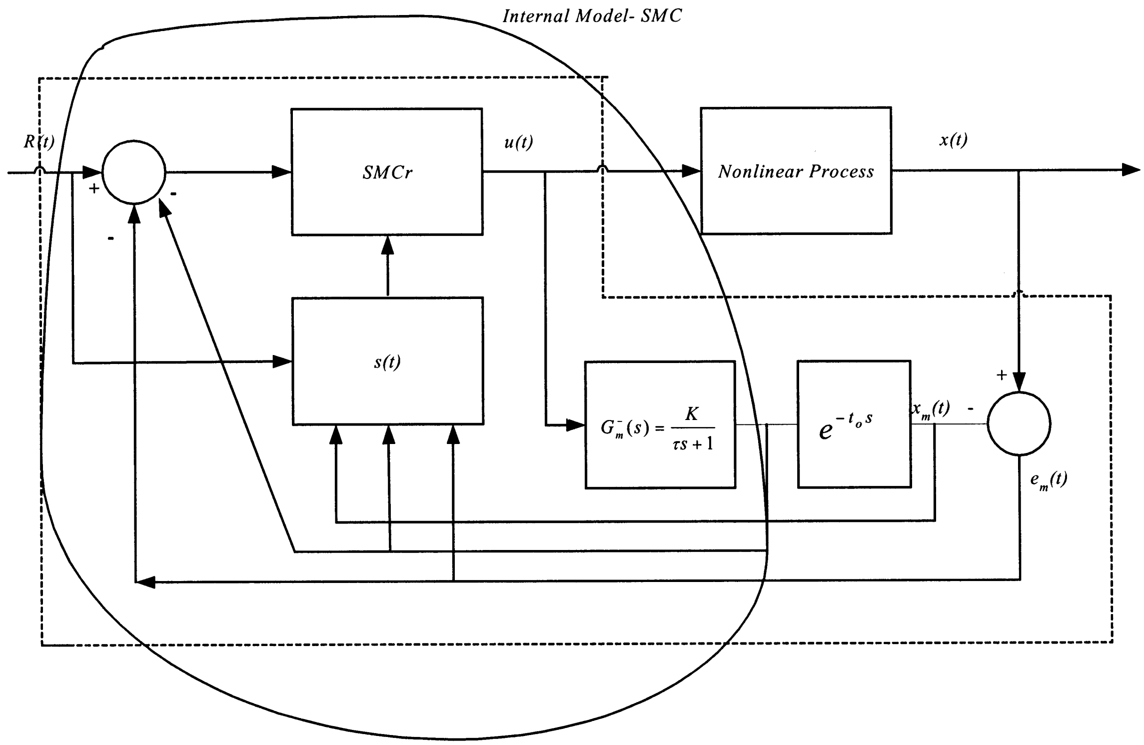

Figure 2. Proposed IM-SMC scheme.

The sliding surface equation can be rewritten as

$$s(t) = e_{\mathrm{m}}^{-}(t) + \lambda \int_0^t e(t)\,\mathrm{d}t \tag{10}$$

The previous equation represents a new sliding surface, which contains a predictive component.

From the sliding condition, $\mathrm{d}s(t)/\mathrm{d}t = 0$.

$$\frac{\mathrm{d}s(t)}{\mathrm{d}t} = \frac{\mathrm{d}R(t)}{\mathrm{d}t} - \frac{\mathrm{d}x_{\mathrm{m}}^{-}(t)}{\mathrm{d}t} + \lambda e(t) = 0 \tag{11}$$

From eq 7 and putting it into differential equation form, which represents the process model

$$\tau \frac{\mathrm{d}x_{\mathrm{m}}^{-}(t)}{\mathrm{d}t} + x_{\mathrm{m}}^{-}(t) = Ku(t) \tag{12}$$

Adding eqs 11 and 12 results in

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} + \frac{x_{\mathrm{m}}^{-}(t)}{\tau} + \lambda e(t) = \frac{K}{\tau}u(t) \tag{13}$$

and the equivalent control law is given by

$$u_{\mathrm{e}}(t) = \frac{\tau}{K}\left[\frac{\mathrm{d}R(t)}{\mathrm{d}t} + \frac{x_{\mathrm{m}}^{-}(t)}{\tau} + \lambda e(t)\right] \tag{14}$$

The SMCr, based on the invertible part of the model process, is given by the following equation:

$$u(t) = \frac{\tau}{K}\left[\frac{\mathrm{d}R(t)}{\mathrm{d}t} + \frac{x_{\mathrm{m}}^{-}(t)}{\tau} + \lambda e(t)\right] + K_{\mathrm{D}}\frac{s(t)}{|s(t)| + \delta} \tag{15}$$

To complete the SMCr, it is necessary to have a set of tuning equations. For the tuning equations as first estimates, using the Nelder−Mead searching algorithm,[11,12] the following equations were obtained:

$$\lambda \leq \frac{1}{\tau + t_0} \tag{16}$$

$$K_{\mathrm{D}} \geq \frac{0.8}{|K|}\left(\frac{\tau}{t_0}\right)^{0.76} \tag{17}$$

$$\delta = 0.68 + 0.12|K|K_{\mathrm{D}}\lambda \tag{18}$$

and the controller, with the derivatives of the reference value discarded,[12] can be rewritten as

$$u(t) = \frac{\tau}{K}\left[\frac{x_{\mathrm{m}}^{-}(t)}{\tau} + \lambda\frac{[R(t) - x_{\mathrm{m}}(t) - e_{\mathrm{m}}(t)]}{e(t)}\right] + K_{\mathrm{D}}\frac{s}{|s| + \delta} \tag{19}$$

$$s(t) = \mathrm{sign}(k)\,[e_{\mathrm{m}}^{-}(t) + \lambda \int_0^t e(t)\,\mathrm{d}t] \tag{20}$$

These equations present advantages from the process control point of view: first, they have a fixed structure depending on the λ parameter and the characteristic parameters of the FOPDT model and, second, the action of the controller is considered in the sliding surface equation, by including the term sign(K). Note that sign(K) only depends on the static gain; therefore, it never switches. From an industrial application point of view, eq 20 represents a PI algorithm.[13]

Proof. The reaching condition is given by

$$s\frac{\mathrm{d}s(t)}{\mathrm{d}t} < 0 \tag{21}$$

$$\frac{\mathrm{d}s(t)}{\mathrm{d}t} = \frac{\mathrm{d}R(t)}{\mathrm{d}t} - \frac{\mathrm{d}x_{\mathrm{m}}^{-}(t)}{\mathrm{d}t} + \lambda e(t) \tag{22}$$

$$\frac{\mathrm{d}s(t)}{\mathrm{d}t} = \frac{\mathrm{d}R(t)}{\mathrm{d}t} - \left[\frac{K}{\tau}u(t) - \frac{x_{\mathrm{m}}^{-}(t)}{\tau}\right] + \lambda e(t) \quad (23)$$

$$\frac{\mathrm{d}s(t)}{\mathrm{d}t} = \frac{\mathrm{d}R(t)}{\mathrm{d}t} - \left[\frac{K}{\tau}\left[\frac{\tau}{K}\left[\frac{\mathrm{d}R(t)}{\mathrm{d}t} + \frac{x_{\mathrm{m}}^{-}(t)}{\tau} + \lambda e(t)\right] + K_{\mathrm{D}}\frac{s}{|s| + \delta}\right] - \frac{x_{\mathrm{m}}^{-}(t)}{\tau}\right] + \lambda e(t) \quad (24)$$

When these are solved, the following result is obtained:

$$\frac{\mathrm{d}s(t)}{\mathrm{d}t} = -K^{*}\frac{s}{|s| + \delta} \quad (25)$$

where

$$K^{*} = KK_{\mathrm{D}}/\tau > 0 \quad (26)$$

Therefore

$$s\,\frac{\mathrm{d}s(t)}{\mathrm{d}t} < 0 \qquad \text{for all } t > 0 \quad (27)$$

and the reachability condition is satisfied.

The next part illustrates the controller performance

4. Simulations

In this section, the IM-SMCr features are shown by some simulation examples. To compare the results obtained with IM-SMCr against the results obtained with IMC and SMC, first it is shown how they work in ideal conditions. Three different models with a relationship of $t_0/\tau > 1$ are considered for simulations:

$$G_1(s) = -0.78\frac{\mathrm{e}^{-3.45s}}{2s + 1} \Rightarrow \text{FOPDT} \quad (28)$$

$$G_2(s) = \frac{\mathrm{e}^{-5s}}{(1 + s)(1 + 0.5s)(1 + 0.25s)(1 + 0.125s)} \Rightarrow \text{higher order} \quad (29)$$

$$G_3(s) = \frac{-\,0.4(s - 0.5)\mathrm{e}^{-5s}}{(s + 1)(s + 0.2)} \Rightarrow \text{inverse response} \quad (30)$$

For all of the processes, their characteristic parameters have been obtained; see Table 1. The values obtained by using the reaction curve procedure.[2]

In Tables 2−4 are shown the tuning values for each of the controllers. These values will be kept constant in all of the simulations. It is important to recall that those tuning values represent starting values that can be adjusted, if it is preferred for the operator, to obtain the desired response. The tuning values in Table 3 were obtained by using the equations given by Camacho.[10,12] IMCr tunings are given in work by Marlin,[1] where τ_{m} is the model time constant, K_{m} is the model static gain, and τ_{f} is the robustness filter time constant.

Figures 3 and 4 shows the process response and controllers response for $G_1(s)$, when set-point and disturbance changes are applied to the process. It is observed that IM-SMCr and IMCr present a very close behavior, while the SMCr presents a slower and oscillatory response, with a higher overshoot than the other two. Figures 5 and 6 are for $G_2(s)$, in which the responses obtained are similar to the previous ones. Figures 7 and 8 are for the nonminimum phase model; also in this case the responses given by IM-SMCr and IMCr are smoother and faster than those of the other controller, SMCr. Up until now, without modeling errors IMCr and IM-SMCr have a better performance than SMCr, but it is important to recall that SMCr is not a predictive structure; therefore, the comparison is done

Table 1. Obtained Characteristic Tuning Parameters

	$G_1(s)$	$G_2(s)$	$G_3(s)$		$G_1(s)$	$G_2(s)$	$G_3(s)$
K	−0.78	1	1	t_0	3.45	4.95	7.8
t	2	1.5	6	t_0/τ	1.73	3.3	1.3

Table 2. IM-SMCr Tuning Parameters

tuning	$G_1(s)$	$G_2(s)$	$G_3(s)$	tuning	$G_1(s)$	$G_2(s)$	$G_3(s)$
λ	0.125	0.10	0.06	δ	0.69	0.69	0.68
K_{D}	0.90	0.90	0.93				

Table 3. SMCr Tuning Parameters

tuning	$G_1(s)$	$G_2(s)$	$G_3(s)$	tuning	$G_1(s)$	$G_2(s)$	$G_3(s)$
λ_0	0.16	0.19	0.02	K_{D}	0.43	0.21	0.43
λ_1	0.79	0.87	0.29	δ	0.71	0.70	0.69

Table 4. IMCr Tuning Parameters

tuning	$G_1(s)$	$G_2(s)$	$G_3(s)$	tuning	$G_1(s)$	$G_2(s)$	$G_3(s)$
τ_{m}	2	1.5	6	τ_{f}	3	1.8	5
K_{m}	−0.78	1	1				

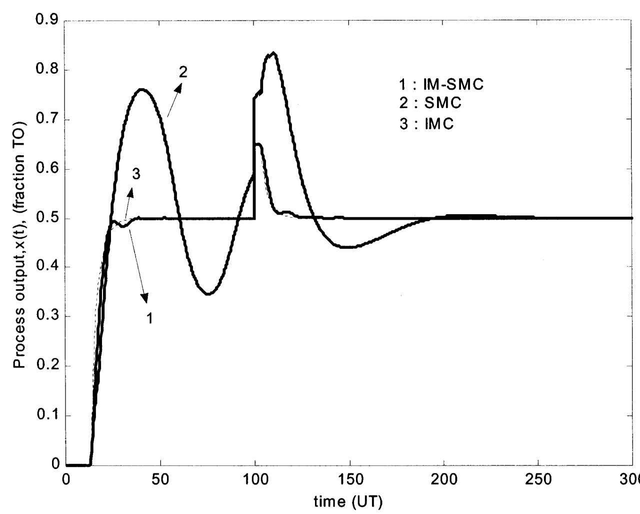

Figure 3. Process response for set-point and disturbance changes for $G_1(s)$.

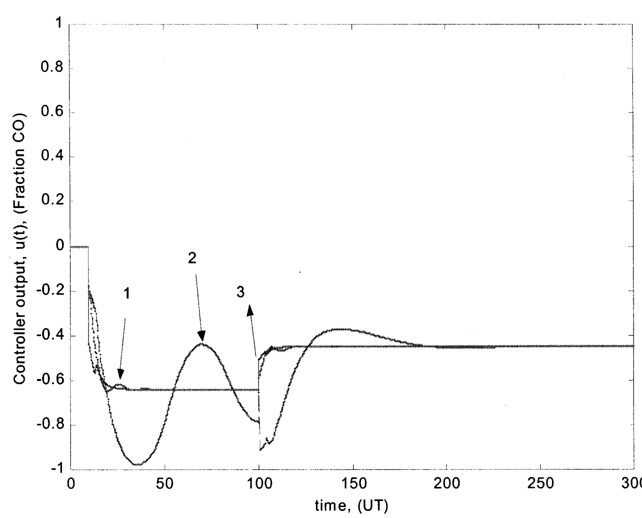

Figure 4. Controllers response when a set point and disturbance occurs in $G_1(s)$.

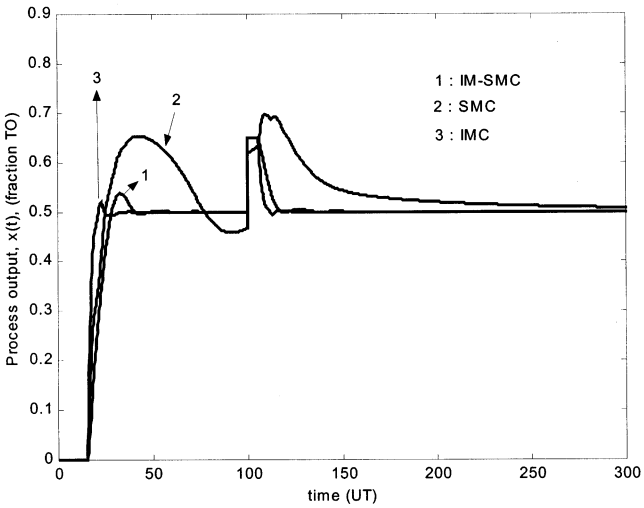

Figure 5. Process response for set-point and disturbance changes for $G_2(s)$.

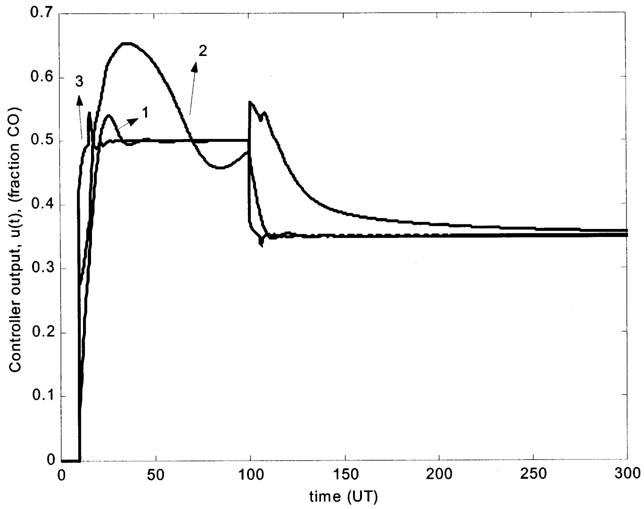

Figure 6. Controllers response when a set point and disturbance occurs in $G_2(s)$.

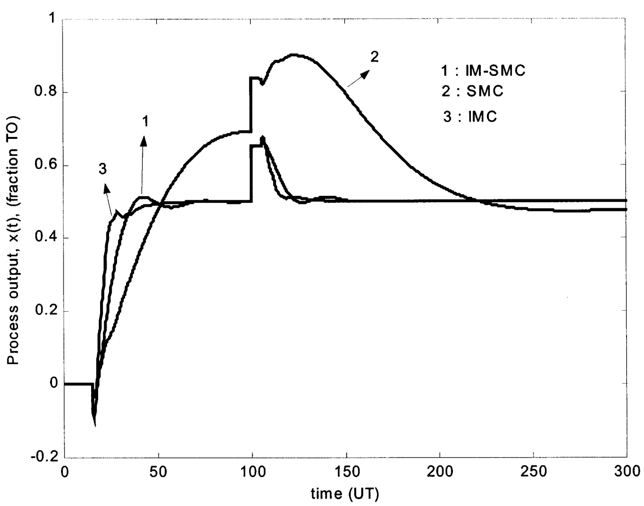

Figure 7. Process response for set-point and disturbance changes for $G_3(s)$.

just to show that the new structure (IM-SMCr) is better for processes with $t_0/\tau > 1$ than the original SMCr. However, in real life there are modeling errors (Figures 9−12); for the case of modeling errors, IM-SMC$_r$ provided a better response than IMCr and SMCr. The new

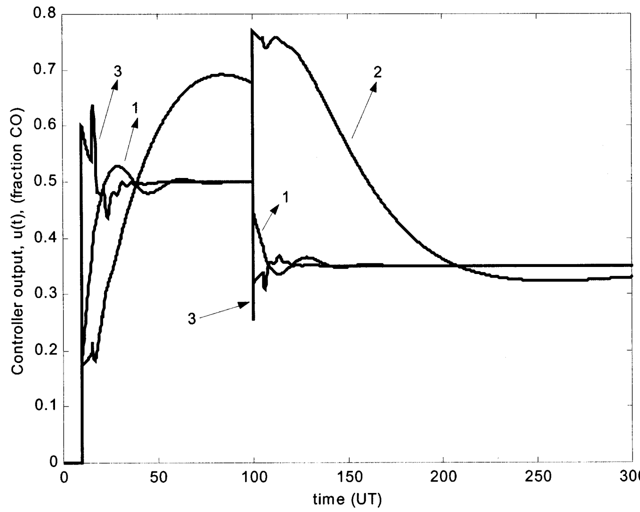

Figure 8. Controllers response when a set point and disturbance occurs in $G_3(s)$.

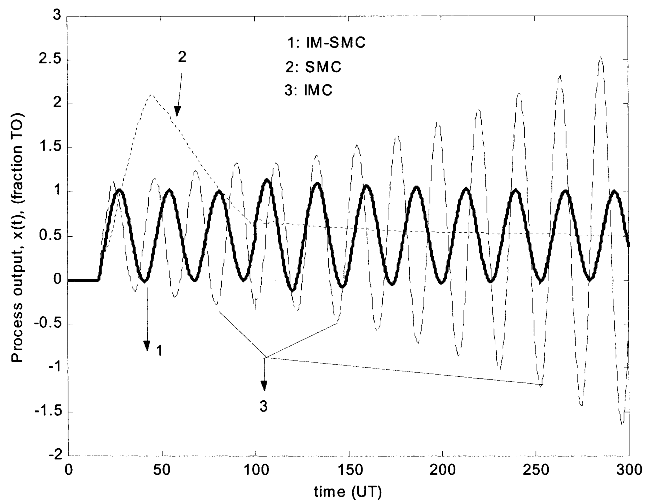

Figure 9. Process response for set-point and disturbance changes in $G_1(s)$, +90% errors in K and t_0.

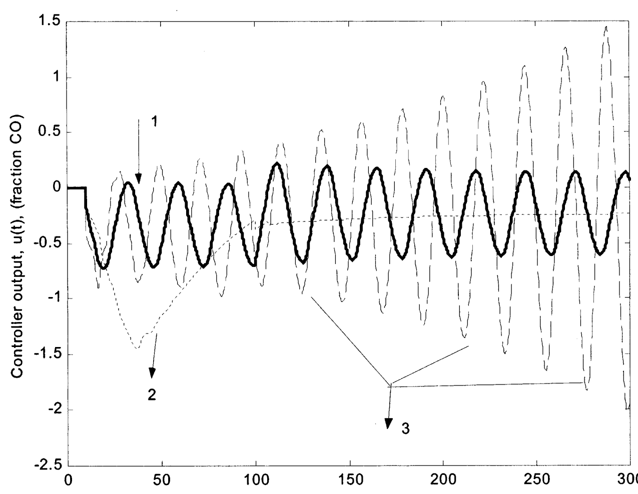

Figure 10. Controllers response when a set point and disturbance occurs in $G_1(s)$, +90% errors in K and t_0.

scheme shows more robustness than IMC for large modeling errors. Also, those figures show that SMCr is more robust than the other two controller schemes when large modeling errors are presented.

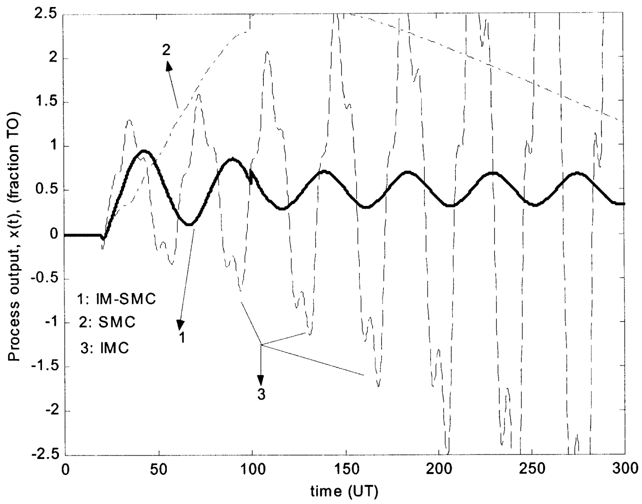

Figure 11. Process response for set-point and disturbance changes in $G_3(s)$, +90% errors in K and t_0.

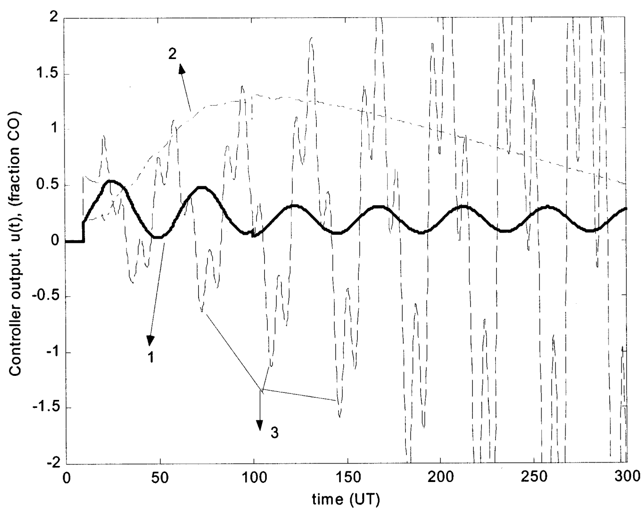

Figure 12. Controllers response when a set point and disturbance occurs in $G_3(s)$, +90% errors in K and t_0.

5. Conclusions

A new control scheme, mixing the internal model approach and the SMC concept for processes with a large deadtime/time constant relationship, has been proposed. The way to design a SMCr from an invertible model of the actual process has been developed, and a new sliding surface has been proposed. The new control scheme was simulated and its performance compared with IMC and SMC. Simulation results showed that IM-SMCr outperforms SMC in nominal conditions for all processes, all with a controllability relationship of $t_0/\tau \gg 1$, and similarly IM-SMC presented better results than IMC when modeling errors appeared. From the results it can be concluded that the controller designed can work for processes with elevated deadtime, despite modeling errors, and thus the hypothesis is confirmed.

IM-SMC acquires some instability from IMC and is less robust than SMC when modeling errors are present. Therefore, the principal deficiency coming from IMC to the new controller is that the new version presents oscillations when modeling errors occurred and could be unstable in some cases.

The controller is of fixed structure, which allows a unique controller of adjustable parameters that can easily be implemented using DCS.

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