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Technical Report

Use of the Monte Carlo Uncertainty Combination Method for Setpoint Evaluation

ISA-TR67.04.14, Use of the Monte Carlo Uncertainty Combination Method for Setpoint Evaluation

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Foreword

In the preparation of an earlier version of ANSI/ISA-S67.04-Part I-1994 it was determined that a need existed to provide additional guidance with regard to methods for implementing the requirements of the Standard. In order to address this need, Standard Subcommittee SP67.15 formed in 1988 (later incorporated into SP67.04) and prepared a Recommended Practice, ISA-RP67.04-Part II-1994. It was the intent of the SP67.04 that the Recommended Practice's scope be consistent with the Standard's scope. The Recommended Practice represents guidelines and examples of methods for implementing ANSI/ISA-S67.04, Part I, in order to facilitate the performance of instrument uncertainty calculations and setpoint determination for safety-related instrument setpoints in nuclear power plants.

The Recommended Practice provides guidance for the implementation of ANSI/ISA-S67.04, Part I, in the following areas:

a) Alternate methods to calculate total channel uncertainty

b) Common assumptions and practices in instrument uncertainty calculations

c) Equations for estimating uncertainties for commonly used analog and digital modules

d) Methods to determine the impact of commonly encountered effects of instrument uncertainty

e) Application of instrument channel uncertainty in setpoint determination

f) Sources and interpretation of data for uncertainty calculations

g) Discussion of the interfaces between setpoint determination and plant operating and calibration procedures and accident analysis

h) Documentation requirements

i) Computer simulation uncertainties

However, the Recommended Practice could not adequately cover all of the topics related to setpoint uncertainties without becoming too voluminous a document. This Technical Report is one in a series that supplements the Recommended Practice and the Standard.

In addition, the Committee agreed that the topics discussed in this report were outside the scope of the Standard; therefore, this Technical Report provides more information concerning the principles and appropriate usage of the Monte Carlo (MC) technique for combination of uncertainties as an alternative to the common Square Root Sum of Squares (SRSS) approach.

The MC technique was first devised in response to the difficult problem of evaluating nuclear shielding effectiveness in the early years of nuclear reactor design evolution. The problem arose in evaluating the shielding effectiveness given a source spectrum of radiation species and energies. The number of interactions possible and the resulting angular dispersion within the shielding material further complicated the analytical assessment of effectiveness. In order to make the solution manageable, it was decided to follow individual particles on their passage through the shielding. The particles that successfully passed through the shielding and that possessed energy within a specific energy band were counted. This analytical process was repeated for a large number of particles with the specified species and energy source distributions, so as to allow a simulation result for shielding effectiveness in which one could be confident.

A number of basic principles underlying the technique should thus become apparent. First, in order to get a reliable result, a large number of particles have to be simulated; the number of simulations can be considered comparable to a sample size. In a statistical analysis, the larger the sample, the greater the confidence in the result. Second, the distribution of input sample species and their associated energies must be realistic in order to assure that the results are valid.

Abstract

This Technical Report supplements ANSI/ISA-S67.04, Part I, and ISA-RP67.04, Part II, in the area of Monte Carlo (MC) uncertainty combination techniques for use as an alternate uncertainty combination method. The report presents the MC uncertainty combination technique for use as one possible alternate for the Square Root Sum of Squares (SRSS) technique.

Key Words

Error Determination, Instrument Setpoints, Instrumentation, Monte Carlo, Setpoint Calculation, Simulation, Standards, Stratified Sampling, Uncertainty, and Uncertainty Determination.

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1 Scope

This Technical Report provides guidance on recognizing the conditions under which Monte Carlo (MC) techniques should be considered both in safety and nonsafety-related applications and begins with the basic principles underlying MC techniques. Section 5 addresses the question, "When should the methodology be used?" The application sequence and the results interpretation are then discussed and, finally, available tools are identified. Annex B illustrates the application of the methodology with sample instrument uncertainty evaluation problems.

2 Purpose

The purpose of this Technical Report is to supplement the information provided in ANSI/ISA-S67.04-Part I-1994 and ISA-RP67.04-Part II-1994 for the performance of instrument uncertainty calculation and instrument setpoint determination. Specifically, the topic addressed in this Technical Report is the use of Monte Carlo (MC) techniques as an alternate technique for uncertainty combination in setpoint calculations. The purpose of this technical paper is to develop the basis for assessing the need for MC, applying MC methodology, and interpreting results of an MC uncertainty analysis. This paper will address both single module and instrument channel simulation applications.

3 Definition

pseudo random numbers: Sequences of numbers that are generated by a software program to simulate true random numbers. In very large numbers of generated pseudo random numbers, the randomness may be violated by repetitions.

4 Basic principles underlying the MC methodology

The following is an illustrative example of the underlying principles involved in an MC simulation. There are two new instrument suppliers in the market that claim to have extremely accurate products; supplier "A" supplies temperature sensors, and supplier "B" supplies transmitters. Both suppliers have produced large quantities of their respective products in the past. An instrumentation and control engineer would like to use the new temperature sensor and the new transmitter in an instrument channel, but the engineer does not have access to the data from the respective suppliers to support their accuracy claims (an assumption in this illustrative example).

The engineer is concerned about using the new instruments without a test of the suppliers' claims and also would like an assessment of how the instruments will work together in an instrument channel, so that a setpoint uncertainty analysis can be conducted. Assume that the engineer has access to a large sample of each instrument type and time to experiment with them on a test bench.

With the availability of time and access to the instruments, the engineer can do a simple experiment. First, the engineer randomly chooses one sensor and one transmitter from the large supplies of each, makes the appropriate electrical connections, and measures the output signal, given that each sensor is put in a constant temperature bath. The measurement of the indicated temperature can then be compared with the actual temperature of the bath to generate a difference measurement. If this process is repeated for hundreds of sets of instruments chosen in a similar manner, the resulting difference measurements can be plotted in histogram form. The plot might look like Figure 1, where the x-axis spans the range of the difference measurement values, and the y-axis spans the range of the number of times each difference value was observed ("Delta T").

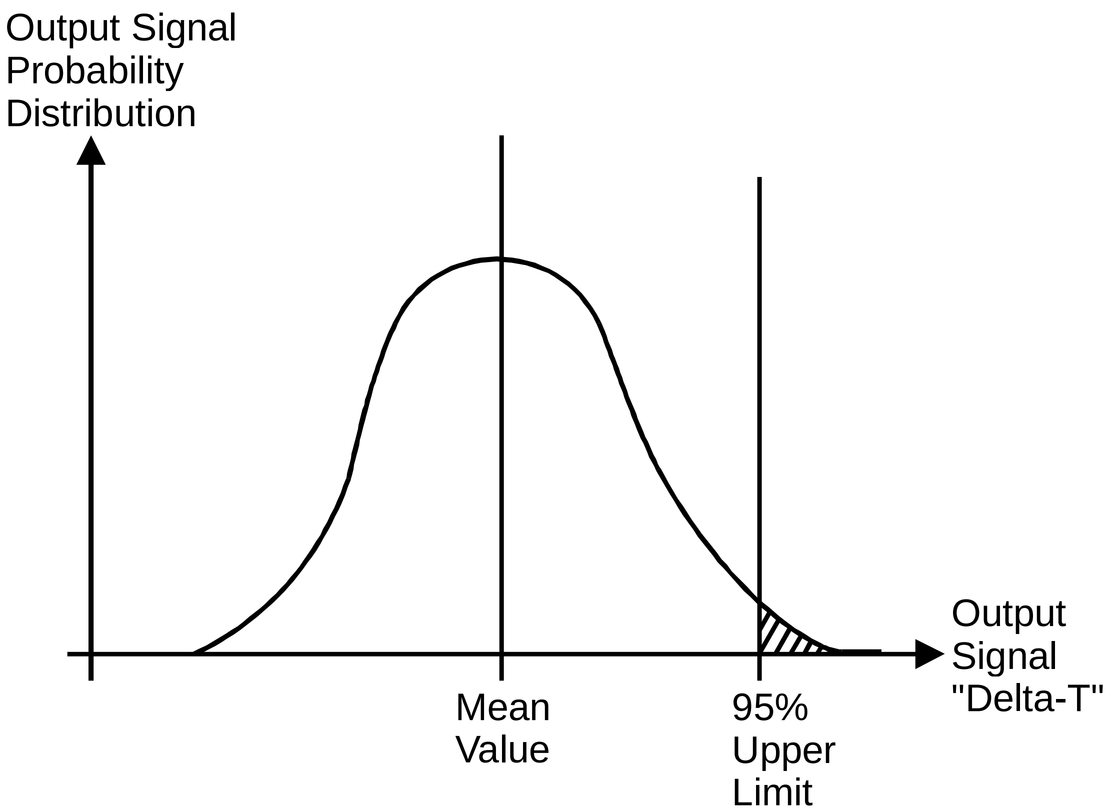

Figure 1 — Typical anticipated output of an experimental evaluation of resultant uncertainty in "Delta T"

Consider why the evaluation process described was utilized. The instruments were chosen at random to eliminate (1) any slow variability in the manufacturing process that might have manifested itself in sensors at the top of the packing box, having a better accuracy than those at the bottom of the packing box, and (2) any variability that might have occurred in the production of the transmitters. The random selection will remove any systematic effects that might be present.

The plot shown in Figure 1 is an approximation to an uncertainty distribution of the resultant error to be associated with the instrument channel output temperature signal. The engineer can use the plot to determine the upper 95 percent probability limit on the indicated temperature by finding the value of the temperature difference measurement (indicated temperature minus the actual temperature), which is the upper bound on 95 percent of the observed differences (see Figure 1). This theoretical experiment illustrates the principles upon which the MC technique is based.

The above experiment is seldom performed in practice, and limited data is used to determine accuracy and other statistical characteristics such as drift. Yet this experimental process can be readily duplicated in a computer program with just a few more bits of information. It should be noted that no mention was made in this experiment concerning the uncertainty distributions associated with either the sensor or the transmitter. It was unnecessary because the actual instruments used in the experiment had the inherent manufacturing and component uncertainties built into their response. The overall uncertainty distribution associated with the sensor or the transmitter might have been Gaussian (also called Normal), Uniform, or Triangular. It did not matter since it was taken care of in the experiment. On the other hand, when an MC computer program or other combination technique is used to simulate this type of experiment, it is critical to know and utilize the appropriate uncertainty distribution for each element of the instrument channel.

Consider the same evaluation process, assuming that an MC-based computer code simulation will be used to determine the uncertainty distribution of the instrument channel output temperature signal. Once this distribution has been established, the resulting upper 95 percent limit associated with the distribution can be readily determined.

In the simulation to be performed, the inherent uncertainty associated with the sensor and the transmitter, respectively, must be known or estimated (see ISA-dTR67.04.10 Reference). Let us assume that the sensor and the transmitter suppliers have data sheets for their products that list a "reference accuracy" value. Typically, the reference accuracy is assumed to be a two sigma value (sigma is the symbol for the standard deviation); i.e., the nominal indicated reading, plus or minus this two sigma value, yields a band of indicated temperatures that should encompass the true temperature reading 95 percent of the time. There are actually two assumptions here: the first is that the reference accuracy is a two sigma value, and the second is that the associated uncertainty distribution is Gaussian. Given these assumptions, how is the expected output response of the sensor and the transmitter simulated so that each resulting uncertainty distribution has the correct attributes; i.e., mean, standard deviation, and Gaussian "shape"?

4.1 The simulation of the uncertainty distribution associated with the sensor and the transmitter utilizes the following sequence.

Standard uncertainty distributions have an associated mathematical formula relating the probability that values in the distribution will lie below a given limit value. For example, the temperature sensor population will yield a range of indicated temperatures for a given, fixed bath temperature. Given a temperature difference between the actual and the indicated temperature of say +3°F, the probability that the observed difference is less than +3°F can be found by the following formula (where the upper limit “b” would be +3 degrees and the lower limit would be zero, and “x” would be a value of the temperature measured). The formula is called the Cumulative Distribution Function (CDF). The CDF will be different for each uncertainty distribution. For a Gaussian distribution with a mean of "μ" and a standard deviation of "σ", the CDF formula is given in Equation 1.

$$p\,(x < b) = \int_{-\infty}^{b} (1/((2\pi)^{\frac{1}{2}})\,\sigma)\,(e - \frac{(x - \mu)^2}{2\sigma^2})\,dx \qquad \text{(Eq.1)}$$

The inverse of this equation can be solved by inputting a value of "p" and solving for the corresponding "b" value. (See the graphical representation of this process in Figure 2.) In order to generate a Gaussian distribution of say 100 "b" values that have the correct mean and standard deviation (μ,σ), 100 values of "p" are generated by a random number generator subroutine, and the corresponding 100 values of "b" are produced by solving the inverse of Equation 1. The resulting "b" values have the correct mean and standard deviation (μ,σ). This process is completed for the sensor and repeated for the transmitter to generate 100 values of the sensor uncertainty indications and 100 values of the transmitter uncertainty indications, respectively.

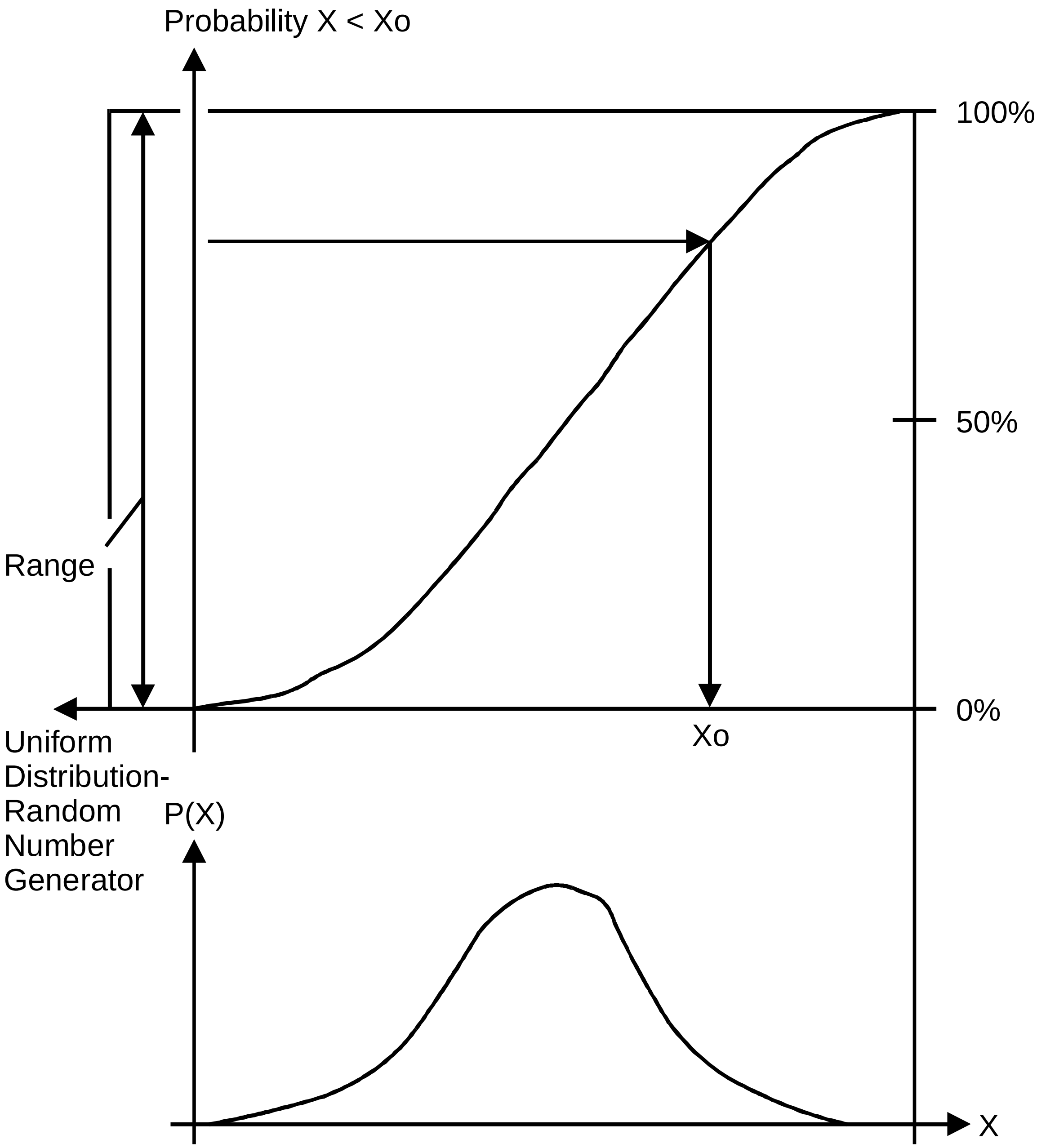

Figure 2 — Cumulative distribution function generation of input variable uncertainty distribution simulation data

NOTE — Using the inverse of the cumulative distribution function to generate the simulation data is one way of accomplishing this task. Other techniques that can be utilized include the following:

a) Using a look up table (see Hald Reference) for the cumulative normal distribution based on the standard normal variable

In this approach, the generation of Gaussian uncertainty distribution simulation points would involve dividing the range of the distribution into a number of intervals, then determining from the table the probability of data points falling within each interval, and, finally, using a random number generator to create a uniform distribution of the required number of simulation points within each interval. The final set of simulation points for all intervals must then be randomized to assure that each variable—so simulated—approximates the random sample discussed in the previous experiment (see Berté Reference).

b) Using an Acceptance - Rejection Method for generating the uncertainty distribution simulation points

In this method, the given uncertainty distribution function is used directly by the following general procedures. Simulation points are generated and then tested to determine if they "fit" the desired distribution. If they "fit," they are kept and utilized in the simulation data set. If not, they are eliminated. (See Rubinstein and Press, et al, References where this method is described in detail.)

4.2 The methods listed in 4.1 produce the simulation equivalent of 100 sensor outputs and 100 transmitter outputs with the specified output uncertainty characteristics. Next, the simulation equivalent for the wiring of the instrument channel circuit must be addressed. This part is modeled by the functional relationship between the sensor and the transmitter. In this case, the output of the sensor is fed into the transmitter. The output of the transmitter is the resultant variable of interest for which we desire the uncertainty distribution. The functional relationship of the sensor and the transmitter signal processing is represented by Equation 2. Recall that the temperatures in Equation 2 are really differences between actual and indicated temperatures.

$$T(_{output}) = T(_{sensor}) + T(_{transmitter}) \qquad \text{(Eq. 2)}$$

Since 100 sets of sensor signals and 100 sets of simulated transmitter signals are analytically summed using Equation 2, a hundred values of "$T(_{output})$" are generated. Figure 3 is a histogram illustrating the results obtained, where the "y" axis is the number of times a small range of $T(_{output})$ values occurred in the 100 simulation runs.

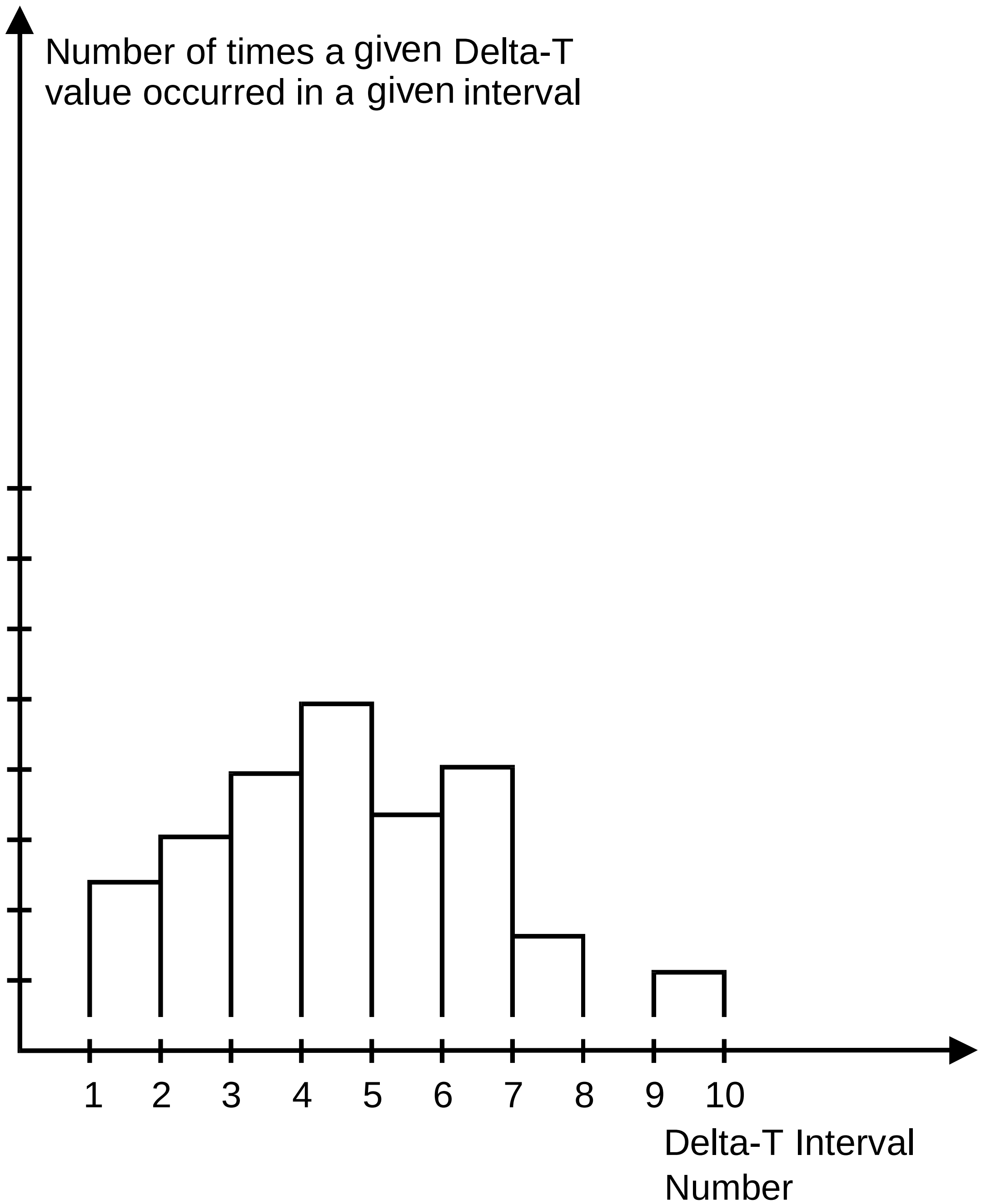

Figure 3 — Sensor/Transmitter Delta Temperature (Delta T) histogram from experimental evaluation procedure

4.3 Once the frequency histogram has been generated, questions concerning probability limits can be evaluated. For example, what is the upper 95 percent probability limit of the instrument channel output temperature? To determine this limit, find the point where the limit would have to be in order for the largest 5 data points in the output histogram to fall above the limit.

4.4 Questions about the output uncertainty distribution parameters can also be answered, such as the following:

a) What is the mean value of the output distribution?

b) What is the standard deviation of the output distribution?

c) Is the distribution Gaussian?

4.4.1 There are many reasons that this methodology is so useful; such as

a) it can readily handle any type of input variable probability distribution, assuming the cumulative distribution function is known;

b) it can readily handle any functional relationship between the input variables and the resultant; and

c) the number of samples simulated is only limited by the computer time available, and accuracy increases with increasing simulation sample size.

5 When should MC techniques be used?

Since SRSS combinatorial techniques are widespread in setpoint evaluations and are fairly straightforward to apply, the engineer should examine the conditions under which the SRSS technique tends to become inaccurate and should use these conditions as markers in considering the use of MC as an alternative.

Before discussing these conditions, consider when the SRSS technique should be used. In the ideal case, SRSS should be used when a resultant variable is a function of a linear combination of random variables. SRSS can be used to find the standard deviation associated with the resultant, given that the standard deviations of the input variables are known. If, in addition, each input variable is known to have an associated Gaussian probability distribution with a known mean and standard deviation, the resultant will also have a Gaussian distribution. Applying the SRSS technique yields the standard deviation of the resultant Gaussian probability distribution. The mean value of the resultant distribution is the sum of the input variable mean values.

This criterion, as found in ISA-RP67.04, Part II, on the distributions of the input variables and their functional relation to the resultant, ideally should be fulfilled for SRSS to be allowed. In actual practice, some relaxation of this criteria is permitted when justifiable approximations are carefully made.

The breakdown in these approximations indicates the need for alternate techniques such as the MC. The following examples are the types of decisions made relative to the need to use alternate combination techniques:

a) A large number of input variables are summed linearly, and some of the associated probability distribution functions are symmetrical but not Gaussian, e.g., if a few variables had uniform distributions.

   In this case, the central limit theorem can be applied and the resultant distribution function will be approximately Gaussian, assuming that the calculated standard deviation of each input variable is utilized.

   NOTE — The exception here would be when only two or three non-Gaussian variables are summed. In this case, higher order derivatives of the Taylor Series expansion, from which the SRSS technique is derived, would have to be employed or an MC technique might be used.

b) The functional relationship between the input variables and the output variable is not linear, but the input variables are definitely Gaussian.

   In this case, an SRSS with Sensitivities (this would involve the incorporation of the first order derivatives in the analytic expression for the resultant standard deviation) might be employed. If a complex functional relationship such as that inherent in a maximizer or a minimizer module exists, an MC technique would probably be more appropriate because the calculation of higher order derivatives becomes too complex.

c) The functional relationship between the input variables and the resultant is non-linear, and the input variables are decidedly non-Gaussian and also non-symmetrical about their respective means.

   An example of this might be when the outputs of two square root extractor modules are input to a maximizer module. In this case, the use of an MC technique would definitely be indicated.

5.1 Sometimes there are no strict delineation criteria that can be used to definitively indicate the appropriate method. The choice is really up to the analyst. Considerations to be addressed might include the following:

a) How important is the result? (Is there enough margin to accept an approximate answer?)

b) Does the approximation err in the conservative or unconservative direction?

c) How difficult is it to do using the higher order derivatives? (How much time and effort is appropriate?)

d) Is the MC code readily available, and can it be used correctly?

6 MC application sequence

The application sequence starts at the functional relationship between the input variables and the resultant variable (first step). This must be known and be capable of translating to analytical form for incorporating into the MC computer code being utilized or written. In some analyses or module characterizations, the functional relationship may already exist as a computer code. The second step involves the characterizations of the input variable uncertainty distributions. For input variables having Gaussian probability distributions, only the mean and standard deviation are needed. For Uniform probability distributions, only the mean and the range are needed as input. Typical simulation codes should already have the CDFs* for these distribution types built-in. For other input variable distribution probability types, the CDF will be required. The CDF is the most general way of characterizing any distribution. The third step is to decide on the number of simulations required. The number of simulations are affected by the following input and functional relationship considerations:

a) The accuracy required in the resultant's probability distribution

b) The computer time required to propagate the input variables through the functional relationship

(Computer time is a consideration when a large computer code is required to characterize the relationship.) The use of a response surface to generate the output variable as a function of the input variables is sometimes used when the actual functional relationship requires excessive running time on a computer. The response surface is usually prepared to approximate the true output in a very limited region around the nominal output of the result. (See Myers Reference.)

After the accuracy- and runtime-related decisions are made, the simulation software should be validated with respect to the functional relationship, as well as to the generation and propagation of uncertainties through the functional relationship. In order to determine if the functional relationship incorporated in the code is giving correct results, the nominal values of each respective input variable can be propagated through the relationship and the result checked. It should be noted that when input uncertainty distributions are propagated through the functional relationship, the nominal or mean value of the resultant distribution may not necessarily have the same mean value as when only the nominal values of the input variables are entered. This can be due to the nonlinearities in the functional relationship or the characteristics of the input distribution types.

* CDF = Cumulative Distribution Functions

6.1 Verifying the generation and the propagation of the input uncertainties (if they are assumed to be independent) can be readily carried out by substituting a functional relationship that is a linear combination of input variables into the code. The resultant output can then be checked by a simple SRSS calculation of the resultant standard deviation. For dependent input uncertainties more complicated testing is required, which should follow the same philosophy of testing under the condition that a simply calculable result is expected, and which can be checked against the code output, i.e., such as using a correlation coefficient of +1 in the test mode.

Interpreting the obtained simulation result involves the following questions related to both simulation and functional uncertainties:

a) How much of the resultant uncertainty is due to the inherent simulation uncertainty?

   This can be solved by running a number of simulation sets; each set will start running at a different time. Normally a random number seed is taken from the computer clock on which the MC simulation code is running and is used to generate the complete set of random numbers used to drive the simulation data set. Therefore, the characteristics of the resultant random variables in each simulation data set will vary slightly and affect the data set specific distribution parameters of the resultant variable probability distribution. The examination of the slight differences in the resultant probability distribution between the simulation sets is a measure of the inherent simulation uncertainty.

b) How is the uncertainty associated with the use of a response surface accounted for?

   If using a response surface is required, this usage contributes another uncertainty component to the final resultant probability distribution. The magnitude of this error is usually an output of the methodology utilized to generate the response surface and should be included in the overall simulation process utilized. One way to incorporate this uncertainty is to generate a random variable that is chosen from an uncertainty distribution (whose characteristics are the same as the response surface error term). This value is added to the resultant output of the response surface at each simulation point.

   NOTE — This outlined pragmatic approach for verifying the generation and the propagation of uncertainties through an algorithm is intended to assist the analyst in informal checking of MC software and does not override the use of standard software Verification and Validation (V&V) Techniques in applications requiring them.

7 Using and interpreting results

The resultant distribution type must be identified before the obtained result is incorporated into a setpoint calculation. If the distribution can be shown to be Gaussian, using the MC analysis result is straightforward. If not, an alternative method must be employed. For example, the question is asked, "What is the 95/95 probability/confidence limit on the resultant obtained?" Assuming the resultant probability distribution can be shown to be Gaussian (via the "W" or "D-Prime tests", see ANSI/N15.15-1974), the number of simulations can be assumed equal to the sample size, and the "Xbar + K Sigma" analytical technique can be used to find the 95/95 limits. If the resultant is non-Gaussian, a set of simulations can be performed, and the distribution of the desired 95 percent probability limit can be obtained. From this distribution, the 95/95 limits can be determined.

8 Available tools that could potentially be used to perform MC analyses

Commercially available general MC software packages include

a) "Sample" software
Provided by the Energy, Science, and Technology Software Center
P.O. Box 1020
Oak Ridge, TN 37831
Phone (615) 576-2606

b) "@Risk" software
Provided by Palisade
31 Decker Road
Newfield, NY 14867
Phone (607) 277-8000

EPRI provides a package of well documented, rigorously tested (nuclear Q/A) software for Monte Carlo and for setpoints analysis:

STARS (Statistical Transient Analysis by Response) NP-7558, January 1992

MOONS (Method of Optimizing Nuclear Setpoints) NP-7543, October 1992

PLANETS (Plant Network Simulation) NP-7557, October 1991

The software is available through

Electric Power Software Center
1930 Hi Line Drive
Dallas, TX 75207

9 Precautions and limitations

If very “large sets” of generated pseudo random numbers are used in an MC simulation, the randomness of these generated number sets may be violated by repetitions. Quantification of the term “large sets” used above is not generically possible since the repetition effect is a function of both the generating algorithm used and computer hardware employed. There are tests that can be run to check for this effect (see Rubinstein Reference).

Annex A — References

AMERICAN NATIONAL STANDARDS INSTITUTE (ANSI)

N15.15-1974	Assessment of the Assumption of Normality
Available from:	**ANSI** 11 West 42nd Street New York, NY 10036 Tel: (212) 642-4900

ISA

ANSI/ISA-S67.04, Part I	Setpoints for Nuclear Safety-Related Instrumentation, 1994
ISA-RP67.04, Part II	Methodologies for the Determination of Setpoints for Nuclear Safety-Related Instrumentation, 1994
ISA-dTR67.04.12	Defining and Handling Dependent Uncertainties in Instrument Channel Uncertainty Analyses, 1995, Draft 3
ISA-dTR67.04.10	Vendor Data
Available from:	**ISA** 67 Alexander Drive P.O. Box 12277 Research Triangle Park, NC 27709 Tel: (919) 549-8411

MISCELLANEOUS

Berté, F.J., “The Application of Monte Carlo and Baysesian Probability Techniques to Flow Prediction and Determination”; presented at the Flow Measurement Symposium, National Bureau of Standards, Gaithersburg, MD, February 1977.

Hald, A., *Statistical Theory with Engineering Applications*, John Wiley & Sons, Inc., 1952.

Myers, R.H., *Response Surface Methodology*, Boston, MA: Allyn and Bacon, 1971.

Press, W.H., et al., *Numerical Recipes in Fortran, 2nd Edition*, Cambridge University Press, 1992.

Rubinstein, R.Y., *Simulation and the Monte Carlo Method*, John Wiley & Sons, Inc., 1981.

Annex B — Applications of the MC technique

B.1 Single module application example

In a typical instrument string used to evaluate core "Delta T," the cold temperature is taken as the maximum of the two loop "T_{cold}" measurements. Recalling the criteria for assessing when SRSS is usable (Gaussian inputs and a linear combination), the maximizer violates the linear combination criteria, even though the individual loop "T_{cold}" values may have an associated Gaussian uncertainty distribution. This would indicate the need for an alternative calculational technique to evaluate the resultant maximized "T_{cold}" uncertainty distribution. If an MC technique is chosen, the following analysis sequence is utilized. Decide on the number of simulations to be utilized. In this case, the functional relationship is simple, since all that is required is for the program to decide which loop temperature, chosen at random from the simulated loop 1 and loop 2 "T_{cold}" values, is larger and then to place that value into the resultant bin.

The number of simulations can be 1000 or more since the computer run time will not be excessive, and the accuracy of the result at the 95 percent probability level will be assured since 50 simulation points are expected to fall above the upper 95th percentile of the resultant distribution, assuming that 1000 simulation points are used. The result of this analysis is shown in Figure B-1, along with the resultant maximized "T_{cold}" uncertainty distribution that would have been obtained if a simple SRSS calculation had been completed. Note that the mean of MC distribution is shifted (biased) toward the higher values and further, that the standard deviation is smaller than that of the SRSS distribution. Using the resulting bias and the standard deviation (if the maximized "T_{cold}" distribution was Gaussian) in an instrument channel uncertainty calculation would then proceed as normal.

B.2 Instrument string application example

The instrument channel example is a little different from the individual module example given previously in that the resultant from one module, the sensor, may be fed into the input of the next module, the transmitter etc., in the channel. In this case, it is assumed that the whole channel is simulated in the MC analysis. The Maximizer module is used in this example. Figure B-2 is a schematic drawing of the instrument channel modules for determining core "Delta T," showing sensor signal processing.

The input data for these MC calculations is given in Table B-1.

Table B-1 — Input variable data

Variable Name	Mean	Standard Deviation	Correlation Coefficient
T_{hot}^{-1} (loop 1)	542.0	1.0	+0.3 (Th1 correlated to Th2)
T_{hot}^{-2} (loop 2)	542.0	1.0	+0.3 (Th1 correlated to Th2)
T_{cold}^{-1} (loop 1)	528.0	1.0	+0.7 (Tc1 correlated to Tc2)
T_{cold}^{-2} (loop 2)	528.0	1.0	+0.7 (Tc1 correlated to Tc2)

NOTES

1. It is assumed that the Averager, Maximizer, and Delta-T Calculator modules do not contribute additional uncertainties but do affect the overall signal processing uncertainty through their functions (as indicated).

2. It is assumed that the Correlation Coefficients have been determined from experimental analyses as discussed in ISA-dTR67.04.12 (see Reference).

The calculation sequence follows the schematic in Figure B-2. Table B-2 lists the functional relationship for each module.

Table B-2 — Functional relationships and constraints in each module

Module Name	Functional Relationship	Constraint
Averager	Th-Avg.= (Th1 + Th2)(1/2)	Th1 and Th2 Correlated
Maximizer	Tc-Max. = (Larger value of Tc1 or Tc2) Selected and Propagated	Tc1 and Tc2 Correlated
Delta T Calculator	(T_{cold} Max. - T_{hot} Avg.)	none

The data from Table B-1 was entered into the appropriate equations in Table B-2. The results of the calculations are shown in Table B-3.

Table B-3 — Results

Calculation	Mean	Standard Deviation	Distribution Type
a. T_{hot} Avg.	541.96	0.81	Gaussian
b. T_{cold} Max.	528.32	0.96	Non-Gaussian
c. Delta T	13.64	1.26	Non-Gaussian

If the individual module output signal Distribution Types are Gaussian and the combination of uncertainties for the modules in the instrument string is a linear sum, the intermediate calculation results can be combined by the SRSS technique with this incorporation. In this example this is not the case, since the bias is the mean value caused by the maximizer in the "T_{cold}" module; in addition the resultant distribution of the maximizer was not Gaussian, as shown in Figure B-1. If any of these conditions required for using SRSS are not met, the simulated output signal distribution values of the "upstream" (nearest the signal source) modules must be the input to the "downstream" modules.

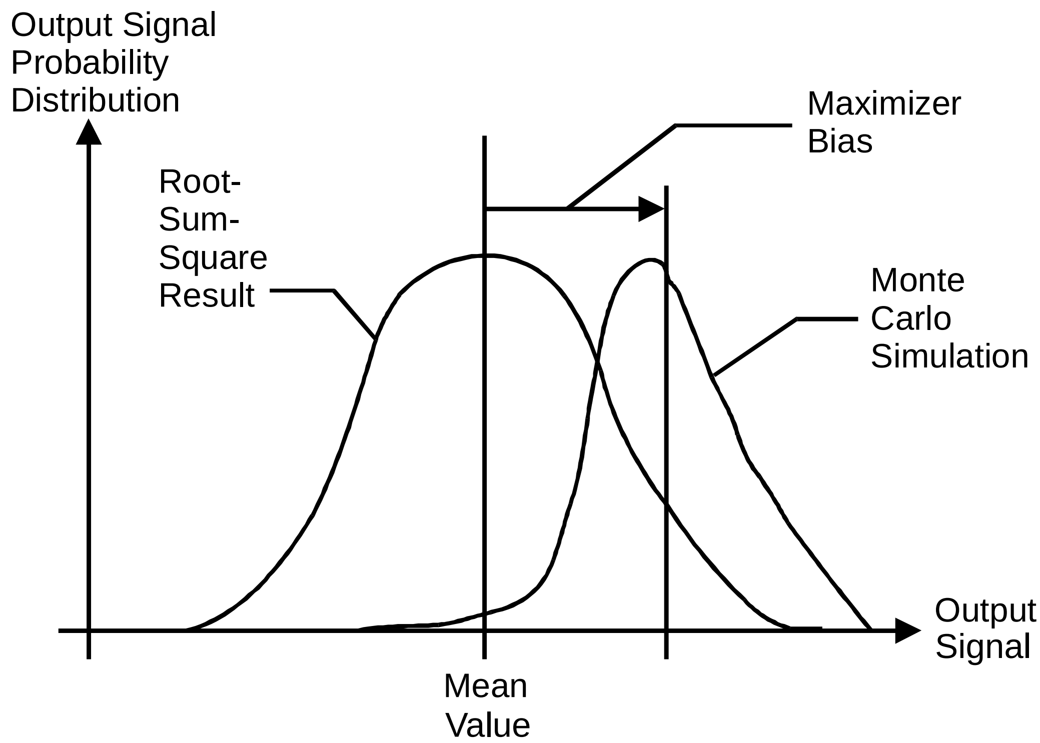

Figure B-1 — Maximizer output signal for SRSS and MC uncertainty combination methods

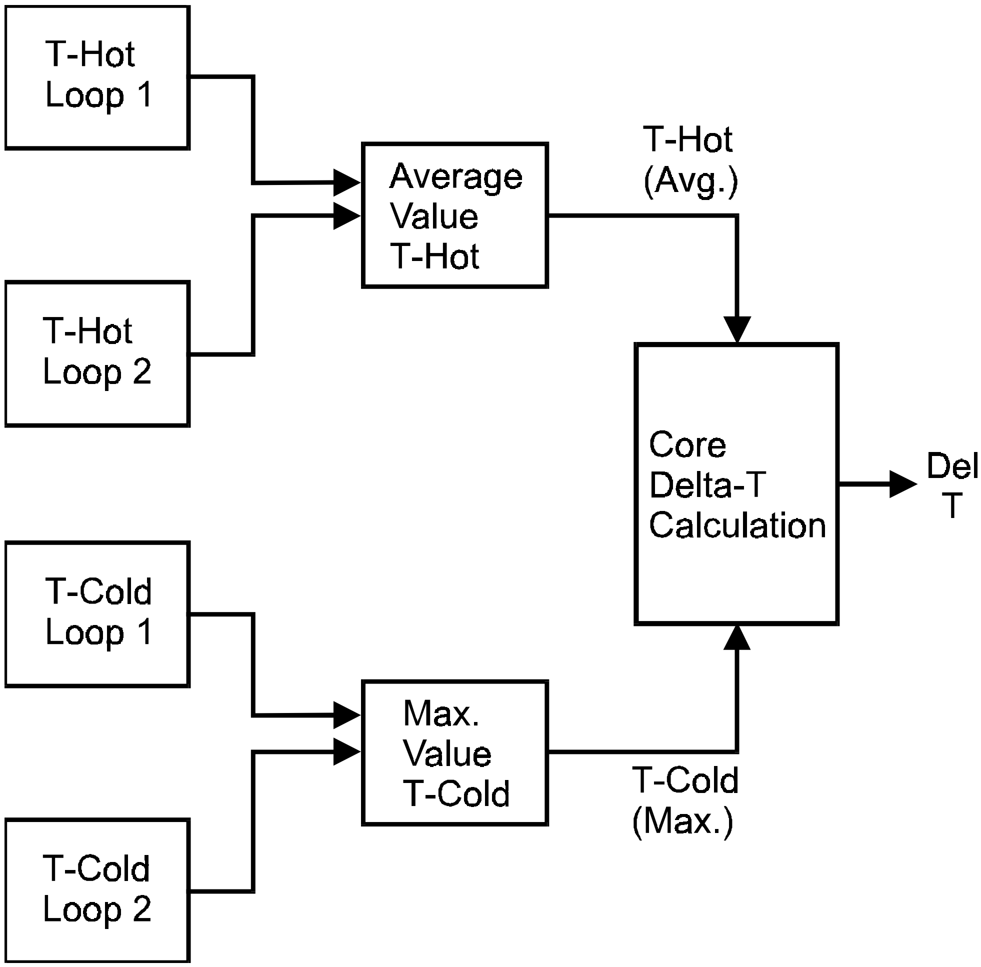

Figure B-2 — Core "Delta T" determination—instrument channel schematic

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