Identidades Hiperbólica

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Definición

(1) $\operatorname{senh}(x) = \dfrac{e^x - e^{-x}}{2}$, $x \in \mathbb{R}$

(2) $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$, $x \in \mathbb{R}$

(3) $\operatorname{tgh}(x) = \dfrac{\operatorname{senh}(x)}{\cosh(x)}$, $x \in \mathbb{R}$

(4) $\operatorname{sech}(x) = \dfrac{1}{\cosh(x)}$, $x \in \mathbb{R}$

(5) $\operatorname{csch}(x) = \dfrac{1}{\operatorname{senh}(x)}$, $x \in \mathbb{R} \setminus \{0\}$

(6) $\operatorname{ctgh}(x) = \dfrac{\cosh(x)}{\operatorname{senh}(x)}$, $x \in \mathbb{R} \setminus \{0\}$

Identidades hiperbólicas

(1) $\cosh^2(\alpha) - \sinh^2(\alpha) = 1$

(2) $1 - \operatorname{tgh}^2(\alpha) = \operatorname{sech}^2(\alpha)$

(3) $-1 + \operatorname{ctgh}^2(\alpha) = \operatorname{csch}^2(\alpha)$

(4) $\sinh(\alpha \pm \beta) = \sinh(\alpha)\cosh(\beta) \pm \sinh(\beta)\cosh(\alpha)$

(5) $\cosh(\alpha \pm \beta) = \cosh(\alpha)\cosh(\beta) \pm \sinh(\alpha)\sinh(\beta)$

(6) $\operatorname{tgh}(\alpha \pm \beta) = \dfrac{\operatorname{tgh}(\alpha) \pm \operatorname{tgh}(\beta)}{1 \pm \operatorname{tgh}(\alpha)\operatorname{tgh}(\beta)}$

(7) $\sinh(2\alpha) = 2\sinh(\alpha)\cosh(\alpha)$

(8) $\cosh(2\alpha) = \cosh^2 + \sinh^2(\alpha)$

(9) $\operatorname{tgh}(2\alpha) = \dfrac{2\operatorname{tgh}(\alpha)}{1 + \operatorname{tgh}^2(\alpha)}$

(10) $\sinh^2(\alpha) = \dfrac{\cosh(2\alpha) - 1}{2}$

(11) $\cosh^2(\alpha) = \dfrac{\cosh(2\alpha) + 1}{2}$

(12) $\sinh\left(\dfrac{\alpha}{2}\right) = \sqrt{\dfrac{\cosh(\alpha) - 1}{2}}$

(13) $\cosh\left(\dfrac{\alpha}{2}\right) = \sqrt{\dfrac{\cosh(\alpha) + 1}{2}}$

(14) $\tanh\left(\dfrac{\alpha}{2}\right) = \sqrt{\dfrac{\cosh(\alpha) - 1}{\cosh(\alpha) + 1}}$

(15) $\sinh(\alpha) + \cosh(\beta) = 2\sinh\left(\dfrac{\alpha + \beta}{2}\right)\cosh\left(\dfrac{\alpha - \beta}{2}\right)$

(16) $\cosh(\alpha) + \cosh(\beta) = 2\cosh\left(\dfrac{\alpha - \beta}{2}\right)\cosh\left(\dfrac{\alpha - \beta}{2}\right)$