Inelastic Effective Length Factor of Nonsway Reinforced Concrete Columns

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Abstract: This paper proposes a new equation for the effective length factor (*k*-factor) for reinforced concrete columns in braced frames. The new formula is valid both for normal and high-strength concrete. The equation was obtained from a sensitivity analysis performed on a two-dimensional nonlinear finite-element numerical model that takes into account the inelastic behavior of the concrete columns (cracking, yielding, and second order effects). The numerical model was calibrated with 44 experimental tests performed by the writers' research group. A comparative study was carried out between the numerical model and different national design codes, displaying important differences with respect to all of them: the ACI code (from 37 to −3%), the Spanish code EHE (from 26 to −9.26%), and the Eurocode 2 (from 14 to −14%). It was decided to propose two additional simplified equations: one for checking and the second for design.

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Introduction

The evaluation of the effective length factor (*k* factor) in real concrete columns is not properly studied at the present time. This is due to the fact that most of the equations to obtain such factor are developed assuming a linear elastic material behavior or a reduced stiffness *EI* of the column, which is far from the real behavior of reinforced concrete columns, where the strength of concrete, the reinforcement ratio, the slenderness, and the stiffness of the joints have an important effect on the curvature of the support.

As is well known, the *k* factor transforms the buckling of a column with different stiffness restraints at the ends in the buckling of another equivalent pinned-pinned column with an effective buckling length ($L_{\text{eff}}=kL$). The differential equations of both problems have been widely solved for an elastic material (Duan and Chen 1999) and they are implemented in the national design codes (both for steel or concrete structures) through the use of simplified equations of the effective length factor or the well-known alignment charts.

Typically the *k*-factor depends on the relative stiffness of the joints Ψ_i, also called "end restrain factor" (the sum of the column stiffness divided by the sum of beam stiffness). This factor is used in the American code ACI_(American Concrete Institute 2005) or the Spanish EHE (2001), which can vary from 0 to infinite

$$\Psi_i = \frac{\sum \left(\frac{EI}{L}\right)_{\text{columns}}}{\sum \left(\frac{EI}{L}\right)_{\text{beams}}} \tag{1}$$

But Aristizábal-Ochoa (1994) proposed a "fixity factor" (ρ_i) that varies from 0 to 1

$$\begin{cases} \rho_1 = \dfrac{1}{1+\dfrac{3}{R_1}} \\ \rho_2 = \dfrac{1}{1+\dfrac{3}{R_2}} \end{cases} \quad \begin{cases} R_1 = \dfrac{K_1}{\left(\dfrac{E_c I_g}{L}\right)} \\ R_2 = \dfrac{K_2}{\left(\dfrac{E_c I_g}{L}\right)} \end{cases} \tag{2}$$

where E_c=elastic modulus of concrete; I_g=gross moment of inertia of the column section; L=unsupported length of the column; K_1 and K_2=stiffness of each spring (end restraint condition). These springs represent the stiffness of the two beams and the exterior column that arrive to the joint. Typically *EI* and K_1 and K_2 are considered elastic.

Both the end restraint factors and fixity factor result in the same effective length factor because they are the solutions to the same differential equation in the elastic range, but use different nomenclature. The elastic *k*-factor for nonsway columns varies from 0.5 (clamped-clamped) to 1 (pinned-pinned).

For reinforced concrete structures there were a lot of studies in the elastic range regarding the design of slender columns. Cran-

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ston (1972) proposed simplified equations for the effective length factor. Also, a lot of work studying the influence of different end restraint conditions was completed by Hu et al. (1993) who proposed a new equation based on the partial fraction model.

Inelastic Behavior

Moreover, the theoretical problem of inelastic buckling was pointed out a long time ago with the tangent elastic modulus theory and employing the reduced modulus theory or the Shanley's theory (Shanley 1947).

For steel structures, Yura (1971) and Disque (1973) presented an inelastic *k*-factor method for steel structures using the concept of tangent elastic modulus, employing the same alignment charts but modifying the end restrains factors.

Besides, the problem for reinforced concrete structures has not been deeply studied, although MacGregor et al. (1970) and Breen et al. (1972) pointed out the necessity to get deeper in this subject. A simple but good manner to include the inelastic behavior of beams and columns is the one implemented in the ACI code (American Concrete Institute 2005), where the stiffness of beams and columns for calculating the end restraint factor is reduced using a fixed factor (i.e., for columns $EI=0.7E_cI_g$).

But the influence of the *k*-factor on the behavior of reinforced concrete structures is dependent on the strength of concrete (Broms and Viest 1961), the slenderness, the fixity factors, and the steel reinforcement ratio because they contribute to the nonlinearity of the column. Moreover, if a column is cracked or yielded, its stiffness is lower than the elastic one. In this case the *k*-factor will be lower than the elastic one. Conceptually this makes sense because it is as though the rotational springs are relatively more rigid, having a tendency toward the behavior of the clamped-clamped column (for which the elastic *k* is 0.5).

With the actual sophistication of the numerical models, the concrete can be modeled closer each time to the real behavior. Thereupon, Bazant and Xiang (1997) studied the inelastic buckling of concrete columns in braced frames but focused the study to improve the method of analysis and not to obtain the *k* factor. They assumed a sine curve as the deflection curve of the column and implemented all the nonlinearities of concrete. The improvement consisted in considering the wavelength as unknown and variable during loading. Conceptually this is the same as the effective length factor. Later, Furlong (1998) discusses about the interest for practitioners to include a very complex method (although more realistic) in the codes.

Moreover, concrete technology has been improved considerably and now high-strength concrete (HSC) can be easily obtained, whose mechanical behavior cannot be extrapolated simply from that of normal strength concrete (NSC). The different simplified methods that can be used for analysis in failure for slender columns therefore need to be checked, so that their application might be extended to HSC from NSC.

The objective of this paper is therefore to establish an improved *k*-factor equation which includes the complicated behavior observed for reinforced concrete structures, in which inelastic deformations are combined with tensile cracking and bond slip. The equation is limited to nonsway columns.

Numerical Simulation

To simulate this behavior a nonlinear finite-element software was selected and calibrated with 44 experiments performed by this research group. The required numerical software had to be two-dimensional (2D) (membrane elements and trusses) to recreate: 2D strain-softening constitutive relations, distributed cracking, bond-slip, plastic behavior of steel and concrete, the appearance of plastic hinges, second order effects, etc. The software selected was ATENA (Cervenka 1998). The model included a biaxial fracture criteria, tension stiffening, and quadratic isoparametric finite elements with 4 G integration points. So as not to extend the paper too much, the complex task of calibration can be read in Bendito (2006), where an error of 2.03% was achieved. Hereinafter, this virtual laboratory allowed performing more tests to propose a new equation for the buckling length.

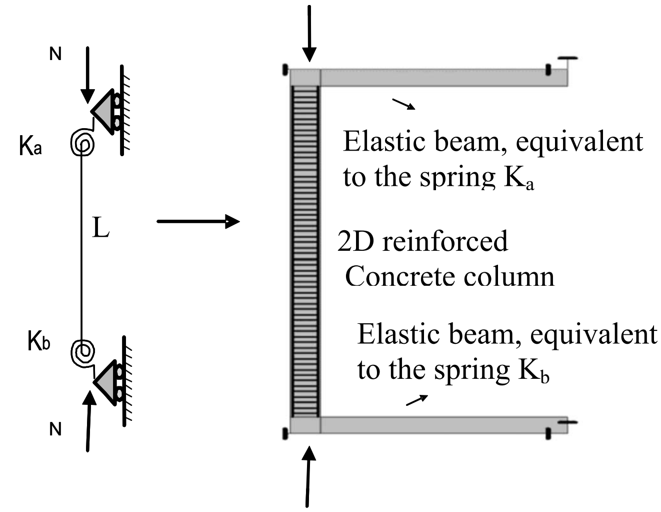

Fig. 1. Geometry of the problem

The 2D finite-element software with membranes elements cannot simulate directly the classical rotational spring, because it has only 2D degrees of freedom (u and v). To do that, special purpose geometry was created: a column with two elastic beams. These beams represent the rotational springs (Fig. 1).

A preliminary study to test the geometrical 2D model was developed in three steps to verify that the buckling behavior is acceptable.

1. Column and springs (that is, beams) were modeled initially using elastic materials to compare the numerical *k* factor with the theoretical elastic solution. Both *k* factors were similar to the second decimal.
2. The second test was accomplished supposing an inelastic behavior for columns but the springs were modeled with 0 stiffness. Load-slenderness graphs were obtained to compare with the elastic Euler's hyperbola values. Also normalized load-slenderness curves were created to check coherence in

Table 1. Parameters of Study in the Sensitivity Analysis

Parameters of study	Scope
Strength of concrete f'_c	30, 60, and 90 MPa
Longitudinal reinforcement ratio ρ_g	2, 3, and 4%
Geometric slenderness $\lambda=L/h$	20,30,35, 40, and 50
Yield stress of steel, f_y	400 and 500 MPa
Fixity factor of the rotational springs, ρ_1 and ρ_2	$\rho_1=0.2$ and $\rho_2=0.2$
	$\rho_1=0.2$ and $\rho_2=0.8$
	$\rho_1=0.8$ and $\rho_2=0.8$

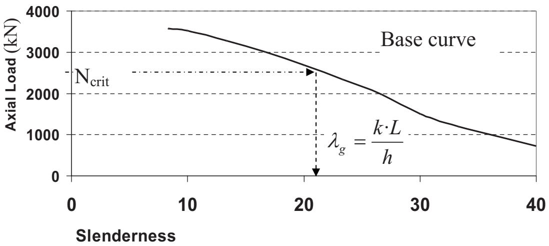

Fig. 2. Procedure to obtain the inelastic buckling coefficient

type of concrete and the reinforcement ratio influence. A statement from Broms and Viest (1961) was numerically verified: "An increase in the proportion of the load carried by the reinforcement leads to a more stable column, i.e., high-strength concrete columns, or those with less longitudinal reinforcement ratio tend to be more affected by length." Geometrical model with 0 stiffness value for beams shows similar behavior to that of a pinned-pinned column, so they could be used to define the buckling length for different column lengths and different stiffness of beams simulating springs. Load-slenderness curves with beam stiffness equal to 0 and inelastic columns will be called "base curves." There was generated a base curve for each parameter combination (f'_c, f_y, and ρ_g).

3. Analyze inelastic columns with different elastic stiffness springs. There were found important differences between elastic and inelastic effective length factors (lower *k*-factor values). This preliminary study made necessary a deeper study of sensitivity to detect the parameters of major influence on the effective length factor.

Sensitivity Numerical Study

The variables that were studied are presented in Table 1. Both, column and beams had a square section of 30×30 cm^2. The reinforcement was four bars, located at each corner of the column in a symmetric distribution. The mechanical reinforcement cover was fixed at 10% of the height and the width of the section. As it was said in the previous section, a real curve of the critical axial load versus slenderness was obtained for each section configuration with the numerical model for a pinned-pinned column (base curve). This curve improved the elastic Euler's hyperbola, because it includes the nonlinearities of the model. It was adjusted with a fifth degree polynomial.

The maximum load under compression was obtained for each parameter using the numerical model and also including the equivalent rotational springs. Substituting this critical axial load in the base curve, the equivalent slenderness was obtained. The inelastic *k*-factor was obtained from it, $[k=(\lambda h)/L]$ (see Fig. 2). The hypothesis that the effective length factor of the inelastic, pinned column is equal to unity, i.e., the same as for elastic columns is accepted. Many graphs were generated in the sensitivity study but only some of them are presented in this paper (Figs. 3 and 4). From the complete sensitivity study it can be inferred that *k* factor increases with the concrete strength and the longitudinal reinforcement ratio, and decreases with the increment of the fixity factor (obvious). Both parameters had the same influence in the inelastic effective length factor, around 35 and 37%. However, only 1% of difference is observed when the strength of steel was modified between 400 and 500 MPa. So, the steel strength was fixed to 500 MPa.

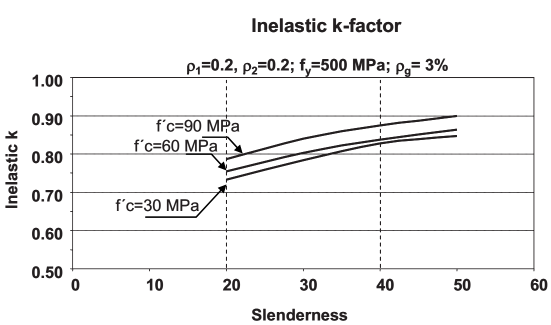

Fig. 3. Curve inelastic $k-\lambda$ for $\rho_g=3\%$, $f_y=500$ MPa, changing f_c

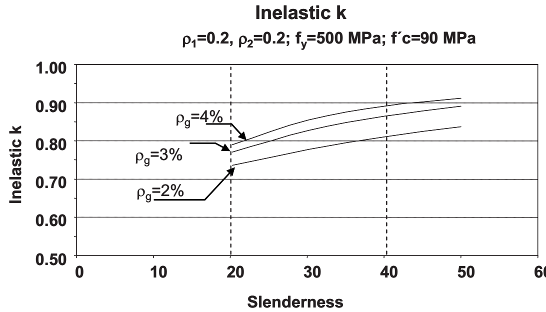

Fig. 4. Curve inelastic $k-\lambda$ for $\rho_g=4\%$, $f_y=500$ MPa, changing f_c

Comparison between the Numerical Model and the Design Codes

The inelastic *k* factor was compared with the codes ACI-318 (American Concrete Institute 2005), the Spanish code EHE (1999), the Eurocode 2 (EC2) (European Committee for Standardization 2004), and with a previous equation proposed by Traver and Bonet (2002). This last equation comes from a one-dimensional finite-element analysis.

It was deduced that for all the cases and for any slenderness, the inelastic *k*-factor was lower than that obtained using the equation from the ACI and Spanish EHE code. Regarding the EC2 and concerning some of the slenderness, however, the inelastic effective length was higher (Figs. 5 and 6).

Higher differences were observed for the lower strength of concrete (f'_c), lower reinforcement ratio (ρ_g) and for the lower stiffness rotational springs (ρ_1 and ρ_2). It is important to find out

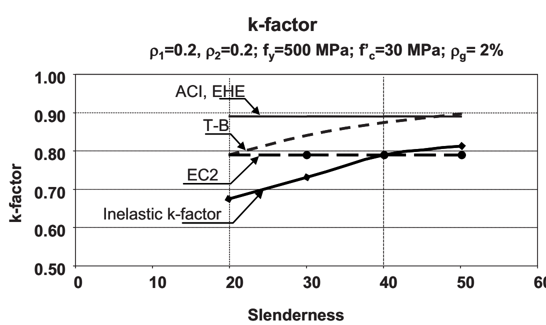

Fig. 5. Comparative curves for ACI code, Spanish EHE, Eurocode 2 (EC2), and Traver and Bonet (TB), for NSC

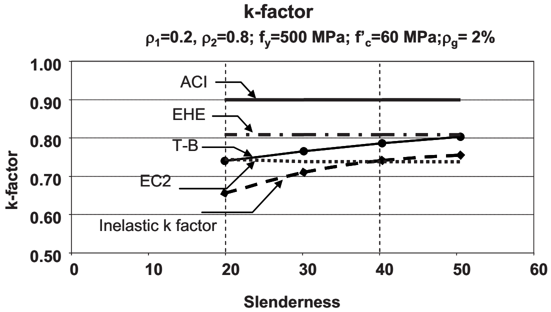

Fig. 6. Comparative curves for ACI code, Spanish EHE, Eurocode 2 (EC2), and Traver and Bonet (TB), for HSC

that the buckling coefficient of the ACI and EHE codes does not take into account the reinforcement ratio; hence for different percentages of it, the values of the k factor are the same. However, the simplified equation of Traver and Bonet (2002), the equation of the EC2, and the inelastic k factor change with f'_c, reinforcement ratio and the slenderness. The errors are shown in Table 2. Because there are representative differences with respect to all of them: the ACI code (between 37 and −3%), with the Spanish code EHE (26 and −9.26%), with the Eurocode 2 (between the 14 and −14%), and regarding Traver and Bonet (14 and −7%), it was decided to propose a new equation for the effective length factor for nonsway columns.

Equation of the Inelastic Effective Length Factor

It is better to adjust the equation in terms of the fixity factors ρ [Eq. (2)], which varies between 0 and 1, whereas the rotational stiffness varies between 0 and infinity. Thereby, the k factor is calculated in terms of the fixity factors "ρ" initially with respect to a fixed slenderness ($\lambda=35$) to include in the main part of the equation only the variables of the strength of concrete and the longitudinal reinforcement ratio. Later on, the slenderness will be included using a correcting parameter.

For a particular case of $f'_c=60$ MPa and a 2% of longitudinal reinforcement ratio the procedure to obtain an equation of k is explained below. A graph of $k-\rho_2$ is obtained, with fixed ρ_1 (not presented for simplicity). The first graph is for $\rho_1=0.2$ and the second one is for $\rho_1=0.8$.

Only these two graphs are obtained to create initially a very simple equation of k, which is a linear interpolation of ρ_2

$$k = a\rho_2 + b \quad (3)$$

But what is very important is that the coefficients "a" and "b" are not constant; they vary with ρ_1.

If the values of a and b are presented in terms of ρ_1, it can be inferred that they are also linear functions. These coefficients, shown in Table 3, are obtained from a trend line.

Table 2. Analysis of Errors

Values	TB	EHE	ACI	EC2
Maximum (%)	14.1	26.2	37.7	14.2
Minimum (%)	−7.7	−9.2	−3.6	−13.8
Standard deviation	0.056	0.07	0.093	0.062
Average (%)	2.9	5.4	16.7	−1.6

Table 3. Values of a and b

ρ_1	a	b
0.2	−0.25	0.86
0.8	−0.16	0.69

Replacing the values of a and b, the following equation is obtained:

$$k = (0.15\rho_1 - 0.28)\rho_2 + (-0.2817 + 0.9167) \quad (4)$$

Simplifying the previous equation, Eq. (5) is obtained

$$k = 0.15\rho_1\rho_2 - 0.28(\rho_1 + \rho_2) + 0.92 \quad (5)$$

The same procedure is performed for each case of longitudinal reinforcement ratio and strength of concrete, reaching the values of k presented in Table 4.

Correcting Factor "α"

The fixity factors ρ_1 and ρ_2 depend on the rotational stiffness of the beams K_1 and K_2, the unsupported length L, and the stiffness E_cI_g of the column [Eq. (2)]. In the real behavior of the columns, the stiffness EI is not elastic because it will vary due to the cracking of concrete, the creep, the reinforcement, etc., but to include a complex equation of EI will complicate extremely the method. The influence of f'_c, f_y, and ρ_g was included in the previous section in the equations of "k." Hereby, it is necessary to complement the previous equations with a correction parameter to include the effect of the geometric slenderness because in the previous step it was fixed to $\lambda=35$. Eq. (2) is reformulated as

$$\rho_i = \frac{1}{1 + \dfrac{3}{R_i}} \qquad R_i = \frac{K_i}{\alpha\left(\dfrac{E_cI_g}{L}\right)} \quad (6)$$

where α = slenderness correcting factor.

The parameter α can be calculated by performing the following steps in sequence, for the other cases of slenderness ($\lambda=20$, 30, 40, and 50):

- The values of the fixity factors ρ_1 and ρ_2 are analytically replaced in the corresponding equation of Table 4.

Table 4. Equations of the Inelastic k Factor with respect to the Stiffness Factors, with $f_y=500$ MPa

ρ_g	$f'_c=30$ MPa
2%	$k=0.20\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.90$
3%	$k=0.20\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.92$
4%	$k=0.20\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.95$
ρ_g	$f'_c=60$ MPa
2%	$k=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.92$
3%	$K=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.95$
4%	$K=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.97$
ρ_g	$f'_c=90$ MPa
2%	$K=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.95$
3%	$K=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.98$
4%	$k=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+1$

Table 5. Proposed Equation of k Factor for Checking

Normal strength concrete (up to 50 MPa)	$k=0.2\rho_1\rho_2-0.28(\rho_1+\rho_2)+A$ where $A=0.025\rho_g+0.85<=1$
High-strength concrete	$k=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+B$
(between 50 and 90 MPa)	$B=0.03\rho_g+f_c/70<=1$
(higher than 90 MPa)	$B=0.025\rho_g+f_c/100<=1$
where ρ_g=longitudinal reinforcement ratio	
Slenderness correcting factor	$\alpha=0.04\lambda-0.4$
where λ=geometric slenderness	

- The value of inelastic k factor is known because it was previously obtained numerically. In this case, the value of k can be related to only one unknown, the parameter α.
- As the equation of k depends on α in a quadratic form; an iterative procedure is performed. The first value of α will be termed "alpha-trial."
- The values of α are adjusted until both values of k are matched. Doing that, the effect of the last variable λ is included in the procedure.
- The values of α are obtained in terms of the slenderness. A relationship is obtained $\alpha=0.04\lambda-0.40$.

Simplified Equations for Design and Checking the Codes

In this section, simplified equations valid for implementation in national codes are presented. The equations are valid for normal and HSC both for design and checking.

Proposed Equations for Checking

The equations for checking are in terms of strength of concrete f'_c, the longitudinal reinforcement ratio ρ_g, and the fixity factors ρ_1 and ρ_2 (Table 5).

Comparison between the Proposed Simplified Equation for Checking and the Exact Inelastic k Factor

Fig. 7 compares the proposed equation for checking with the exact inelastic effective length factor to demonstrate that it has better accuracy than that existing in the codes. The errors have been diminished from the initial 14% until 5.7%.

If some random cases are computed for casual values of slenderness, rotational stiffness, and strength of concrete, the maximum error is as low as 1.8%. In conclusion, the proposed equation has a good accuracy for the calculation of the k factor for inelastic columns and elastic rotational springs.

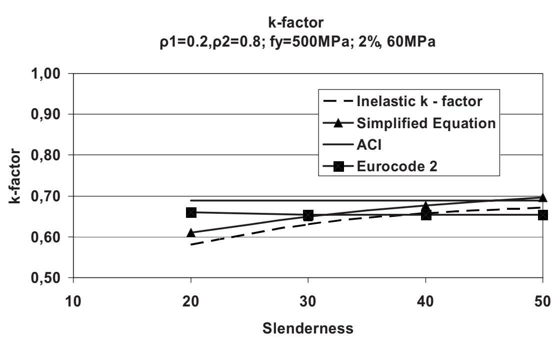

Fig. 7. Comparison between the proposed equation for checking and the inelastic k-factor

Table 6. Proposed Equation for Design

Normal strength concrete	$k=0.2\rho_1\rho_2-0.28(\rho_1+\rho_2)+0.95$
High-strength concrete	$k=0.15\rho_1\rho_2-0.28(\rho_1+\rho_2)+1$
where ρ_g=longitudinal reinforcement ratio	
Slenderness correcting factor	$\alpha=0.04\lambda-0.4$
where λ=geometric slenderness	

Proposed Equation for Design

The proposed equation for design is simplified to not depend on the longitudinal reinforcement ratio. They depend on f'_c and ρ_1 and ρ_2 (Table 6). Table 7 summarizes the errors for both equations (checking and design). It can be noticed that the average error is 2.6% for checking and around 7.6% for design in the safe side, which improves greatly the existing methods in the codes.

Conclusions

The calculation of the effective length factor in real concrete columns is not properly addressed now. The reason is, most of the research to obtain such length assumes a linear elastic material behavior, which is not the case for reinforced concrete. There is no research study prior to the present one that uses 2D nonlinear finite-element analysis to study the effective length factor.

- It was demonstrated that if the real behavior of the column is modeled, the k-factor is lower than the elastic one.
- If a sensitivity study is performed, the strength of concrete and the longitudinal reinforcement ratio have the same influence on the inelastic k-factor coefficient, around 35 and 37%. However the yield stress of steel has not any influence.
- If a comparative study is performed between the numerical model and the different codes, it can be shown that there are representative differences with respect to all of them: the ACI code (between 37 and −3%), with the Spanish code EHE (26 and −9.26%), with the Eurocode 2 (between the 14 and −14%), and regarding Traver and Bonet (14 and −7%). It was decided to propose a new equation for the effective length factor for nonsway columns.

Three types of equations were proposed for the inelastic k-factor: one complete and two simplified (checking and design). It can be noticed that the medium error is 2.6% for checking and around 7.6% for design in the safe side, which improves greatly on the existing methods in the codes.

Table 7. Error for Both Methods

Values	Error for checking (%)	Error for design (%)
Maximum	6.19	19.02
Minimum	0.00	0.00
Typical deviation	0.02	0.04
Average	2.60	7.63

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Notation

The following symbols are used in this paper:

E_c = elastic modulus of concrete;
f_c' = cylinder strength of concrete;
f_y = yield stress of steel;
h = height of the column cross section;
I_g = gross moment of inertia;
K_1 and K_2 = stiffness of end springs (end restrain condition);
k factor = effective length factor;
L = unsupported length of the column;
α = slenderness correcting factor;
λ = geometric slenderness$=L/h$; and
ρ_i = fixity factors.

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