SIMPLIFIED MODEL OF LOW CYCLE FATIGUE FOR RC FRAMES

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ABSTRACT: This note describes a simplified model of damage for reinforced concrete (RC) frame members. The model is different from those proposed in the literature because it combines the concept of inelastic hinge and the methods of fracture and continuum damage mechanics. The model allows the characterization of the following effects: (1) unsymmetrical cross section with different yield capabilities at positive and negative bending; (2) influence of the axial force on the flexural member behavior, stiffness, and strength degradation due to cracking of the concrete; (3) plastic deformations due to the yield of the reinforcement; and (4) different stiffness under positive and negative flexure due to unsymmetrical cracking even in symmetrical sections and low cycle fatigue. The modeling of the low cycle fatigue effects, which is the main object of this note, is made using concepts from fracture mechanics. Numerical simulations of tests reported in the literature validate the model.

INTRODUCTION

The modeling of RC frame behavior under cyclic loading still remains an open problem. The existing models can be classified into three approaches: (1) lumped plasticity models; (2) distributed plasticity models; and (4) multilayer models. The first one involves the use of the concept of inelastic hinge or some other similar concept. The second is based on phenomenological moment-curvature relationships. The third is based on uniaxial stress-strain relations.

The model proposed in this note can be classified as belonging to the first category. Most of those models are based on concepts of the plasticity theory. However, one of the most important goals of nonlinear frame analysis is to make a damage evaluation (e.g., due to earthquakes). This damage analysis is often made by postprocessors that evaluate the state of the structure using the concept of "damage index."

On the other hand, in continuum mechanics, structural damage analysis is made with specifically designed models that are mainly based on fracture mechanics or continuum damage mechanics. The concept of damage index does not exist, or is included as a very particular case of the more general "damage variable." Even if these models use concepts from plasticity, they belong to quite a different category.

The objective of this note is to explore the possibilities of an approach that combines the concept of inelastic hinges with the methods of fracture and continuum damage mechanics. Additionally, an attempt is made to solve some of the drawbacks related to the use of the lumped inelasticity concept, specifically, those associated with the determination of the model parameters.

This note is based on the results presented in Flórez-López (1995). The model proposed there introduced two kinds of internal variables: (1) plastic rotations of the hinges; and (2) damage variables. The latter are not indices and are related to concrete cracking and its influence on the flexural behavior of the member. It was assumed that damage evolution was described by the Griffith criterion. That model showed a good correlation with experimental results in some monotonic and hysteretic examples. However, it presented some important limitations, among them, the fact that the model did not represent low cycle fatigue. The constitutive equations proposed in this note generalize that model because they include low cycle fatigue effects. This modification is obtained using exclusively the concepts from fracture and damage mechanics.

NOTATION

Let us consider a frame member between nodes i and j. The same notation introduced in Flórez-López (1995) will be used in order to characterize the behavior of the member. This notation presents the analysis of frames using the same scheme of continuum mechanics, i.e., the behavior of the member is described in terms of "stresses and deformations," "internal variables," and "thermodynamic forces." In order to differentiate these variables from their equivalents of the continuum mechanics, we add the adjective "generalized."

The conventional lumped inelasticity model is used to describe the inelastic behavior of the member, i.e., the frame member is assumed to be composed of an elastic beam column and two inelastic hinges at the ends i and j. The variables are introduced as follows:

Generalized deformations represented by the matrix $\mathbf{\Phi}^t = (\phi_i, \phi_j, \delta)$, where ϕ_i and ϕ_j are the rotations of the element ends with respect to the chord i-j and δ is the elongation of the chord.

Generalized stresses $\mathbf{M}^t = (m_i, m_j, n)$, where m_i and m_j are the moments at the end of the member and n is the axial force.

Generalized plastic deformations $(\mathbf{\Phi}^p)^t = (\phi_i^p, \phi_j^p, 0)$, where ϕ_i^p and ϕ_j^p correspond to the plastic rotations of hinges i and j, respectively. It can be noticed that the permanent elongation of the chord is being neglected in the model.

The damage variables $\mathbf{D}^+ = (d_i^+, d_j^+)$ and $\mathbf{D}^- = (d_i^-, d_j^-)$. The damage parameters can take values between 0 (no damage) and 1 (total damage). They measure the influence of cracking in the flexural behavior of the member and are associated to hinges i and j, respectively. Hinge i, or hinge j, behaves as a conventional rigid plastic hinge when d_i, or d_j, takes the value of 0. If the damage variable takes the value of 1, the hinge works as an internal hinge of conventional elastic frames, i.e., no moment can be resisted by the hinge. The variables with the superindex +/− characterize concrete cracking under positive/negative moments.

The plastic deformations and the damage are the internal variables of the model proposed in this note.

A constitutive model for a frame member can be defined as the set of equations that relates generalized stresses and deformations. These constitutive equations, with the conventional compatibility and equilibrium relations determine the behavior of any frame.

As in continuum mechanics, the state laws presented in the

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following section and the evolution laws described in the next section compose the constitutive model proposed in this note.

STATE LAWS

The complementary strain energy of a damaged member can be expressed, as a function of the variables defined in the previous section as

$$U^* = \frac{1}{2}\langle \mathbf{M}\rangle_+^t \mathbf{F}(\mathbf{D}^+)\langle \mathbf{M}\rangle_+ + \frac{1}{2}\langle \mathbf{M}\rangle_-^t \mathbf{F}(\mathbf{D}^-)\langle \mathbf{M}\rangle_- \quad (1)$$

where $\langle Z\rangle_+ = \langle Z\rangle$, $\langle Z\rangle_- = -\langle -Z\rangle$, and $\langle Z\rangle$ = MacAuley brackets, i.e., $\langle Z\rangle = 0$ if $Z < 0$, $\langle Z\rangle = Z$ if $Z \geq 0$. The term $\mathbf{F}$ corresponds to the flexibility matrices of the member whose expression was proposed in Flórez-López (1995). These expressions were obtained on the basis of the concepts of damage mechanics. The flexibility coefficients in $\mathbf{F}$ increase with the damage. In this way stiffness degradation due to the cracking of the concrete is represented. It is noticed that the member has different flexibilities under positive and negative bending. Therefore, the model represents, in a simplified way, the crack closure effect under the reversal of the loading. The following state laws are derived from the complementary strain energy:

$$\mathbf{\Phi} - \mathbf{\Phi}^p = \mathbf{F}(\mathbf{D}^+)\langle \mathbf{M}\rangle_+ + \mathbf{F}(\mathbf{D}^-)\langle \mathbf{M}\rangle_- \quad (2a)$$

$$\mathbf{G}^+ = -\frac{\partial U^*}{\partial \mathbf{D}^+} = (G_i^+, G_j^+);\ \mathbf{G}^- = -\frac{\partial U^*}{\partial \mathbf{D}^-};\ G_i^+ = \frac{\mathrm{F}_{11}^0\langle m_i\rangle_+^2}{2(1-d_i^+)^2} \quad (2b)$$

Equation (2*a*) represents the relationship between generalized deformations and stresses. This equation cannot be considered as a constitutive model since it includes the two internal variables. Therefore, two additional equations must be added. These expressions will be called internal variable evolution laws.

The plastic rotation evolution laws were derived from two yield functions, one for each hinge of the member. An expression for the yield function of a damaged hinge can be found in Flórez-López (1995). Another simpler expression was proposed in Flórez-López (1998).

Equation (2*b*) defines the crack driving forces, or energy release rates, associated with the cracking due to positive and negative moments and for hinges *i* and *j*. The damage evolution laws presented in the next section are expressed as a function of the energy release rates.

Damage Evolution Laws

In Flórez-López (1995) it was assumed that cracking in the member followed a Griffith criterion. That is, damage evolution is only possible if an energy release rate G_i reaches a critical value or "crack resistance" R. As accepted in conventional fracture mechanics, crack resistance was assumed to be a function of the corresponding damage variable $R = R(d_i)$. The crack resistance term was identified with experimental results on RC specimens and an expression was proposed in Cipollina and Flórez-López (1995). A procedure for the computation of the constants of the model from well known properties of the member, such as the first cracking moment M_{cr}, the yield moment M_p, the ultimate moment M_u, and the ductility factor, was also proposed in that paper.

However, it is also well known that the Griffith criterion cannot represent fatigue effects. This is due to the fact that in this model damage (or more precisely, crack extension) is assumed to be a function of the maximum value of the energy release rate. Therefore, in fatigue tests with constant maximum force or displacement, damage would keep the value reached during the first cycle.

Several models can be found in the literature that generalize the Griffith criterion to include fatigue or low cycle fatigue effects. From these, we have chosen that proposed in Marigo (1985). This model has the following characteristics:

- No additional internal variables are introduced.
- It still relates damage evolution to energy release rate.
- During a monotonic loading it gives the same response as the Griffith criterion.
- It assumes that there is no damage evolution during elastic unloadings.
- Damage increments are possible during loading phases even if the value of the energy release rate is inferior to the crack resistance. That is, damage evolution due to fatigue effects is possible.
- Only one additional constant is needed.
- It includes the Griffith criterion as a particular case.

The model was adapted to the framework considered in this paper, giving the following damage evolution law:

$$\dot{d}_i = \frac{G_i^\alpha}{R^\alpha \dfrac{\partial R}{\partial d_i}}\langle \dot{G}_i\rangle \quad \text{if } G_i \geq G_{cr} \quad (3a)$$

$$\dot{d}_i = 0 \qquad \text{otherwise} \quad (3b)$$

where $R(d_i)$ = crack resistance term; and G_i = energy release rate, $G_{cr} = R(0)$. A procedure for the computation of the crack resistance in RC members is described in Cipollina et al. (1995) and Flórez-Lopez (1995). The dots over the damage and the energy release rate represent time derivatives and α is the additional constant introduced by the fatigue law. This constant can take values between 0 and $+\infty$. The preliminary results obtained so far seem to indicate that for RC frame members, and as a first approximation, this constant can be considered independent of the member properties or the axial load.

During a monotonic loading, (3) and the Griffith law gives the same damage evolution, whatever the value of α as shown in the following:

For $\dot{d}_i > 0$ we have, after the Griffith criterion

$$R(d_i) = G_i;\ \text{or equivalently}\ \frac{(G_i)^{\alpha+1}}{\alpha+1} = \frac{[R(d_i)]^{\alpha+1}}{\alpha+1};\ \forall\, \alpha \geq 0 \quad (4)$$

It can be noticed by taking a time derivative of (4), that this equation becomes, indeed, the fatigue law (3).

In nonmonotonic loadings, the fatigue law (3) and the Griffith criterion differ, since there may be damage evolution after (3), even if the energy release rate has not yet reached the crack resistance R_i.

Eq. (3) characterizes a damage evolution that is always + or 0 ($\dot{d}_i \geq 0$), since $R > 0$ and $\partial R/\partial d_i > 0$.

It can be noticed that the fatigue law becomes, again, the Griffith criterion, when α tends to infinite. Indeed, for values of the energy release rate smaller than the crack resistance R, the relation $(G_i/R(d_i))^\alpha$ tends to 0 when α tends to infinite. In this case, damage evolution is only possible if the energy release rate is equal to R. Therefore, in that particular case, the damage evolution given by the fatigue law matches the damage values predicted by the Griffith criterion in any case, even nonmonotonic loadings.

In fact, the constant α can be considered as a "fatigue brake." The maximum fatigue effect is obtained for $\alpha = 0$ and there is no fatigue at all, only brittle damage, for $\alpha = \infty$. In the numerical simulations performed so far, it has been found that for values of α close to 30 or higher, the fatigue effects are negligible.

Numerical simulations of displacement-controlled fatigue tests done in the laboratory verified this model, see Bendito

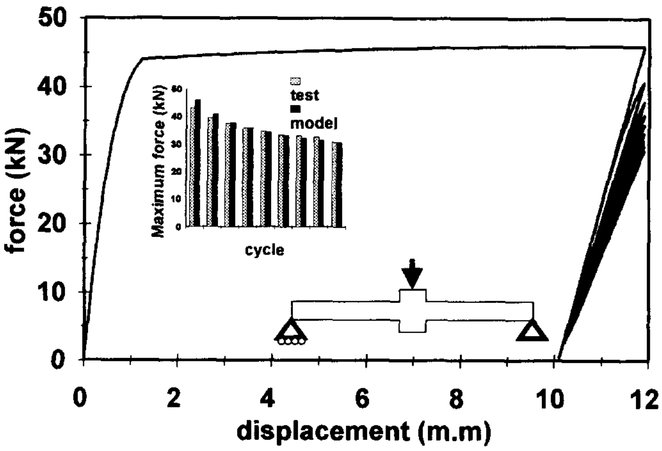

FIG. 1. Numerical Simulation of Test Reported in Bendito (1997)

(1997). The specimens represented beam-column joints as indicated in Fig. 1. The loading consisted of increasing monotonically the deflection until reaching the maximum of the curve displacement versus force; and then the specimen was cycled between force 0 and the previous maximum displacement.

The numerical simulation of this test is shown in Fig. 1. The properties M_{cr}, M_p, M_u, and ϕ_u^p were not calculated but taken directly from the experimental results ($M_{cr} = 5/L$, $M_p = 44/L$, $M_u = 46/L$, $\phi_u^p = 10/L$ where L is the length of the beam). The constant α was chosen for the best fit between model and test ($\alpha = 2$). The experimental results are not shown in this note but a comparison between the maximum force reached at each cycle in the test and the simulation is also shown in Fig. 1.

It can be seen that the model predicts very accurately the loss of strength due to fatigue damage at each cycle. However, small increments of plastic deformations were observed experimentally that are not represented by the model.

Numerical Verification

In this section, the model is validated by the numerical simulation of three tests reported in the literature. The first one consists of a circular column under a constant axial load and cyclic lateral loads as indicated in Ghee et al. (1989). The experimental results are also not shown in this note.

The numerical simulation of the test is shown in Fig. 2. The following cross section properties were used: $M_{cr}^+ = M_{cr}^- = 1{,}000$ cm/kN, $M_p^+ = M_p^- = 32{,}000$ cm/kN, $M_u^+ = M_u^- = 45{,}000$ cm/kN, $\phi_u^{p+} = \phi_u^{p-} = 0.08$, $\alpha^+ = \alpha^- = 2$. These values were not computed, but estimated from the experimental results. It can be noticed that the same value of α identified in the fatigue test of Fig. 1 was chosen. In Fig. 2, some representative points of the experimental results (maximum and minimum values as well as the plastic deflection of each cycle) are indicated with emptied rectangles. It can be seen that there is a good agreement, qualitatively and quantitatively, between the test and model except at the last few cycles. In these cycles, a higher loss of strength can be observed in the test than the one predicted by the model. In Ghee et al. (1989), it is indicated that in these few cycles buckling of the longitudinal reinforcement occurred. This observation is discussed in the last section of this note.

The second example corresponds to a test on a specimen of an unsymmetrical cross section in Scribner and Wight (1978). The numerical simulation is shown in Fig. 3. The parameters of the simulation were $M_{cr}^+ = M_{cr}^- = 500$ cm/kN, $M_p^+ = 5{,}900$ cm/kN, $M_p^- = 5{,}070$ cm/kN, $M_u^+ = 9{,}000$ cm/kN, $M_u^- = 7{,}600$ cm/kN, $\phi_u^{p+} = 0.065$, $\phi_u^{p-} = 0.07$, $\alpha^+ = \alpha^- = 2$.

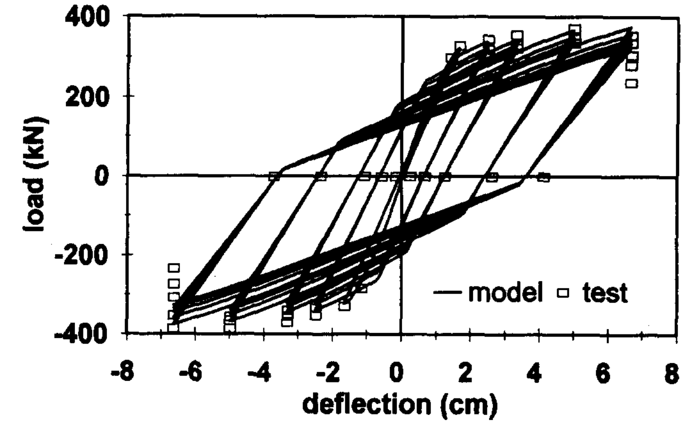

FIG. 2. Numerical Simulation of Test Reported in Ghee et al. (1989)

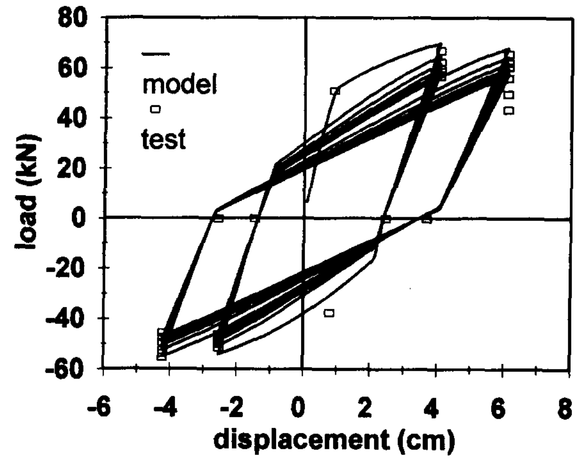

FIG. 3. Numerical Simulation of Test Reported in Scribner and Wight (1978)

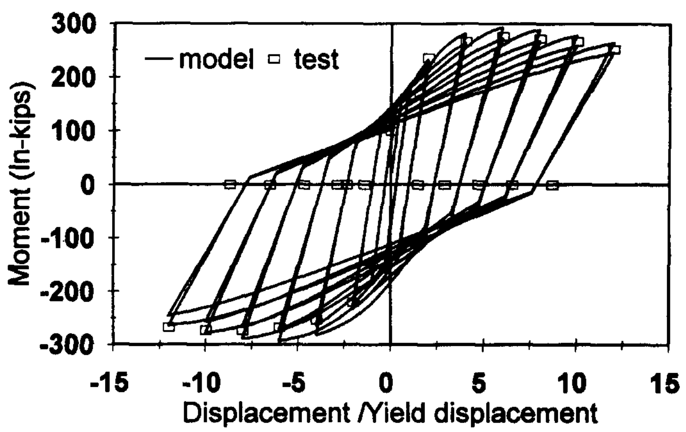

FIG. 4. Numerical Simulation of Test Reported in Lim and McLean (1991)

In the last example, a column of circular cross section was again subjected to a constant axial load and then to a lateral loading as reported in Lim and McLean (1991). The numerical simulation is presented in Fig. 4. Another simulation of this test, using the model without fatigue effects, is presented in Flórez-López (1995). This simulation and the experimental results are not presented in this note.

The comparison of both simulations allows the evaluation of the model improvement by including low cycle fatigue effects. Again, the cross member properties were chosen rather than computed.

CONCLUDING REMARKS

It is important to note that all the fatigue simulations presented in this note were done with the same value of α, i.e., the value identified in the test of Fig. 2. These four simulations correspond to tests on different specimens, with or without axial load, and symmetrical or unsymmetrical cross sections. Other simulations, not presented here, also gave good results with the same α. It seems that a value of $\alpha \cong 2$ could be adopted, at least as a first approximation, as a constant for RC frame members in general, independently of the characteristics of the structure under consideration or even the axial force.

However, it appears that buckling of the longitudinal reinforcement takes place for high values of damage and that this results in an acceleration of the fatigue effects. The model, such as presented in this note, cannot capture this sudden decreasing of the strength of the member. It has been found that using a variable α (i.e., an α that tends to 0 as a function of the damage) can be used to model this effect. Yet, there would be no physical justification for the modeling of strength degradation due to buckling of the reinforcement using an energy release rate as a driving force.

In conclusion, the writers of this note believe that taking this parameter as a constant for any RC frame member is a very simple criterion, that allows the modeling of low cycle fatigue effects without the introduction of new member dependent parameters and, that gives good enough approximations for engineering purposes.

Another point that frequently is raised is in which way models based on fracture mechanics concepts are better than the classic ones. The writers of this note think that stiffness and strength degradation in structural members can be modeled in a more physically sound manner if the constitutive equations are based on energy concepts, specifically those related with continuum damage and fracture mechanics. The framework used in the note could also be used to model more complex situations, for instance, damage in a three-dimensional case or time-dependent damage, and perhaps resulting in much simpler models. However, the authors cannot state that the present model will give more exact results when compared with the classic methods. Pretest simulations by independent analysts would probably be necessary to make possible such a statement.

The writers of this note believe that in any case, the nonlinear analysis of RC frames using inelastic hinges and the concepts of fracture mechanics is an interesting alternative approach that deserves to be, at least, explored.

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