Ejercicios propuestos 6
Matemáticas de la Física 3

1. Muestre que el problema de Neumann

$$\nabla^2 u = -q \quad \text{en } \Omega \ ; \quad \frac{\partial u}{\partial n} = f \quad \text{en } \Gamma$$

debe satisfacer la condición

$$\int_\Omega d\xi \, d\eta \, q = \int_\Gamma ds \, f$$

2. Resuelva

$$\nabla^2 u = 0 \qquad a < r < b \qquad 0 < \theta < 2\pi$$
$$u(a,\theta) = f(\theta) \quad u(b,\theta) = g(\theta) \quad 0 \leq \theta \leq 2\pi$$

3. Resuelva

$$\nabla^2 u = 0 \qquad a < r < b \qquad 0 < \theta < 2\pi$$
$$\partial_r u(a,\theta) = f(\theta) \quad \partial_r u(b,\theta) = g(\theta) \quad 0 \leq \theta \leq 2\pi$$

donde se satisface:

$$\int_{r=a} f \, ds + \int_{r=b} g \, ds = 0$$

4. Resuelva

$$\nabla^2 u = 0 \qquad 0 < x < 1 \qquad 0 < y < 1$$

$$u(x,0) = x(x-1) \qquad u(x,1) = 0 \quad 0 \leq x \leq 1$$

$$u(0,y) = u(1,y) = 0 \quad 0 \leq y \leq 1$$

5. Resuelva

$$\nabla^2 u + u = 0 \ , \quad 0 < r < a \ , \quad 0 < \theta < \alpha$$

$$u(r,0) = u(r,\alpha) = 0 \quad 0 \leq r \leq \alpha$$

$$u(0,\theta) = acotada \quad u(a,\theta) = f(\theta) \quad 0 \leq \theta \leq \alpha$$