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Valuation of exploration and production assets: an overview of real options models

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Abstract

This paper presents a set of selected real options models to evaluate investments in petroleum exploration and production (E&P) under market and technical uncertainties. First are presented some simple examples to develop the intuition about concepts like option value and optimal option exercise, comparing them with the concepts from the traditional net present value (NPV) criteria. Next, the classical model of Paddock, Siegel and Smith is presented, including a discussion on the practical values for the input parameters. The modeling of oil price uncertainty is presented by comparing some alternative stochastic processes. Other E&P applications discussed here are the selection of mutually exclusive alternatives under uncertainty, the wildcat drilling decision, the appraisal investment decisions, and the analysis of option to expand the production through optional wells.

Keywords: Real options; Investment under uncertainty; E&P investments; Project valuation under uncertainty; Real options in petroleum

1. Introduction and intuitive examples

This paper focuses on investments in exploration and production and how oil companies can maximize value by managing optimally the real options embedded in their portfolio of projects and other real assets. Fig. 1 displays the sequential options process for the typical E&P investment decisions phases. During the exploration phase managers face the wildcat drilling decision, which is optional in most cases. In case of success (oil/gas discovery), the firm has the option to invest in the appraisal phase through delineation wells and additional 3D seismic, in order to get information about the volume and quality of the reserves, reducing the technical uncertainty. When the remaining technical uncertainty does not justify additional investment in information, the firm has the option to develop the reserve by committing a large investment in the development phase. The firm can also relinquish the undeveloped reserve to the governmental agency or wait for better conditions (until a certain date). Finally, the firm has operational options during the reserve producing life, such as the option to expand the production (e.g., by adding optional wells), the option of temporary suspension of the production, and even the option to abandon the concession.

Real options approach (ROA) is a modern methodology for economic evaluation of projects under

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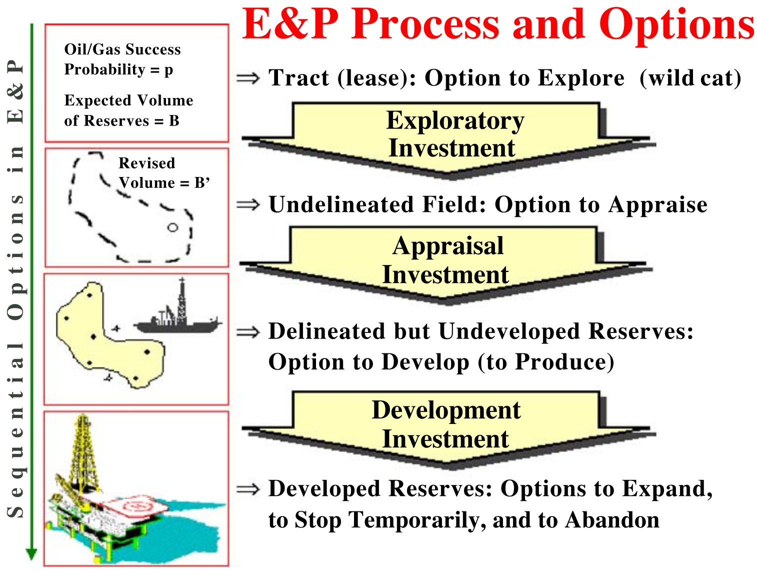


Fig. 1. Exploration and production as a sequential real options process.

uncertainty. It highlights the managerial flexibility (the "option") value to respond optimally to the changing scenario characterized by the uncertainty. At least for while, ROA complements (not substitutes) the traditional corporate tools for economic evaluation, namely the discounted cash flow (DCF) and the net present value (NPV) rule. The diffusion in corporations of sophisticated tools like ROA takes time and training. Although the corporative interest for ROA started in the 1990s, recent research (see Graham and Harvey, 2001, mainly the Table 2) with 392 American and Canadian CFOs showed a growing interest in this technique at top management level, with 26.59% answering that have incorporated real options in the economic evaluation of projects.

Illustrating with examples all phases shown in Fig. 1, this article presents a comprehensive—although incomplete—set of real options applications for valuation of exploration and production assets. This paper is organized as follows. Section 2 presents some simple examples to develop the intuition on the main real options concepts. Section 3 includes a selected bibliography on real options in petroleum, focusing the classical Paddock, Siegel and Smith's model. Section 4 discusses some stochastic processes used to model oil prices in real options applications. In the next four sections, some ideas and applications that later were fully developed through research projects coordinated by the author are presented. Section 5 presents an application of selection of alternatives to develop an oilfield under uncertainty with help of the concept of economic quality of a developed reserve. Section 6 analyzes the wildcat drilling case considering the information revelation issue. Section 7 presents a simple way to include technical uncertainty into a dynamic model, to evaluate options to invest in information. Section 8 presents the option to expand the production through optional wells. Section 9 presents the concluding remarks.

2. Simple examples on real options in petroleum

Real options models give two linked outputs, the investment opportunity value (the real option value) and the optimal decision rule (the threshold for the optimal option exercise). In order to illustrate these concepts, we present below some simple examples to develop the intuition and to compare with values and decisions using the traditional NPV rules. However, ROA can be viewed as one optimization under uncertainty problem. In most practical cases, we have:

Maximize the NPV (typical objective function) subject to:

- Relevant options (managerial flexibilities);
- Market uncertainties (oil price, rig rate, etc.); and
- Technical uncertainties (petroleum existence, volume and quality).

Among the relevant options we can mention the option to defer the investment (timing option), the option to expand the production, the option to learn (investment in information), and the option to abandon.

We start with one example from exploration, working with the uncertainty on the petroleum existence. Suppose that one oil company has exploratory rights over a tract with two distinct prospects in the same geologic play. The chance factor CF (probability of success) is 30% for both cases. The drilling cost in this deepwater area is I_W=\$30 million for each exploratory well. In case of success, the conditional development NPV is \$95 million in each case (so, economically equivalent prospects). Both prospects have the same negative expected monetary value[2] (EMV) given by:

$$\mathrm{EMV} = -I_W + [\mathrm{CF} \cdot \mathrm{NPV}] = -30 + [0.3 \times 95]$$
$$= -\$1.5 \text{ million}$$

Are the prospects worthless? In order to answer this question we need to include at least two features not considered in the above traditional EMV calculus: the information revelation and the optional nature of the drilling. These prospects are dependent so that we need to consider the sequential drilling case. In case of success in the first prospect (positive information revelation), the chance factor for the second prospect CF_2^+ increases, whereas in case of a dry hole in the first drilling, the chance factor for the second prospect CF_2^- decreases. Imagine that a geologist using a Bayesian approach found that in the former case the chance factor increases to $CF_2^+ = 50\%$. To be consistent, the law of iterated expectations[3] tells that we must have $CF_2^- = 21.43\%$, because $30\% = [0.7 \times CF_2^-] + [0.3 \times 50\%]$. Fig. 2 illustrates the information revelation from the first drilling and the conditional chance factors for the second prospect.

If we drill the first prospect and get a negative information revelation, the EMV for the second prospect is even more negative. However, we do not need to drill the second prospect because drilling is an option, not an obligation. So that in case of negative revelation we stop losses and the second prospect values zero. However, in case of positive revelation from the first prospect (with only 30% chances), the revised EMV^+_2 is positive: $\mathrm{EMV}^+_2 = -30 + [0.50 \times 95] = +\$17.5$ million. So, the value of the tract considering the information revelation of the first drilling and the optional nature of the second drilling is:

$$\mathrm{EMV}[\text{optional well 2} \mid \text{well 1 outcome}]$$
$$= -1.5 + [(0.7 \times \text{zero}) + (0.3 \times 17.5)]$$
$$= +3.75 \text{ million}$$

An apparent worthless tract is much better than a traditional analysis can indicate. In this simple example the optimal decision rule is: "Drill the first prospect. Exercise the option to drill the second prospect only in case of positive information revelation from the first one". The real option value of \$3.75 million in this simple example is clearly linked with this decision rule. The key factors for this positive value were the optional nature of the prospect drilling and the information revelation effect of dependent prospects.

There are important business consequences for the oil company from this example in both farm-in (buying rights) and farm-out (selling) of tracts. In addition, the oil company could design some special

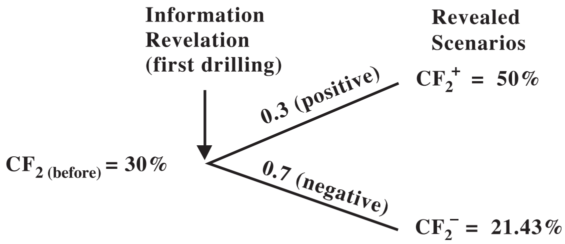


Fig. 2. Information revelation for the second prospect.

[2] Term largely used in exploratory economics for the value of a prospect considering the chance factor.

[3] The expectation (or mean) of a conditional expectation is equal to the prior expectation. In this case, the new information (first drilling outcome) will change CF_2, but the mean of the possible CF_2 values, $E[CF_2 \mid \text{new information}]$, must be equal to the original chance factor $CF_2 = 30\%$. See the proof and other properties for the information revelation in Dias (2002).

partnership operations. In the example, the oil company could give 100% of the rights of the first prospect for free to a Company Y, except that the Company Y must drill immediately the (first) prospect and give all the information to original owner of the tract (Company X). In this case, the expected value for the Company X tract rises to +\$5.25 million because Company X does not drill the first prospect with $EMV_1 = -\$1.5$ million. This kind of dealing is possible because firms have different evaluations for the same prospect.

Now we consider the development investment decision and the market uncertainty represented by the oil prices. The NPV rule tells to invest if NPV>0 and reject the investment if NPV < 0. The ROA can recommend very different actions, as we will illustrate in the following set of related simple examples. But before the examples, we need to establish a simple NPV equation as function of key parameters.

The NPV from the exercise of the development option is given by the difference between the value of the developed reserve V and the development cost D.

$$\mathrm{NPV} = V - D \tag{1}$$

In the oil industry, the value of the developed reserve V is given by the market transactions on developed reserves or, most commonly, by the discounted cash flow approach. With the DCF approach, V is the present value of the revenues net of operational costs and taxes, whereas the investment D is the present value of the investment flow net of tax benefits. In real options nomenclature, V is the underlying asset and the investment cost D is the exercise price of the development option. In the next section we will see that if we have V and D (e.g., from the DCF model) and a few more parameters, we can apply the classical model of Paddock et al. in order to obtain the real option value. However, for our intuitive examples here and for some applications presented later, we will present a parametric version of V as function of the oil price P and other few key parameters like the volume and quality of the reserve.

The two main fiscal regimes in the petroleum upstream (E&P) industry are the concessions regime (used in countries like USA, Brazil, UK, etc.) and the production-sharing regime (used for example in Africa). See for example Johnston (1995) for a discussion of fiscal regimes. For concessions, is very reasonable to assume that the NPV is a linear function[4] of the oil price P, so we will focus in this case for simplicity.

The simplest linear model is to consider that the market value of the developed reserve V is proportional to the oil prices. In addition, we can consider the variable reserve volume in order to price the reserve in \$/bbl. Let B be the number of barrels in this reserve. The proportional or "Business Model" is given by:

$$V = qBP \tag{2}$$

The motivation for the name "Business Model" is drawn from the reserves transactions market[5], because the product qP is the value of one barrel of developed reserve. The called "one-third" rule of the thumb is well known in USA, where the average value paid for one barrel of developed reserve is 33% of the well-head oil price (P), see Gruy et al. (1982).[6] The value of 1/3 is only a particular case of q in our business model. In general, we can estimate q using the DCF (see below) or by using data from the reserves market, if available. This parameter is called economic quality of the developed reserve because V increases monotonically with q and because q depends on reservoir rock quality and fluid quality. This value is lower than 1 and it is also function of other issues like discount rate[7] (including country risk), taxes, and operational

[4] This is a stylized fact known by people working with DCF models in the fiscal regimes of concession, when performing the sensitivity analysis NPV × P. However, the same is not true for production sharing contracts. In addition, in our NPV equations are not considered the option to abandon and other operational options, which could introduce non-linearity in the model.

[5] See Adelman et al. (1989, data in Table 2) for data and discussion on the market value of a developed reserve.

[6] The "one-third" rule of the thumb was also used in Paddock et al. (1988) to perform numerical examples.

[7] Being V in present value in Eqs. (1) and (2), all discounting effect is embedded into the quality factor q. Moreover, when using q estimated from developed reserve market, we need to adjust the value of q in order to consider the *time-to-build* the project so that V and D must be in present values at the same date (the date of the option exercise, at the investment start-up).

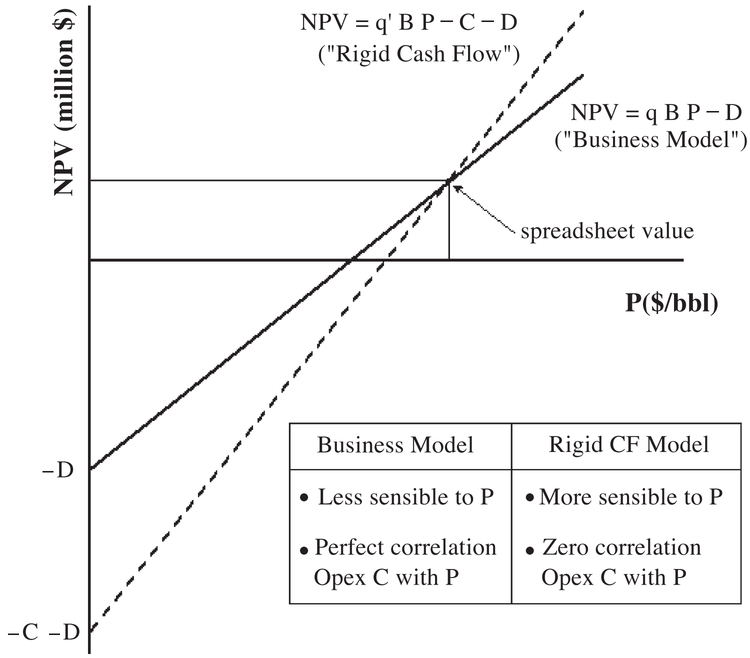


Fig. 3. Two linear models for NPV(P).

costs (opex). For the general case, the economic quality of reserve is defined by:

$$q(P) = \frac{1}{\mathbf{B}} \frac{\partial V(P)}{\partial P}$$

If V is linear with P, the economic quality is independent of P and constant. Other linear function for $V(P)$ is presented in the equation below, called "Rigid Cash Flow Model".

$$V = q'BP - C \tag{3}$$

In this equation, B represents the reserve volume (as before), q' is the economic quality of the reserve for this model, and C is a part of the opex in present value. In general, C is a function of B.[8] The name "Rigid Cash Flow Model" is because it looks more consistent with a standard DCF. Fig. 3 shows these two linear functions for NPV(P), combining Eqs. (1), (2) and (3).

Both models started from the base-case NPV from our DCF spreadsheet (the crossing point in the above figure). The sensitivity analysis using DCF is here named "Rigid Cash Flow Model" (dotted line). However, for very low oil prices is reasonable to think that the opex shall drop due to the lack of demand for oil services in the low oil prices scenarios. Similarly, in the high oil prices scenarios case, it is probable that demand for oil services rises and so the opex. This means that the "Rigid Cash Flow Model" could be "too rigid" in the sense that in fact the variation of NPV with P is not so pronounced as indicated by this model. The alternative "Business Model", also shown in Fig. 3, presents a lower inclination for the NPV(P) function. This model takes an opposite point of view, with all opex inserted into the parameter q, implying a perfect correlation[9] between P and opex. Intuitively is more probable that the NPV will reach an intermediate value between these extreme values predicted by the two models.

[8] See a more rigorous discussion of these and others NPV × P models at http://www.puc-rio.br/marco.ind/payoff_model.html.

[9] If P and opex are perfectly correlated, they have a linear relation. See for example DeGroot and Schervish (2002, p. 218).

The simpler business model has been used in numerical examples of real options (Paddock et al., 1988) and it is recommended by Pickles and Smith (1993, p. 16): "in equilibrium the prices of developed reserves and oil at wellhead must appreciate at the same rate". However, it is not a consensus in the literature (Bjerksund and Ekern, 1990). Typically, this business model generates more conservative real option values (due to the lower sensitivity with P) than the rigid cash flow model.

Using the business model, let us analyze the first intuitive example for development decision with market (represented by the oil price) uncertainty. Assume that the current oil price[10] $P(t=0)$=$18/bbl, the quality $q=20\%$, volume $B=500$ million barrels, and development cost D=$1850 million. Hence, the NPV is negative: $qPB - D = 0.2 \times 18 \times 500 - 1850 = -\$50$ million. Imagine we can defer one period the decision (timing option) and at $t=1$ the oil price can rise or fall only $1/bbl (simple and small market uncertainty), with 50% chances[11] each scenario. See Fig. 4.

Suppose that other company offer $1 million for the rights of this oilfield. Do you sell? Assume a discount rate of 10% per period. Using the traditional DCF approach, we could accept the offer. However, with real options in mind we need to check what happen if we wait one period and exercise the option only if the future scenario is favorable. In this case, if we wait one period and the oil price rises to P^+=$19/bbl, the NPV is positive: $\text{NPV}^+ = qP^+B - D = 0.2 \times 19 \times 500 - 1850 = +\$50$ million. But if the oil price drops to P^-=$17/bbl, the NPV is even worse than at $t=0$: $\text{NPV}^- = 0.2 \times 17 \times 500 - 1850 = -\$150$ million. A rational manager will not exercise the option with this negative NPV, so that if we wait and see the scenario P^-, the development right values maximum(NPV^-, 0) = 0, because we are not obligate to invest—it is an option! Hence, recognizing the option to defer one period, and bringing it in present value, the value of our real option (F) in this stylized example is:

$$F = [50\% \times \max(\text{NPV}^+, 0) + 50\% \times \max(\text{NPV}^-, 0)]/(1 + \text{discount rate}) = +\$22.73 \text{ million}$$

which is much higher than the offer from the other company. The very different value obtained here is because we recognize explicitly both the market uncertainty and mainly the value to defer the investment decision (the option/managerial flexibility). Note that we just used the NPV rule in each scenario at $t=1$. The reader can check that if we increase the variance at $t=1$ (e.g., the oil prices can rise or fall by $2/bbl) the value of real option is even higher. In general, market volatility increases the real option value.

Let us see a second example using the same Fig. 4. The only difference here is a lower investment cost: assume D=$1750 million. In this case the NPV is positive ($0.2 \times 18 \times 500 - 1750 = +\$50$ million) at the current market prices (or current market expectations). Do you exercise immediately or wait and see?

If we use the traditional NPV rule, we could exercise the option. However, it is necessary to compare the value of immediate exercise (+50 million) with the mutually exclusive alternative of deferring one period, exercising the option only in the favorable scenarios. It is easy to see that in this case the option value is:

$$F = [50\% \times 150 + 50\% \times 0]/(1.1) = +\$68.18 \text{ million}$$

which is higher than the immediate exercise value, NPV($t=0$)=$50 million. Hence, in this second example the optimal rule is: wait one period, exercising the option at P^+ and not exercising at P^-. This example showed that can be optimal to defer a positive NPV

[10] Alternatively, we can think the current *long-term* expectation on the oil price, e.g., using a futures market value as proxy.

[11] For while, we omit the discussion on probability mapping (stochastic process) and discount rate features for simplicity.

project. In this case, the NPV is not high enough to ignore the option to defer. In addition, this flexibility to postpone investment increases the value of the rights on this undeveloped oilfield.

Finally, let us see a third example where the option to defer is worthless. Consider again Fig. 4 and the same previous values except that the investment cost is lower: D=\$1700 million. In this case, the NPV from immediate investment is +\$100 million ($=0.2 \times 18 \times 500 - 1700$). The same question: Do you exercise immediately or wait and see? It is easy to check that if we defer, the option value is:

$$F = [50\% \times 200 + 50\% \times 0]/(1.1)$$
$$= +\$90.91 \text{ million}$$

Hence, here the waiting value is lower than the immediate investment. So, the optimal rule is to exercise the option immediately (here the traditional NPV rule holds). We say in this case that the real option is "deep-in-the-money", because the current NPV is high enough to justify the immediate investment.

Comparing the two last examples, we could imagine that there exists an intermediate NPV* at $t=0$ so that value of waiting = value of immediate exercise. This is a threshold NPV* because is optimal the option exercise the option if NPV ≥ NPV*. The concept of threshold for the optimal option exercise, for a given D, could be also the value of developed reserve V^* or even the oil price P^*. This threshold also can change with the time. Note that if we have a "now-or-never" situation to invest (no option), the threshold is NPV* = 0, or the traditional break-even for P^*. These simple concepts of option value and threshold will be exploited in more complex applications presented in the next sections.

3. Real options in petroleum: literature overview and the classical model

In a well-known paper of 1977, Stewart Myers (Myers, 1977) coined the term "real options" observing that corporate investments opportunities could be viewed as call options on real assets. Most of the earlier real options models were developed for natural resources applications, perhaps due to the availability of data for commodities prices. See Dias (1999) for a bibliographical overview on real options. Some important earlier real options models in natural resources include Tourinho (1979), first to evaluate oil reserves using option pricing techniques, Paddock et al. (1988), a classical model discussed below, Brennan and Schwartz (1985), analyzing interactions of operational options in a copper mine, and Ekern (1988), valuing a marginal satellite oilfield.

A sample of other important real options models for petroleum applications is briefly described in sequence, highlighting the main individual contribution. Bjerksund and Ekern (1990) showed that for initial oilfield development purposes, in general is possible to ignore both temporary stopping and abandonment options in the presence of the option to delay the investment.[12] Kemna (1993) described some case studies in her long consultant for Shell. Dias (1997) combined game theory with real options to evaluate the optimal timing for the exploratory drilling. Schwartz (1997) compared oil prices models that are discussed in the next section. Laughton (1998) found that although oil prospect value increases with both oil price and reserve size uncertainties, oil price uncertainty delays all options exercise (from exploration to abandonment), whereas exploration and delineation occur sooner with reserve size uncertainty. Cortazar and Schwartz (1998) applied the flexible Monte Carlo simulation to evaluate the real option to develop an oilfield. Pindyck (1999) analyzed the long-run behavior of oil prices and the implications for real options. Galli et al. (1999) discussed real options, decision-tree and Monte Carlo simulation in petroleum applications. Chorn and Croft (2000) studied value of reservoir information. Saito et al. (2001) evaluated development alternatives by combining reservoir simulation engineering with real options. Kenyon and Tompaidis (2001) analyzed leasing contracts of offshore rigs. McCormack and Sick (2001) discussed valuation of underdeveloped reserves. The real options textbooks of Dixit and Pindyck (1994, mainly chapter 12), Trigeorgis (1996, pp. 356–363), and Amram and Kulatilaka (1999, chapter 12), analyze investment models for the oil and natural resources industry.

[12] This simplification for development decisions will be assumed in the classical model that is presented in this section.

In the beginning of the 1980s, Paddock, Siegel and Smith started a research in the MIT Energy Laboratory using options theory to study the value of an offshore lease and the development investment timing. They wrote a series of papers, two of them published in 1987 and 1988. The Paddock, Siegel and Smith (PSS) approach is the most popular real options model for upstream petroleum applications. This classical model is useful for both learning purposes and as first approximation for investment analysis of development of oil reserves, even thinking that in the real life are necessary models that fit better the real world features. The book of Dixit and Pindyck (1994, see chapter 12) describes this model in a more compact and didactic way. This model has practical advantages (when compared with others options models) due to its simplicity and few parameters to estimate. One attractive issue is the simple analogy between Black–Scholes–Merton financial option and the real option value of an undeveloped reserve, which is illustrated in the Table 1.

The analogy above is also useful for other real options applications. Instead, developed reserve value is possible to consider any operating project value (V) as the underlying asset for this option model. For example, F could be an undeveloped urban land, V the market value of a hotel, and D the investment to construct the hotel. In absence of a direct market value for V, it is possible to compute V as the present value of the revenue net of operational costs and taxes. Remember from Eq. (1) that the traditional net present value is given by NPV = $V - D$, where D is the present value of the investment cost to develop the project. Here, the investment D is analog to the exercise price of the financial option because it is the commitment that the oil company faces when exercising the real option to develop the oilfield.

Table 1
Analogy between financial options and real options

Black–Scholes–Merton's financial options	Paddock, Siegel and Smith's real options
Financial option value	Real option value of an undeveloped reserve (F)
Current stock price	Current value of developed reserve (V)
Exercise price of the option	Investment cost to develop the reserve (D)
Stock dividend yield	Cash flow net of depletion as proportion of V (δ)
Risk-free interest rate	Risk-free interest rate (r)
Stock volatility	Volatility of developed reserve value (σ)
Time to expiration of the option	Time to expiration of the investment rights (τ)

The time to expiration (τ) of this real option is the deadline when the investment rights expire. There is a relinquishment requirement and the oil company faces a "now-or-never" opportunity at this date: or the firm commits an immediate development investment plan or returns the concession rights back to the National Petroleum Agency or similar governmental institution. This time varies from 3 to 10 years. In other applications (like development of urban lands), we have perpetual options (no expiration date).

Volatility (σ) is the annual standard deviation of dV/V. In the case that V is proportional to P is possible to use the same value of the volatility of oil price,[13] which has more available data for estimations. Dixit and Pindyck (1994, chapter 12) recommend σ between 15% and 25% per annum. Some authors (e.g., Baker et al., 1998, p. 119) use a value higher than 30% p.a. for σ. Another possibility to estimate σ of V is the Monte Carlo simulation of the stochastic processes of key market components like P, opex elements, and taxes. With this approach we get a combined variance of V at $t = 1$, and with a formula we get the volatility. See Copeland and Antikarov (2001, chapter 9) for details on this alternative approach.

In the dividend yield analogy,[14] the cash flow yield δ is the (annual) operating net cash flow value as a percentage of V. For petroleum reserves, there is a depletion phenomenon due to the finite quantity of petroleum in the reservoir. This generates a decline rate (ω) in the oilfield output flow-rate along the reserve

[13] If both V follows a geometric Brownian motion (GBM) and V is proportional to P (that is $V = kP$), then P also follows a GBM with the same parameters (σ, δ, α). See Dixit and Pindyck (1994, p. 178) or the proof with Itô's Lemma at Dias (2002).

[14] The project cash flows are like "dividends" earned only if the option is exercised and the underlying asset start operations. So, the dividend yield is an *opportunity cost of waiting* that faces the investor who has the option but not the underlying asset. In contrast, the interest rate r rewards the waiting policy (imagine the investor put the amount D in the bank). So, r and δ have opposite effects in both option value and optimal exercise thresholds, and it is frequent to see in equations the difference $r - \delta$.

life. The equation to estimate δ including the depletion issue is presented in both Paddock et al. (1988) and in Dixit and Pindyck (1994),[15] but two more practical ways to estimate δ are presented below.

For the model that assume the value of the developed reserve proportional to the (wellhead) price of oil is possible to see δ as the oil price (net) convenience yield by using data from the futures market. With the notation $\Phi(t)$ for the oil futures price delivering at time t, P for the spot oil price (or the earliest futures contract, at t_0), r for the risk-free interest rate, and Δt = time interval (or $t - t_0$), the equation is:

$$P = \Phi(t)\exp[-(r-\delta)\Delta t] \tag{4}$$

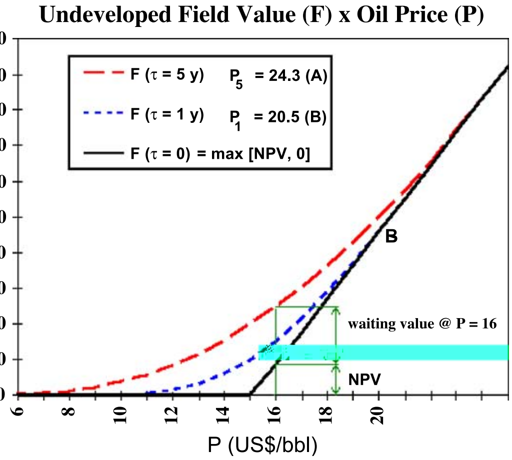
The second way is a practical rule using a long-term perspective that is useful for real options models. What is a good practical value for the net convenience yield δ? Pickles and Smith (1993) suggest the risk-free interest rate. They wrote (pp. 20–21): "We suggest that option valuations use, initially, the 'normal' value of net convenience yield, which seems to equal approximately the risk-free nominal interest rate".[16]

One interesting feature of the PSS model is the resultant partial differential equation for the real option value (the value of undeveloped reserve) is identical to the known Black–Scholes–Merton equation with continuous dividend. Only at the boundary conditions appear two additional conditions, to take account of the earlier exercise feature of this American call option (American options can be exercise at any time up to expiration, whereas European options can be exercised only at the expiration). See the equation and the boundary conditions for example in the book of Dixit and Pindyck (1994, chapter 12). By solving this partial differential equation numerically we obtain two linked answers, the real option value of the undeveloped reserve (F) and the decision rule (invest or wait for better conditions), recall our simple intuitive examples in the previous section. Here, thanks to the analogy, any good software that solves American call options (with continuous dividend-yield) also solves our PSS real options model.[17]

The decision rule is given by the critical value (or threshold) of V, named V^*, which the real option is "deep-in-the-money". It is optimal the immediate exercise of the option to develop the oilfield only when $V(t) \geq V^*(t)$. For models where V is proportional to P, it is easier to think with P^*, the oil price that makes a particular undeveloped oilfield to be "deep-in-the-money". In this case the rule is to invest when $P \geq P^*$.

Fig. 5 presents a typical solution for an undeveloped oilfield with 100 million barrels (and $q = 0.187$, so that $V = 18.7 \times P$). The curves represent the real option values for the cases of 5 years to expiration of rights ($\tau = 5$ years), 1 year ($\tau = 1$ year), and at expiration ($\tau = 0$). For the last situation, known as "now-or-never" case, the NPV rule holds and the real option value is the maximum between the NPV and zero.

Fig. 5 shows that for P=\$15/bbl the NPV is zero. Hence, \$15/bbl is the break-even oil price. It also shows that at \$16/bbl the NPV is positive, but the net waiting value, also called option premium (the differ-



[15] A detailed proof for the Paddock, Siegel and Smith equations is available at www.puc-rio.br/marco.ind/petmode1.html.

[16] In the risk-neutral valuation, largely used in options pricing, the risk-neutral drift is $r - \delta$ and using the suggestion of Pickles and Smith ($r = \delta$) we get a *driftless* risk-neutral process, which sounds reasonable for the long-run equilibrium. Schwartz (1997, p. 969) uses an interest rate $r = 6\%$ p.a. and a convenience yield for the *copper* = 12%. However, he suggests (footnote 34) a *long-term* convenience yield of 6%, that is equal the interest rate value, as Pickles and Smith suggested!

[17] For example, the spreadsheet "Timing", a full functional shareware available at www.puc-rio.br/marco.ind/.

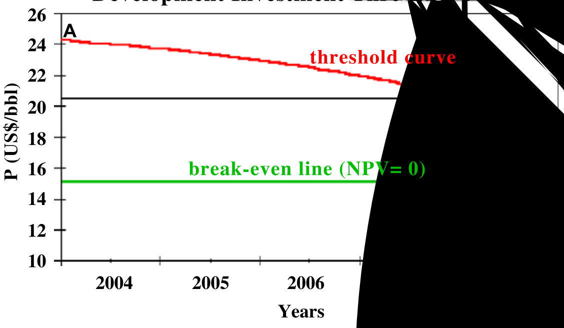
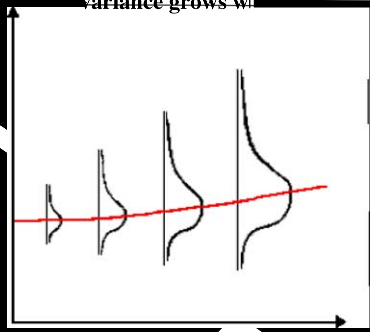


Fig. 6. The real option decision rule: inv[redacted]



ence F − NPV), for $\tau = 5$ years is [redacted] higher than the NPV. So, at $16/bb[redacted] for this field-example is "wait an[redacted] premium is zero only at the tange[redacted] the option curve and the NPV line [redacted] which occurs only at the point A, at [redacted] bbl. If $P \geq 24.3$, even with 5 years to[redacted] oilfield is "deep-in-the-money" (the NP[redacted] in this example and it is optim[redacted] th[redacted] exercise of the option to invest. Fig. 5[redacted] the real option curve for 1 year to ex[redacted] option is less valuable than the case of 5 y[redacted]ars)[redacted] the threshold value P^* drops to $20.5/bb[redacted] (see p[redacted] B). Fig. 6 presents the threshold curve for t[redacted]is oilfield[redacted] from 5 years to expiration until the "now[redacted]or-never" deadline.

Note in Fig. 6 the correspondence with the points A and B from Fig. 5. Note also that at the expiration (now-or-never case), the real option rule collapses to the NPV rule, that is, invest if the oil price is higher than the break-even price for the oilfield (in this case about $15/bbl).

4. Stochastic processes for oil prices

Like Black–Scholes–Merton financial options model, Paddock, Siegel and Smith model assumes that the underlying asset (here the developed reserve value V but could be the oil price P itself) follows a kind of random-walk model named geometric Brownian motion (GBM). Under this hypothesis, the underlying asset V at a future date t has lognormal distribution with variance that grows with the fore-

[redacted]

The curve starting at P_0 represents the expected value of $P(t)$, which decreases with the forecasted time horizon if $P_0>\bar{P}$. This drift curve points toward the equilibrium level $\bar{P}$ for mean-reversion models.[18] Although the distribution of futures prices is also lognormal as in the GBM, note that the variance grows until a certain time t_i and remains constant after that. The reason is the mean-reversion force effect at work that, even in a very distant future, does not permit (makes unlikely) values of P too far from $\bar{P}$.

The mean-reversion model (MRM) is more consistent with futures market, with long-term econometric tests and with microeconomic theory. However, GBM is much simpler to use. The natural question that arises: Is wrong the GBM used in the Paddock, Siegel and Smith to model oil prices in real options applications? Is significant the error by using GBM instead a MRM? What is a good value for the equilibrium level $\bar{P}$? This paper discusses these and other related questions by analyzing recent articles of two important researchers in commodity prices, Robert Pindyck (MIT) and Eduardo Schwartz (UCLA).

Pindyck (1999) discusses the long-run evolution of the oil prices using 127 years of data. He found mean-reversion, but the reversion speed is slower (half-life[19] of 5 years) than presented in some other papers (from 1 to 2 years). The oil prices revert to a quadratic U-shaped trend line (instead a fixed level as shown in Fig. 8), which is argued as consistent with models of exhaustible reserves incorporating technological change. He presents a model for oil prices named multivariate Ornstein–Uhlenbeck where the oil prices revert toward a long-run equilibrium level that is itself stochastic and mean-reverting.

In other words, Pindyck argues that the mean-reversion model is better for oil prices but the future equilibrium level is uncertain and the reversion is slow.[20] He concludes that for applications like real options "the GBM assumption is unlikely to lead to large errors in the optimal investment rule". This conclusion is reaffirmed in his more recent study (Pindyck, 2001). So, the simple PSS model is not bad.

Schwartz (1997) compares four models for the stochastic behavior of oil prices: geometric Brownian motion, pure mean-reversion model, two-factor model and three-factor model. Pure reversion model is the MRM for a fixed (non-stochastic) level and without any other additional stochastic variable. The original two-factor model is due to Gibson and Schwartz (1990), which oil price (P) follows a geometric Brownian motion but the oil convenience yield (δ) follows a mean-reversion process. The three-factor model is like the two-factor but with the risk-free interest-rate (r) as the third stochastic variable (also mean-reverting).

Schwartz (1997) prefers the two and three factors models, which are less predictable than the pure mean-reverting model. He presents a real options example in which the pure reversion model induced higher error (compared with the two and three factors model) than the GBM in the investment decision. But he alerts that GBM can induce too late investment because it neglects mean-reversion.

Considering both Pindyck and Schwartz points of view for real options models, I think that GBM is a good approximation in several cases. However, GBM could be inadequate when the spot price is too far from a more reasonable long-run equilibrium level (actually this level could be in the range of \$22–28/bbl). The slope of the futures term-structure is a good intuitive way to see if spot price is too far or not.

For the GBM paradigm, every change in oil price is a permanent shock in the long-run price drift, whereas the pure mean-reversion assumes the opposite, that is, that every price oscillation is just a temporary deviation from the predictable long-run equilibrium level. Both points of view seem too drastic. A more reasonable point of view could be a model not "too unpredictable" as GBM, neither "too

[18] This issue is more consistent with the *backwardation* phenomenon in futures market, when the price of a futures contract is inferior to both spot price and shorter-maturity contracts. See Bessembinder et al. (1995, pp. 3373–374) for this point and econometric tests. With GBM, we need a time-varying (mean-reverting) δ to explain futures market term structures.

[19] Half-life is the expected time for a variable to reach the half of the distance between the current and the equilibrium level.

[20] For high prices case, one reason for the slow reversion is that E&P investments take many years to put new oil in the market (time-to-build effect). In general, we can explain the *inertia* (lack of options exercise) in the supply side with real options tools. See a discussion of the *hysteresis* effect in Dixit and Pindyck, chapter 7.

Table 2
Stochastic models for oil prices in real options applications

Class of stochastic model	Name of the model	Main reference
"Unpredictable" model	Geometric Brownian motion (GBM)	Paddock et al. (1988)
"Predictable" model	Pure mean-reversion model (MRM)	Schwartz (1997, model 1)
More realistic models	Two and three factors model	Gibson and Schwartz (1990), and Schwartz (models 2 and 3)
	Reversion to uncertain long-run level	Pindyck (1999) and Baker et al. (1998)
	Mean-reversion with jumps	Dias and Rocha (1998)

predictable" as the pure MRM. Three types of models with this intermediate point of view are discussed below. Table 2 summarizes alternative stochastic models for oil prices.

As in Pindyck (1999) and in Baker et al. (1998), Schwartz and Smith (2000) also present a model of mean-reversion towards a stochastic long-run level. Schwartz and Smith conclude that this model is equivalent to the two-factor model. Curiously, they also conclude that for many long-term investments, we may be able to safely evaluate investments using stochastic equilibrium prices only, modeled as GBM! Hence, we can use a GBM with low volatility for long-term oil prices expectation in real options models.

The third model presented from the "more realistic" class is due to Dias and Rocha (1998). They consider mean-reversion for oil prices in normal situations[21] but allow for large jumps due to abnormal (rare) news, modeled with a Poisson process. In other words, sometimes occur abnormal news like war, market crashes and OPEC surprises, generating a radical change of expectations about the supply versus demand balance. These kinds of news cause large variations in weeks or few months. Fig. 9 shows the nominal oil prices (average monthly prices) historic since 1970, and with this time-scale we can see some large jumps.

In Fig. 9, we see both jumps-up (1974, 1979, 1990, 1999, 2002) and jumps-down (1986, 1991, 1997, 2001) with this adequate (for E&P investments) long-term scale. Like the two others "more realistic" models, the jump-reversion one is not too predictable as the pure mean-reversion model (due to the jumps component), neither too unpredictable as GBM (due to the reversion component).

The particular model feature of recognizing the (large) jumps possibility, in some cases can induce better corporate decisions. For example, in oil-linked credit securities and in others oil prices linked agreements, the jumps feature highlights the convenience to put "cap" and/or "floor" in the credit spread. In December 1998, the Brent oil prices had dropped to \$10/bbl. At this time Petrobras and credit institutions considered a mean-reversion model with jumps and set cap and floor in an important 10 years "win–win" contract linked to oil prices. One year later the oil prices rose about 150% and the cap protected Petrobras from paying more than the desirable due to the jump-up in the year of 1999. In the year 2000, the oil prices reached \$30/bbl, three times the oil prices at the contract date, and the cap protection remained important.

5. Selection of mutually exclusive alternatives under uncertainty

In Sections 5–8, some models designed for important upstream applications are summarized. The ideas here presented were later developed in research projects at Petrobras[22], some of them with the PUC-Rio collaboration. The model presented in this section is fully detailed in Dias et al. (2003).

Consider a delineated but undeveloped oilfield, being the oil prices the relevant source of uncertainty. In addition to the dilemma "invest now" × "wait and see", we can also choose one from a set of mutually

[21] Dias and Rocha (1998) reversion is toward a fixed long-run equilibrium of \$20/bbl. However, as the main future improvement for the model, they suggest to consider the equilibrium level as stochastic (perhaps a GBM with low volatility).

[22] From Pravap-14, a research program on valuation of projects under uncertainty at Petrobras.

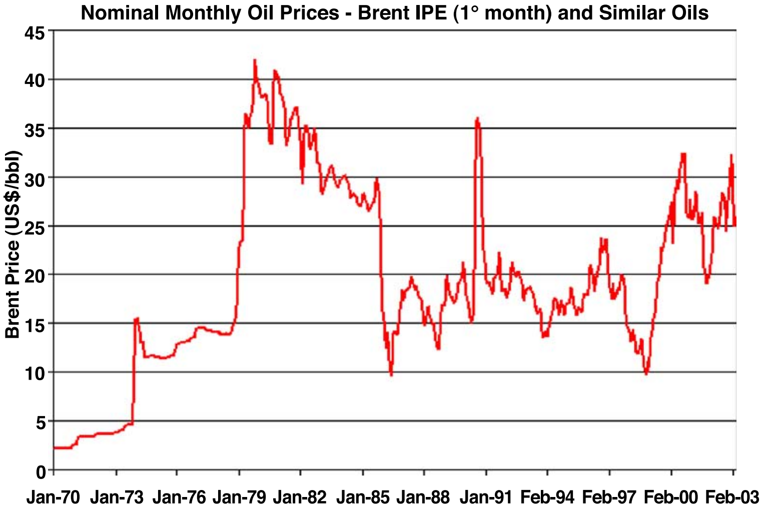


Fig. 9. Oil Prices and large jumps, Jan/1970–Mar/2003.

exclusive alternatives of scale to develop this oilfield. Assume that the reserve volume B is approximately the same for all alternatives.

Let us work with the simple "Business Model" (see Section 2, Eq. (2)) to develop this application, but we could work with the "Rigid Cash Flow Model" in a similar way. In this application, the economic quality of developed reserve (q) plays a key role. We saw that this parameter depends on many factors like fluid quality, reservoir quality, opex, discount rate, etc. In addition, it also depends on the amount of capital placed to develop the reserve (number of wells, plant capacity, pipeline diameter, etc.). Larger quantity of wells means faster production (higher present value for the revenues) and so a higher value for q than with lower number of development wells. However, more wells means higher development cost D, so there is a trade-off between q and D when choosing the number of wells to develop a reserve. So, if we exercise the option to develop the oilfield with the alternative j, we obtain the following NPV:

$$\text{NPV}_j = V_j - D_j = (q_j PB) - D_j \quad (5)$$

The concept of quality q eases the analysis of mutually exclusive alternatives to develop an oilfield under uncertainty because the trade-off V with investment D is captured with a single variable q. The selection of the best alternative to develop an oilfield for different oil prices is exemplified considering three alternatives, $A_1(D_1, q_1)$, $A_2(D_1, q_1)$, $A_3(D_3, q_3)$, where $D_1 < D_2 < D_3$ and $q_1 < q_2 < q_3$. What is the alternative with the current higher NPV? The answer depends on the oil price. Fig. 10 presents the NPVs from the three alternatives as functions of the oil prices, according to Eq. (5). In a real options model, this graph corresponds to the "now-or-never" situation (at the option expiration).

Each alternative has a specific angular coefficient and a specific X-axis intercept. The X-axis intercept corresponds to NPV = 0 case, so this intercept is the break-even price (unitary development cost in \$/bbl) for each alternative. Recall from Fig. 3 that the Y-axis intercept is the development cost, here D_j. While these interceptions are related to the development cost, the angular coefficient (tangent of θ_j) is related with the quality q_j. These parameters are easy to obtain with a DCF spreadsheet (see Section 2).

Fig. 10 shows that, for low oil price like P_1=\$15/bbl, the alternative 1 has the higher NPV, whereas for the intermediate price level P_2 the best one is the alternative 2. Alternative 3 is more attractive for higher oil prices case, as in P_3. So, in this now-or-never situation the best alternative of scale depends on the oil price (or long-run expectation on P). However, the oil prices are uncertain and in general the oil company has an option to develop and some time before the expiration of the rights. What is the best

and acquisitions of tracks in the last 3 years by several oil companies promise an increasing exploratory activity in this basin. Company X has 5 years to explore and commit a development plan (in case of success). This prospect today is marginal because a fair EMV estimate is negative. Another oil company (Company Y) offers US$3 million for the rights of this prospect. Shall the Company X accept the Company Y offer?

Consider again the equation for the expected monetary value (EMV) presented at Section 2:

$$\begin{aligned} \mathrm{EMV} &= -I_{\mathrm{W}} + [\mathrm{CF} \cdot \mathrm{NPV}] \\ &= -I_{\mathrm{W}} + \mathrm{CF}(qPB - D) \end{aligned} \tag{6}$$

The estimated chance factor CF is only 15% and the wildcat drilling cost is I_{W}=$20 million. Assume that in case of success, the discovered reserve has an expected volume B of 150 million bbl, and an expected economic quality q of 20%. Suppose that the oil prices today is $20/bbl and follow a stochastic process. Suppose for simplicity that the optimal development investment is a linear function of the reserve size B, including a fixed cost ($200 million) that represents the minimum development cost to put any production system. The equation for the adequate development investment (in million $) is given by:

$$\begin{aligned} D &= \text{fixed cost} + \text{variable cost} \\ &\quad \times \text{ reserve volume} = \mathrm{FC} + (\mathrm{VC} \cdot B) \\ &= 200 + 2B \end{aligned} \tag{7}$$

Here was assumed that the variable cost of development is $2/bbl. For the expected reserve size B of 150 million barrels, the development cost is $500 million. Using these numerical values into Eq. (6):

$$\begin{aligned} \mathrm{EMV} &= -20 + \{0.15 \times [(0.2 \times 20 \times 150) - 500]\} \\ &= -20 + 15 = -\$5 \text{ MM} \end{aligned}$$

So, the immediate drilling has a negative EMV. However, there is 5 years to expiration of the rights and we know that if the oil price rises to $22/bbl, for example, the EMV becomes positive. In addition, there is a lot of uncertainty in the estimation of the chance factor (CF), reserve volume (B) and reserve quality (q), even more due to the basin status with few wildcats already drilled there. The large uncertainty in the basin leverages the value of long-term option prospects. In 5 years, many wildcats will be drilled in several tracts of this basin, so that new information about the geology of this basin will be revealed. This revelation of geological information shall change the expectations of the variables of Eq. (6). Either free information—from the other companies activities, or costly ones (our investment in 3D seismic and in drilling other prospects at this basin) almost surely will change the geologic models, seismic interpretation, expectation about the reserve volume and quality, etc. The information revelation can be positive (increasing the EMV) or negative (decreasing the EMV), but both revelations are useful: negative information can prevent a probable misuse of $20 million in the wildcat.

In order to ease the prospect valuation under technical and market uncertainties, let us consider the simple case that the decision to drill the wildcat in this tract only will be taken at the expiration, 5 years ahead. So, we simplify the job by considering European type real option. It is a lower bound for the true value of (American) real option. The possible values for the EMV at the expiration can be estimated using a Monte Carlo simulation of the variables shown in Eq. (6). In order to perform the simulation, we need the probability distributions of the variables at $t = 5$ years.

Assume that the (long-term expectation on) oil price follows a GBM. Let us use the risk-neutral framework to evaluate the prospect. In this approach, we penalize the market expectations in order to use a risk-free discount rate.[23] In the risk-neutral simulation, a risk-premium is subtracted from the real drift of the stochastic process (see for example, Trigeorgis, 1996, pp. 102 and 218). The use of risk-neutral distribution permits the use of the risk-free interest rate to discount the simulated EMV at 5 years ahead, in order to get the present value of this EMV. This EMV present value is necessary to compare it with the Company Y offer of US$3 million. By following the Pickles and

[23] The risk-adjusted discount rate of the *option* is not necessarily the same of the underlying asset, and could be a complex problem. Fortunately, we can change (penalize) the underlying asset probability distribution in order to use the risk-free discount rate to calculate the present value of options. This is named *risk-neutral approach*.

[redacted]th's advi[redacted]ble to [redacted]w that thi[redacted]= ze- [redacted]o, so that[redacted]on at [redacted]5 has mea[redacted]ing a [redacted]latility σ =[redacted]ation [redacted]) oil price[redacted]istri- [redacted]ution is (se[redacted]

$$[redacted]ar[P(t)] = [redacted]$$

$$= [redacted]$$

$$= [redacted] \quad (8)$$

The tech[redacted] a careful prob[redacted]ed the prior dist[redacted], and B. In 5 y[redacted]e exploratory [redacted]n and many p[redacted]e expiration, v[redacted] calculate the[redacted] drilling. So,[redacted] expectation [redacted] use the mean[redacted] ble means f[redacted] years constit[redacted] ations (or rev[redacted] the condition[redacted] exploratory a[redacted] revelation dis[redacted] short, the m[redacted] same of the [redacted] ations, see S[redacted] reduction of [redacted] and the distr[redacted] of the prior [redacted] limit of full [redacted] distributions [redacted] reduction of [redacted] 5 years. In o[redacted] the intensity[redacted] years, and h[redacted] information. [redacted] in this estim[redacted]

[24] Dias (20[redacted] *revelation distri*[redacted] and the *marting*[redacted]distributions.

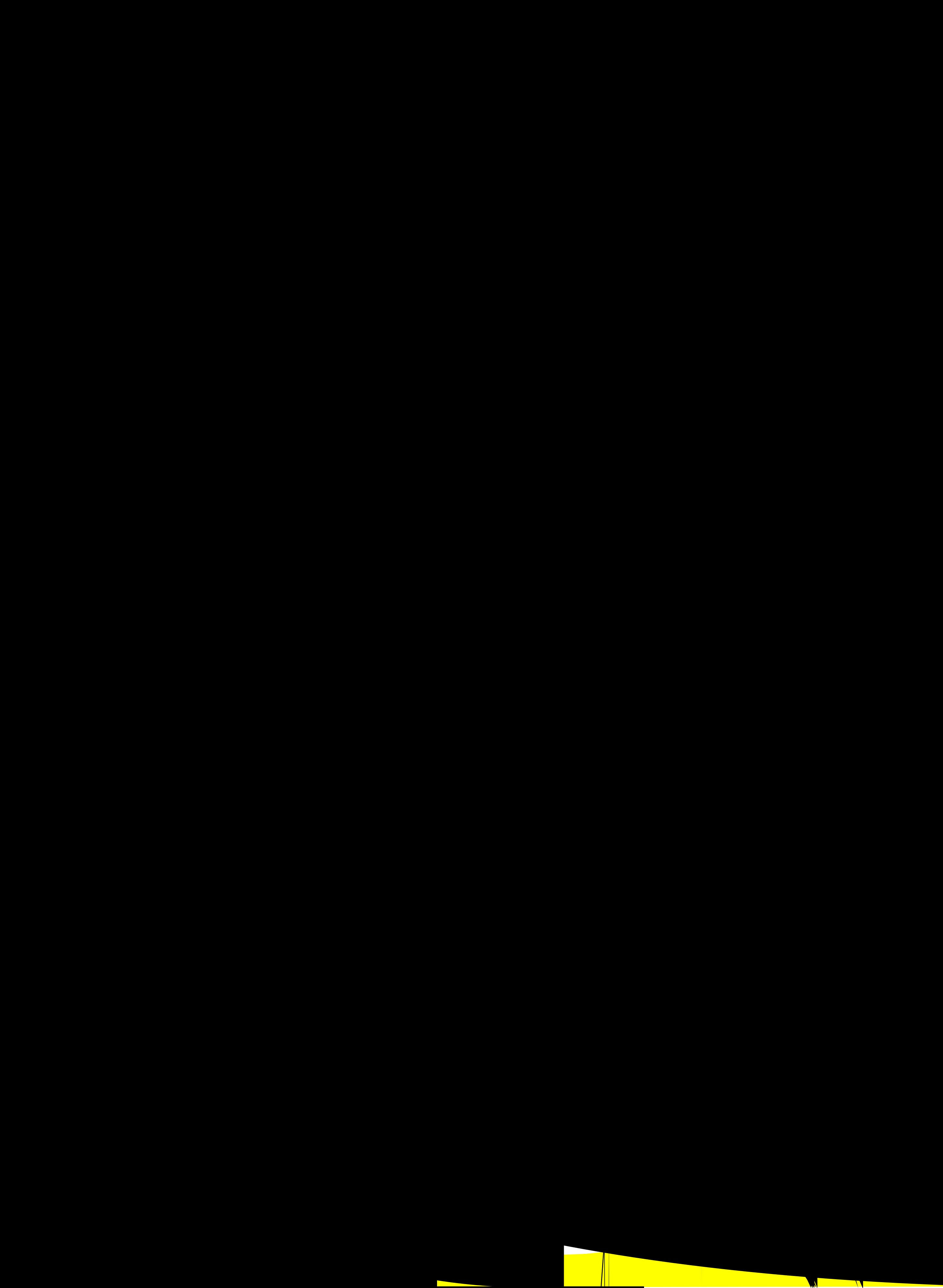
Table 3 shows the revelation distributions for the technical uncertainties and the risk-neutral distribution for the oil prices, in all cases at $t = 5$ years.

Note that we do not penalize the revelation distributions (as in the oil prices case) because these technical risks have no correlation with market portfolio. So, corporate finance theory tells us that technical uncertainty does not require additional risk premium by diversified shareholders of the oil company.

Using Eqs. (6) and (7) and a Monte Carlo simulation to combine these uncertainties, we get the distribution for the benefit value (CF·NPV) and so the EMV distribution at $t = 5$ years. As the expected values of the distributions at $t = 5$ years are the same values used in the today's EMV estimate (so that we are not more optimistic in the future), the expected value of the EMV distribution generated by the Monte Carlo simulation, must be approximately the current one (− \$5 MM). However, the key insight for valuation of the rights is the optional nature of this wildcat investment. In this future date (5 years ahead) a rational manager only will exercise the option to drill the wildcat if the revised EMV is positive. In other words, if the information revealed in the basin exploratory activity combined with the oil prices evolution set a positive EMV for this prospect, the rational manager will exercise the drilling option. If the new information points that the prospect remains with negative EMV, the drilling option will not be exercised and the prospect value is zero. So, real options'

Table 3
Distributions for EMV variables at $t = 5$ years

Stochastic ariable	Distribution	Parameters
Chance Factor for the Wildcat (CF)		
Economic Quality for Developed Reserve ( )		
[redacted] Prices ( ) RI[redacted]		



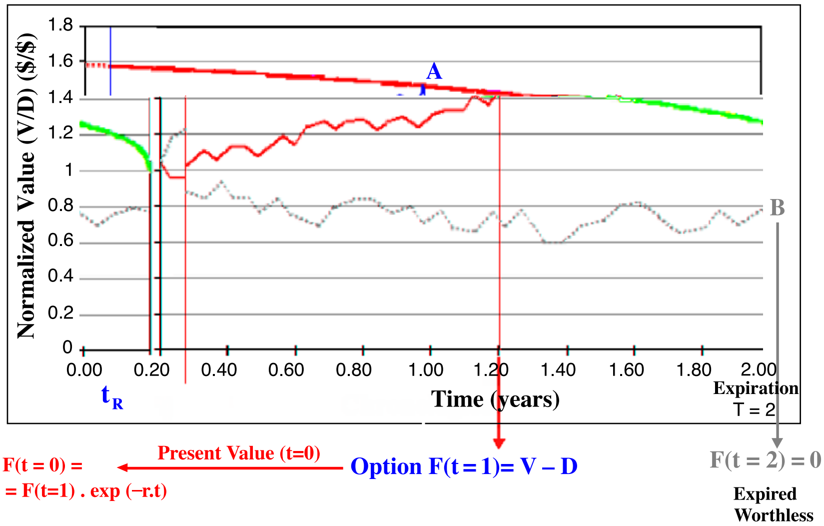


Fig. 13. Normalized threshold and the Monte Carlo valuation of real options.

For each alternative we can simulate the distribution of expectations revealed by the new information (revelation distributions, see the previous section), and for each scenario revealed we have a standard real options problem that could be solved using models like the PSS one. But this could be computationally very intensive because for each simulated scenario of q and B (and so D, function of B) we need to find the threshold curve $P^*(t)$ in order to solve the option problem. Is there a more practical way to do this? Is possible to use a single threshold curve valid (optimal) for any combination of q, P, B and D? Fortunately the answer is yes for both questions! There is a mathematical artifice that helps us in this job.

The mathematical artifice is a normalization of the real options problem that allows a single normalized threshold curve to be the optimal solution for all parameters combinations. This insight is derived from the method used by McDonald and Siegel (1986, especially p. 713) when analyzing a real options problem where both the project value (V) and the development investment (D) follow correlated geometric Brownian motions. In order to reduce the two state variables problem to a problem with a single state, they observed that a normalized threshold $(V/D)^*$ is homogeneous of degree zero, in other words, if both V and D are doubled the optimal exercise of real options is the same. As pointed out by Dixit and Pindyck (1994, p.210), the real options value is homogeneous of degree one,[25] that is, $F(V, D) = Df(V/D, D/D)$. However, this result that simplifies the combination of technical with market uncertainties in real options problems was proved only for geometric Brownian motions. Using a Monte Carlo framework, Dias (2002) applies the normalization idea to combine technical uncertainty (that affects both V and D) together with the stochastic process for the oil prices (that affects linearly V).

With this normalization, we need to solve the real options problem only once in order to estimate the normalized threshold curve $(V/D)^*$ along the time. With this threshold, we perform Monte Carlo simulations for the technical uncertainties revealed by the investment in information, and for the (risk-neutral) stochastic process for the oil prices. For each instant t, we compare the actual V/D with the normalized threshold $(V/D)^*$, previously computed. Let us detail this approach with the help of Fig. 13.

Assume that is optimal the immediate investment in information and that the information is revealed at the revelation time t_R (there is a time to learn), see Fig. 13. Suppose that $P(t=0)$=\$20/bbl, and that the initial

[25] A function is homogeneous of degree n in $\boldsymbol{x}$ if $F(t\boldsymbol{x}) = t^n F(\boldsymbol{x})$ for all $t>0$, $n \in \boldsymbol{Z}$, and $\boldsymbol{x}$ is the vector of variables.

technical parameters expectations are $q = 0.21$ and $B = 100$ million bbl. By using Eqs. (2) and (7), we get $V = 420$ and $D = 400$ (both in million \$), so that $V/D = 1.05$ at $t = 0$. This is the starting point of our Monte Carlo simulation. While we are investing in information, until t_R the value of V/D for this project will oscillate only due to the oil prices evolution, modeled with a risk-neutral GBM. However, at the revelation time t_R we will revise the expectations about the technical parameters q and B (and D). These expectations revision will cause a jump-up (in case of good news on q and/or B) or a jump-down (in case of negative revelation). The jumps sizes are sampled from the distributions of conditional expectations of q and B (or revelation distributions, see the previous section). Fig. 13 illustrates this method, showing two simulated sample-paths with jumps at t_R, one path with exercise at $t = 1$ (the almost continuous line) and the other (dotted-line) reaching the expiration date without crossing the normalized threshold curve $(V/D)^*$.

If $V/D \geq (V/D)^*$ for the path i at any time t, we exercise the option (point A in Fig. 13) and calculate the present value (using the risk-free interest rate) of this option, denoted by $F_i(t = 0)$. In the opposite case, we wait and see. The paths that do not cross the normalized threshold curve $(V/D)^*$ along the 2 years of option are worthless. By summing up all the simulated real options present values $F_i(t = 0)$ and dividing it by the total number of simulations, we get the real options value after the investment in information. Subtracting the cost of information (the learning cost), we obtain the real options value net of the learning cost for the alternative of investment in information under evaluation.

Remember that for each alternative, there are different learning costs, different learning time t_R, and different revelation distributions for q and B. The revelation power (capacity to reduce technical uncertainty) of one alternative is linked to the variance of revelation distribution[26] (Dias, 2002, proposition 3). So, we repeat this procedure for each alternative of investment in information and the resultant net real options values are compared. The higher one is the best alternative of investment in information. In general, large uncertainties in q and B enhance the value of learning and so the real options value. In order to see this, think with the "visual equation for real options". Our simple model captures this idea.

The last point in this section is the equation necessary to perform the risk-neutral Monte Carlo simulation of the oil prices. For the geometric Brownian motion, the risk-neutral sample paths are simulated with the equation below, applied to each instant t using the oil price from the previous instant $t-1$:

$$P_t = P_{t-1}\exp\{r - \delta - 0.5\sigma^2)\Delta t + \sigma N(0,1)\sqrt{\Delta t}\} \quad (9)$$

In this equation, Δt represents the time-step in the simulation, $N(0, 1)$ is the standard Normal distribution (mean = zero and variance = 1), and the remaining variables are as before. We use the recursive Eq. (9) from $t = 0$ until the option exercise or until the expiration, several times obtaining several oil prices paths, and so several V/D paths.

8. The option to expand the production using optional wells

Sometimes the best way to reduce the remaining technical uncertainties in the oilfield is by using the information generated by the cumulated production in the field and/or measuring the bottom-hole pressure after months or few years of cumulated production. In these cases, the investment in information before the oilfield development is not adequate due to the low potential to reveal information relative to the cost to gather the information.

For these cases, the proper way to gather information is by embedding options to expand the production into the selected alternative of development, so that depending on the revealed information we exercise or not the option to expand the production. In case of oilfield development, a good way is to select some optional wells, which will be drilled only in favorable scenarios. In addition, the optimal locations for these wells also depend on the information generated by the initial oilfield production.

To include this option to expand into the development plan, we incur in some costs. For example, at higher cost the processing plant shall be dimensioned considering the possible exercise of this option. An-

[26] For the Alternative 0 of not invest in information, we have single expectations for q and B instead distributions.

other way is by leaving free space in the production unit in order to amply the capacity in case of option exercise with the drilling of optional wells.[27] The third way is by waiting the main production decline (reservoir depletion) in order to integrate these wells when the processing capacity permits (see Ekern, 1988, for a real options valuation). But even in this last case is necessary to consider the possible new load in the platform from the optional new risers and a possible additional cost because the subsea layout must consider the possibility of optional wells flowlines going to the production platform. However, these costs to embed the option to expand are, in general, only a fraction of the potential benefit of drilling these wells in the favorable reservoir scenarios and/or favorable market scenarios for oil prices.

The economic analysis of the option to expand with optional wells requires an in-depth study of marginal contribution from each well in the overall oilfield development. In this study is necessary to identify the candidate wells that can become optional wells. Wells with higher reservoir risk are primary candidates because with new reservoir information these wells could become unnecessary or its optimal location could be different. The other class of optional well candidates is that resultant of the marginal well contribution for wells that presented negative NPV or even a small positive NPV[28] so that the option to drill these marginal wells are not "deep-in-the-money".

One feature that sometimes can be important is the secondary depletion in the optional wells area. The production of the main wells causes a differential pressure in the reservoir, inducing some oil migration from the optional wells area to the producing area. In most cases the oil migration does not mean that the main wells will produce this oil (it could be trapped in the way), or if this oil will be produced, it takes too long time to reach the distant non-optional wells. In many cases this oil will never be produced. This secondary depletion acts as a dividend lost by the option holder, causing the same effect of the dividend yield in the traditional real options models—higher incentive to drill earlier the optional well as higher is the secondary depletion effect.

The general method to analyze the option to expand production through optional wells is:

- Define the quantity of wells "deep-in-the-money" to start the basic investment in development by the marginal analysis of each well using several reservoir simulations;
- Define the maximum number of optional wells;
- Define the timing (or the accumulated production) that the reservoir information will be revealed;
- Define the of marginal production of each optional well as function of uncertain parameters and the possible scenarios (revelation distributions) to be revealed by the cumulative production;
- Consider the secondary depletion in the optional well area if we wait after learn about reservoir;
- Simplify computational effort by limiting the expiration of the option to drill the optional wells—remember the declining NPV due to the secondary depletion and the discount rate effect;
- Add market uncertainty, simulating a stochastic process for oil prices and perhaps for the volatile daily rate of rigs that could drill the optional wells;
- Combine uncertainties using Monte Carlo simulation as shown in the previous section;
- Use a threshold curve to consider the optimal earlier exercise of the option to drill the wells, and calculate option value; and
- Compare this option value with the cost to embed the option to expand. The option value must be higher than the cost to embed the option, to justify this flexibility cost into the development plan.

Fig. 14 presents the timeline of the option to expand problem that help us understand a practical modelling of this problem.

The timeline in Fig. 14 considers a time-to-build of 3 years, from the development investment start-up until the production start-up, at $t = 3$ years. Between the years 3 and 4, the initial production of the wells and eventual production tests give us all relevant technical information for this example. For the period from year 4 until the year 9, the exercise of the option to drill is considered mainly for the optional wells that had reservoir uncertainty before the initial production.

[27] Free area has a relatively high cost for offshore platforms, mainly in case of *floating platforms*.

[28] The direct method to estimate the marginal NPV of a well is by reservoir simulation with and without each well. With the production profiles in each case, the two NPVs are calculated and the marginal NPV is just the difference between these NPVs.

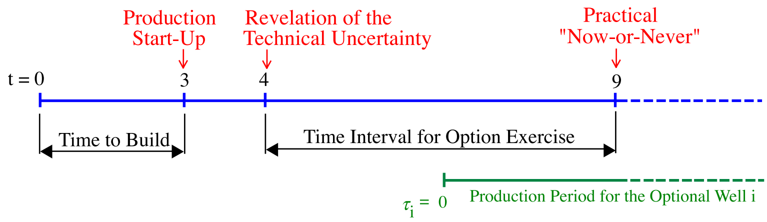


Fig. 14. Timeline for the option to expand the production using optional wells.

The deadline at year 9 is only a practical setting due to both the secondary depletion effect and the fact that distant benefit has low present value, so that the option value is too small after year 9 for practical purposes. Fig. 14 also presents the timeline for the optional well i that can be drilled at any time between years 4 and 9.

In this example, the Monte Carlo simulation for the technical uncertainty is placed only at $t = 4$. The simulation for the oil prices stochastic process starts at $t = 0$ and is necessary only until the exercise of the drilling option (at $\tau_i = 0$ for the optional well i). After the option exercise is necessary only the expected value curve for the oil prices because no other options (like abandonment) are considered in this model, which could introduce asymmetries in the value distribution.

The valuation of the option to expand is another research project under development between PUC-Rio and Petrobras. The implementation of this approach needs an active management in all phases of the project. The communication between the project teams in different project phases is necessary to preserve the embedded flexibility of the production unit and subsea layout, against the desire of "optimization" and "cost reduction" that could destroy the option value in subsequent phases by non-informed project teams.

9. Conclusions

In this article, we presented some simple examples in order to develop the intuition behind important real options concepts such as the optional nature of the investment (and the valuation consequences), the optimal option exercise rule (threshold), and the use of information for optimal option exercise. In addition, was summarized the classical model of Paddock, Siegel and Smith that exploits a simple analogy between American call options and real options model for oilfield development. We also discussed different stochastic processes for oil prices, analyzing recent important papers of Pindyck and Schwartz.

We illustrated with examples some models for the main real options cases that occur in upstream petroleum applications, summarized in Fig. 1. The cases presented were the selection of mutually exclusive alternatives to develop an oilfield; the option to drill a wildcat in an unexplored basin considering the information revelation issue; the option to invest in information in the appraisal phase, using the concept of revelation distributions to combine technical uncertainty with oil price uncertainty; and the option to expand case using the concept of optional wells.

Some upstream real options applications were not shown for brevity. For example, models for valuation of exploratory prospect including the option to abandon the sequential investment plan in the appraisal phase (see Dias, 1997, for a simple model). The valuation of petroleum reserves with models like Brennan and Schwartz (1985) and Dixit and Pindyck (1994, chapter 7), as performed by Oliveira (1990), is useful mainly for mature reserves, where the options of temporary stopping and abandonment can be very important. Other interesting model could be the valuation of deepwater technology using real options approach. This technology not only adds value to the current oil company's portfolio of projects, as it also permits that the firm enters in new areas, obtaining business opportunities that are not available for oil companies that do not have this know-how. The analyses of these models are left for a future article.

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