STABILIZATION AND DETECTION OF TIME VARYING LINEAR SYSTEMS

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Abstract

We introduce the weaker notion of the $\mathcal{P}$-stabilizability and $\mathcal{P}$-detectability of the linear time varying systems. Then, by a naive differential algebraic setting, using earlier results on the characterization of the controllability and observability of those systems by Kalmans type rank conditions, it is proven that a time varying system is $\mathcal{P}$-stabilizable if and only if the non-controllable part of the system is asymptotically stable. Analogously, those are $\mathcal{P}$-detectable by a higher order Luenberger observer if and only if the non-observable part of the systems are asymptotically stable.

1 Introduction

The notions of the stabilizability, detectability, and assignability are fundamental and well-known concepts from the theory of linear time invariant systems. In this paper we shall generalize all of these concepts and results for linear time varying systems, due to earlier characterizations of the controllability and observability of the linear time varying systems by Kalman's rank conditions [7],[8], [9].

1.1 Time varying linear systems.

Let $A:[0,T]\to\mathcal{R}^{n\times n}$, $B:[0,T]\to\mathcal{R}^{n\times p}$, $C:[0,T]\to\mathcal{R}^{q\times n}$ be smooth functions. Consider the system

$$\begin{aligned}\dot{x}(t) &= A(t)x(t)+B(t)u(t),\\ y(t) &= C(t)x(t). \qquad (\Sigma_T)\end{aligned}$$

The extended system

$$\begin{aligned}\dot{x}_1(t) &= x_2(t),\\ &\dots\\ \dot{x}_k(t) &= (A(t)x_1(t)+B(t)u(t))^{(k)}\end{aligned}$$

is the prolongation (of order k) of the original dynamics (Σ_T). This means, that, for example for $k=3$, the last equation is

$$\dot{x}_3=\left(\ddot{A}+2\dot{A}A+A\dot{A}+A^3\right)x_1+ \\ +\left(\ddot{B}+2\dot{A}B+A\dot{B}+A^2B\right)u+\left(2\dot{B}+AB\right)\dot{u}+B\ddot{u}$$

In general, that has the form

$$\dot{x}_k=P_k\left(A,\dot{A},\dots,A^{(k-1)}\right)x_1+ \\ +\textstyle\sum_{i=0}^{k-1}Q_{ki}\left(A,\dots,A^{(k-2-i)},B,\dots,B^{(k-1-i)}\right)u^{(i)},$$

where P_k, Q_{ki} are polynomials.
In terms of the state feedbacks

$$u^{(i)}(t)=K_i(t)x_1(t), \tag{1}$$

we can define a *prolongated* (so-called $\mathcal{P}$-)*feedback system*

$$\begin{aligned}&\dot{x}_1(t)=x_2(t),\\ &\dots\dots\\ &\dot{x}_k=P_k\left(A,\dot{A},\dots,A^{(k-1)}\right)x_1+\\ &+\sum_{i=0}^{k-1}Q_{ki}\left(A,\dots,A^{(k-2-i)},B,\dots,B^{(k-1-i)}\right)K_ix_1.\end{aligned} \tag{2}$$

We notice, that relations (1) are restrictions for the gain matrixes. For example, from the relation $(K_0x)^{\cdot}=K_1x$, $(K_1x)^{\cdot}=K_2x$, ..., we can deduce that

$$\begin{aligned}K_1 &= \dot{K}_0+K_0A+K_0BK_0,\\ K_2 &= \dot{K}_1+K_1A+K_1BK_0,\\ &\dots\\ K_k &= \dot{K}_{k-1}+K_{k-1}A+K_{k-1}BK_0\ .\end{aligned} \tag{3}$$

However, we can think an extension of the state feedback of the prolongated system, getting, instead of the $u^{(i)}$ derivatives the new control variables v_i. Hence, the relations among the gain matrices are omitted. In this setting, the exact prolongation can be approximated in the framework of the sliding control.

1.2 $\mathcal{P}$-stabilizable systems.

System (Σ_T) is *$\mathcal{P}$-stabilizable* if there exist state feedbacks (1) and an asymptotically stable uncontrolled system

$$\dot{x}(t) = A_0(t)x(t), \qquad (4)$$

such that the prolongation of (4) of order k, is the $\mathcal{P}$-feedback system (2).

System (Σ_T) is *$\mathcal{CP}$-assignable* if for all $A_0(t)$ there exist state feedbacks (2) such that the corresponding $\mathcal{P}$-feedback system is the k-th prolongation of system (4). Obviously, if (Σ_T) is $\mathcal{P}$-assignable then it is $\mathcal{P}$-stabilizable.

Roughly speaking, we shall show that if system (Σ_T) is controllable, then that is $\mathcal{CP}$-assignable. See Theorem 4.

We notice, that both notions can be considered supposing relations (3) among the gain matrices, or, ignoring those. The advantage of the generalized setting is that we only obtain linear relations, designing the gain matrices.

If system (Σ_T) is not controllable, then, under weak differential algebraic conditions for the time varying terms of $A(t)$, $B(t)$, we shall prove that system (Σ_T) is $\mathcal{P}$-stabilizable if and only if, the non-controllable part of the system is asymptotically stable.

1.3 $\mathcal{P}$-detectable systems.

Now, we define the prolongation of the *Luenberger's observers*. For this purpose, compute the derivatives of the output mapping of the system:

$$\begin{aligned} y(t) &= C(t)x(t), \\ \dot{y}(t) &= \left(\dot{C}(t) + C(t)A(t)\right) x(t) + C(t)B(t)u(t). \end{aligned}$$

For the second derivative

$$\begin{aligned} \ddot{y} &= \left(\ddot{C} + 2\dot{C}A + C\dot{A} + CA^2\right) x + \\ &+ \left(2\dot{C}B + C\dot{B} + CAB\right) u + CB\dot{u}. \end{aligned}$$

In general, the $k-$th derivative has the form

$$y^{(k)} = P_k(A, C)x + \sum Q_k^i(A, B, C)u^{(i)}.$$

The prolongation of the Luenberger's observer is

$$\begin{gathered} \dot{\hat{x}} = A\hat{x} + Bu + \\ + \sum D_j \left(y^{(j)} - P_j(A, C)\hat{x} - \sum Q_k^i(A, B, C)u^{(i)}\right). \end{gathered}$$

The error equation is

$$\dot{e}(t) = \left(A(t) - \sum D_j(t)P_j(A(t), C(t))\right) e(t).$$

System (Σ_T) is *$\mathcal{P}$-detectable* if there exists a prolongated Luenberger's observer, such that the corresponding error equation is asymptotically stable. System (Σ_T) is $\mathcal{OP}$-assignable, if for all $A_0(t)$ there exists a *prolongated Luenberger's observer* such that its error equation is system (4). Obviously, if system (Σ_T) is $\mathcal{OP}$-assignable then that is $\mathcal{P}$-detectable. If system (Σ_T) is not observable, then under weak differential algebraic conditions for the time dependence of the matrices $A(t)$, $C(t)$, we shall show that system (Σ_T) is detectable if and only if the non-observable part of the system is asymptotically stable.

2 Controllability, observability.

In the papers [7], [8], [9], under different additional conditions for the time dependence of $A(t)$, and $B(t)$, the following generalized Kalman's rank condition was proven to reachability (controllability) of system (Σ_T):

$$\sum \cdots \sum \sum Im\,(A_1^{n_1} \cdots A_k^{n_k} B_j) = \mathcal{R}^n, \qquad (5)$$

$(0 \le n_1 < n, \ldots, 0 \le n_k < n, \quad j = 1, \ldots, m)$, where $A_1, \ldots, A_k$ is a basis of the Lie algebra L, generated by the set $\{A(t) : \ 0 \le t \le T\}$, and $B_1, \ldots, B_m$ is the basis of the vector space V_B, generated by $\{B(t) : \ 0 \le t \le T\}$. Then,

$$A(t) = \sum a_i(t)A_i, \qquad B(t) = \sum b_j(t)B_j \ . \qquad (6)$$

We notice that a matrix Lie algebra $L \subset \mathcal{R}^{n\times n}$ is a vector subspace of $\mathcal{R}^{n\times n}$ which is closed with respect to the Lie bracket of the matrices, defined by $[X, Y] = XY - YX$. The multiplication table of L, with respect to that basis is defined by $[A_i, A_j] = \sum \Gamma_{ij}^l A_l$. On the other hand, let $A \in L$, then the linear mapping $Ad\ A : L \to L$ is defined by $Ad\ AX = [A, X]$. Since, $\Gamma_i = \left(\Gamma_{ij}^l\right)$ is the matrix representation of $Ad\ A_i$ with respect to the basis $A_1, \ldots, A_k$. Let $a(t) = (a_1(t), \ldots, a_k(t))^*$. The Wei-Norman's differential equation for the inverse of the fundamental matrix, is

$$\sum e^{-g_k\Gamma_k} \cdots e^{-g_{i+1}\Gamma_{i+1}} E_i\dot{g} = -a, \quad g(0) = 0 \qquad (7)$$

where E_i is the $0-1$ matrix with the unique 1 at the i-th diagonal element. Then, the Wei-Norman's product of the inverse of the fundamental matrix is

$$\Phi(t)^{-1} = e^{g_1(t)A_1} \cdots e^{g_k(t)A_k}$$

(see [10]). In [7], we have proven that the Kalman's rank condition (5) to the controllability of the system (Σ_T), is necessary and sufficient, if certain differential algebraic conditions hold for the time varying coefficients $a_1(t), \ldots, a_k(t)$, and $b_1(t), \ldots, b_m(t)$, defined in (6). The main idea was the concept of the persistent excitedness of system (Σ_T), by their coefficients. In Theorem 1 we will weaken this condition, replacing it by the condition that the associated Wei-Norman's equation is *generic*.

The Wei-Norman's equation (7) is *generic* if there is no basis of the Lie algebra L, such that the solution has the form

$$g(t) = (g_1(t), \ldots, g_k(t), 0, \ldots, 0)^*, \quad K < k \tag{8}$$

Let $\alpha_1, \alpha_2, \ldots, \alpha_k \in \mathcal{R}^k$ be linearly independent. Then, the matrices

$$\tilde{A}_i = \sum \alpha_{ij} A_j, \quad i = 1, 2, \ldots, k \ ,$$

constitute a basis in L. Let the new table of multiplication be denoted by $\tilde{\Gamma}_i = \left(\tilde{\Gamma}_{ij}^l\right)$. The structure matrix of system (Σ_T) can be expressed in this basis by $A(t) = \sum \tilde{a}_i(t) \tilde{A}_i$. Then, the non genericity of the Wei-Norman's equation means that there exist $\alpha_1, \alpha_2, \ldots, \alpha_k \in \mathcal{R}^k$, such that the Wei-Norman's equation corresponding to this basis, is

$$\left[\sum_{i=1}^{k-1} e^{\tilde{g}_{k-1}\tilde{\Gamma}_{k-1}} \cdots e^{\tilde{g}_{i+1}\tilde{\Gamma}_{i+1}} E_i\right] \begin{pmatrix} \dot{\tilde{g}}_1 \\ \vdots \\ \dot{\tilde{g}}_{k-1} \\ 0 \end{pmatrix} = \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \vdots \\ \tilde{a}_k \end{bmatrix}$$

Now, we shall define a nonlinear system with output equation, identifying the variables $\bar{a}_j \Leftrightarrow u_j$, $-\bar{g}_i \Leftrightarrow x_i$, $1 \leq i, j \leq k-1$ $a_k \Leftrightarrow y$. Hence,

$$\left[\sum_{i=1}^{k-1} e^{x_{k-1}\tilde{\Gamma}_{k-1}} \cdots e^{x_{i+1}\tilde{\Gamma}_{i+1}} E_i\right] \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_{k-1} \\ 0 \end{pmatrix} = \begin{bmatrix} u_1 \\ \vdots \\ u_{k-1} \\ y \end{bmatrix} \tag{9}$$

The genericity can be expressed in terms of the equivalent (to (9)) input-output system. First, we transform (9) into a polinomial differential equation in order to apply the Diop's state elimination procedure.

The matrix-exponentials $e^{x_i\tilde{\Gamma}_i}$ are quasi polynomial matrices, so (9) is a quasi polynomial equation. Now, we introduce new variables in the differential equation (9). In terms of the eigenvalues λ_{ij}, of the matrices $\tilde{\Gamma}_i$. If λ_{il} is real, then $\bar{x}_{il} = e^{\lambda_{il}x_i}$, and if $\lambda_{il} = \alpha_{il} + j\beta_{il}$, then $\bar{x}_{il} = e^{\alpha_{il}x_i}$, $\hat{x}_{il} = \sin\beta_{il}x_i$, $\tilde{x}_{il} = \cos\beta_{il}x_i$ are the new variables.

For the new variables we have the algebraic differential equations $\dot{\bar{x}}_{il} = \lambda_{il}\bar{x}_{il}\dot{x}_i$, $\dot{\bar{x}}_{il} = \alpha_{il}\bar{x}_{il}\dot{x}_i$, respectively, and $\dot{\hat{x}}_{il} = \beta_{il}\tilde{x}_{il}\dot{x}_i$, $\dot{\tilde{x}}_{il} = \beta_{il}\hat{x}_{il}\dot{x}_i$.

The obtained differential equation is algebraic in the states x_i, $\bar{x}_{il}$, $\tilde{x}_{il}$, $\hat{x}_{il}$, and that is equivalent to (9). Now, we can apply the Diop's elimination procedure (see [1]). Hence an external description $\tilde{P}(u, \dot{u}, \ldots, y, \dot{y}, \ldots) = 0$, $\tilde{Q}(u, \dot{u}, \ldots, y, \dot{y}, \ldots) \neq 0$ is obtained, which is equivalent to (9).

From this, we can deduce that

$$P_{\alpha_1,\ldots,\alpha_k}(a, \dot{a}, \ldots) = \tilde{P}(\hat{a}, \dot{\hat{a}}, \ldots, \tilde{a}_k, \dot{\tilde{a}}_k, \ldots) = 0,$$

holds for $\hat{a} = (\tilde{a}_1, \ldots, \tilde{a}_{k-1})^*$, such that

$$Q_{\alpha_1,\ldots,\alpha_k}(a, \dot{a}, \ldots) = \tilde{Q}(\hat{a}, \dot{\hat{a}}, \ldots, \tilde{a}_k, \dot{\tilde{a}}_k, \ldots) \neq 0,$$

if and only if, the solution $\tilde{g}$ of the Wei-Norman's differential equation has the form $\tilde{g}(t) = (\tilde{g}_1(t), \ldots, \tilde{g}_{k-1}(t), 0)^*$. In order to obtain the last equations, we substituted the original terms $a_1(t), \ldots, a_k(t)$.

The Wei-Norman's equation (6) is *generic* if and only if, the coefficients $a_1(t), \ldots, a_k(t)$ do not satisfy the Wei-Norman-Diop's differential equations for any $\alpha_1, \alpha_2, \ldots, \alpha_k \in \mathcal{R}^k$.

Let's combine this statement with our earlier result on the reachability (controllability) of system (Σ_T). Hence the following theorem can be obtained.

Theorem 1. *If the time varying coefficients $b_1(t), b_2(t), \ldots, b_k(t)$ are linearly independent in the time interval $[0, T_0] \subset [0, T]$ and the smooth time varying coefficients $a_1(t), a_2(t), \ldots, a_k(t)$ do not satisfy the Wei-Norman-Diop's equation for any $\alpha_1, \alpha_2, \ldots, \alpha_k \in \mathcal{R}^k$, then the Kalman's rank condition (5) is necessary and sufficient to controllability (reachability) of system (Σ_T).*

In general, under the same differential algebraic condition, the controllability subspace of system (Σ_T) is the image space

$$R = \sum \cdots \sum \sum Im\,(A_1^{n_1} \cdots A_k^{n_k} B_j) \tag{10}$$

The observability of system (Σ_T) can be treated by duality. For this, let $\{C_1, C_2, \ldots, C_l\}$ be a basis of the vector space $V_C = V(C(t) : \quad 0 \leq t \leq T)$. The time varying coefficients $c_1(t), c_2(t), \ldots, c_l(t)$ are defined respect to this basis by

$$C(t) = \sum c_i(t) C_i. \tag{11}$$

Then, the analogous theorem to observability of system (Σ_T) is

Theorem 2. *If the time varying coefficients $c_1(t), c_2(t), \ldots, c_l(t)$ are linearly independent in the time interval $[0, T_0] \subset [0, T]$ and for the smooth time varying coefficients $a_1(t), a_2(t), \ldots, a_k(t)$ the Wei-Norman-Diop's equations do not hold for any $\alpha_1, \alpha_2, \ldots, \alpha_k \in \mathcal{R}^k$, then the Kalman's rank condition*

$$\sum \cdots \sum \sum Im\,(A_1^{*n_1} \cdots A_k^{*n_k} C_i^*) = \mathcal{R}^n$$

is necessary and sufficient to the observability of system (Σ_T).

Under the same differential algebraic condition, the observability subspace of system (Σ_T) is the image space

$$\mathcal{O}_b = \sum \cdots \sum \sum Im\,(A_1^{*n_1} \cdots A_k^{*n_k} C_i^*)\,.$$

Or, equivalently, the unobservable subspace is

$$\mathcal{N}_o = \bigcap \bigcap \cdots \bigcap Ker\,(C_i A_1^{n_1} A_2^{n_2} \cdots A_k^{n_k})\,.$$

Finally, if we suppose that the Wei-Norman's equation is generic, then the following statements are true:

1. The *controllability* and the *reachability* subspaces coincide.
2. The reachability subspace is $A(t)$-invariant.
3. The unobservability subspace is $A(t)$-invariant.

3 Construction of the controllers and observers.

We introduce the notations: $\underline{n} = (n_1, \ldots, n_k)$, $e_i = (0, \ldots, 1, \ldots, 0)$, $A^{\underline{n}} = A_1^{n_1} A_2^{n_2} \cdots A_k^{n_k}$, $A^{*\underline{n}} = A_1^{*n_1} A_2^{*n_2} \cdots A_k^{*n_k}$.

First, we compute the application

$$K = (K_1, K_2, \ldots, K_k) \to$$
$$\to \sum Q_{ki}\left(A, \ldots, A^{(k-2-i)}, B, \ldots, B^{(k-1-i)}\right) K_i, \quad (12)$$

which is associated to the feedback system (2).

The $Q_{ki}(A, B)$ has the form

$$Q_{ki}(A(t), B(t)) = \sum \sum Q^s_{j,\underline{n}}(a(t), b(t)) A^{\underline{n}} B_j \ ,$$

where the coefficients are differential polynomials of the coefficients.

Example. Let the system of the skew symmetric matrices be

$$\dot{x}(t) = \begin{pmatrix} 0 & a_1(t) & a_3(t) \\ -a_1(t) & 0 & a_2(t) \\ -a_3(t) & -a_2(t) & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ b(t) \\ 0 \end{pmatrix} u(t)$$
$$y(t) = c_1(t)(1 \quad 0 \quad 0)x(t) = c(t)x_1(t).$$

For this system the terms Q_{30}, Q_{31}, respectively, are $\begin{pmatrix} 2\dot{a}_1 b + a_1 \dot{b} - a_2 a_3 b \\ \ddot{b} - (a_1^2 + a_2^2) b \\ -2\dot{a}_2 b - a_2 \dot{b} - a_1 a_3 b \end{pmatrix}, \quad \begin{pmatrix} a_1 b \\ 2\dot{b} \\ -a_2 b \end{pmatrix}.$

The genericity condition, in this case is

$$\left(\sum \alpha_{2i} a_i\right) \left(\left(\sum \alpha_{1i} a_i\right)^2 + \left(\sum \alpha_{3i} a_i\right)^2\right) \neq$$
$$2\left(\sum \alpha_{1i} a_i\right)\left(\sum \alpha_{3i} \dot{a}_i\right) - 2\left(\sum \alpha_{1i} \dot{a}_i\right)\left(\sum \alpha_{3i} a_i\right),$$

for all linearly independent vectors, $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{R}^3$. If in $[0, T_0]$, $0 < T_0 < T$, $b \neq 0$ and the Wei-Norman's equation is generic in $[T_0, T]$, then the system is controllable, in consequence of that the Kalman's rank condition (5) holds.

Now, let's choose the basis $B = (0\ 1\ 0)^*$, $A_1 B = (1\ 0\ 0)^*$, $A_2 B = (0\ 0\ -1)^*$.

Then, denoting $A_0 = I$, the matrices $A_i B B^* A_j^* \in \mathcal{R}^{3\times 3}$; $i, j = 0, 1, 2$ constitute a basis. This suggests that we can choose the gain matrices as $K_i(t) = \sum k_{ij}(t) B^* A_j^*$.

Hence, application (12), mappings into $\mathcal{R}^{3\times 3}$ and that only can be onto if $3 \leq k$. Choosing a prolongation of order 3, we obtain that the differential algebraic condition (determinant $\neq 0$) $a_3(a_2^2 - a_1^2) \neq 2(\dot{a}_1 a_2 - a_1 \dot{a}_2)$, $b \neq 0$ is necessary and sufficient that the mapping (12) were invertible, that is, the system were $\mathcal{P}$-assignable.

Let's choose the matrices $A^{\underline{n}_1} B_{j1}, A^{\underline{n}_2} B_{j2}, \ldots, A^{\underline{n}_m B_{jm}}$, such that the matrices $A^{\underline{n}_i} B_{ji} B^*_{jl} A^{*\underline{n}_l}$; $\quad l = 1, 2, \ldots, m$ constitutes a basis of the vector space of the linear mappings of the rank space (10), where $m = dimR$. If it is possible, we say that the *system structure is complete.* Suppose, that it is true. Then, let's agree the matrices $D_1, \ldots, D_{n-m} \in \mathcal{R}^{p\times n}$ such that $KerD_i \supset R$ and that the matrices $A^{\underline{n}_l} B_{jl} D_i$; $l = 1, 2, \ldots, m$; $i = 1, 2, \ldots, n-m$ constitute a basis of the spaces of the linear mappings from $R^\perp$ into R.

The terms $Q_{ki}(A, B)$ suggest that we have to choose $K_i(t) = \sum k_{il}(t) B^*_{jl} A^{*\underline{n}_l} + \sum d_{il}(t) D_i$.

The subspace R is $A(t)$-invariant, thus

$$A(t) = \begin{pmatrix} A_1(t) & A_{12}(t) \\ 0 & A_2(t) \end{pmatrix}$$

with respect to the decomposition $\mathcal{R}^n = R \oplus R^\perp$. The feedback equation over R can be expressed in the basis

$$A^{\underline{n}_i} B_{ji} B^*_{jl} A^{*\underline{n}_l}; \ A^{\underline{n}_l} B_{jl} D_r; \ \begin{cases} i, l = 1, 2, \ldots, m \\ r = 1, \ldots, n-m \end{cases}$$

The assignability over R is equivalent to the non-singularity of the application (12), which can be expressed as the determinant of this linear mapping is different from 0, that is, it can be expressed as a differential algebraic condition (non equality). Thus, for given $A_{10}(t)$, there exist feedback gains such that the last feedback equation is

$$\dot{x}(t) = \begin{pmatrix} A_{10}(t) & 0 \\ 0 & A_2(t) \end{pmatrix} x(t).$$

Theorem 3. *Suppose, that the Wei-Norman's equation is generic and the system structure is complete. Then, if the differential algebraic condition, corresponding to the application (12) holds, system (Σ_T) is $\mathcal{P}$-stabilizable if and only if the non-controllable structure $A_2(t)$ is asymptotically stable.*

Our construction, also implies the result on the $\mathcal{CP}$-assignability.

Theorem 4. *Suppose, that the Wei-Norman's equation is generic and the system structure is complete. Then, if the differential algebraic condition, corresponding to the application (12) holds, system (Σ_T) is $\mathcal{CP}$-assignable if and only if that is controllable.*

The construction of the obsever gains is analogous, using the dual basis $A^{*\underline{n}_r} C^*_{jr} C_{ji} A^{\underline{n}_i}$; $\ i, r = 1, \ldots, m$.

Theorem 5. *Suppose, that the Wei-Norman's equation is generic and system structure is complete. Then, if a differential algebraic condition, corresponding to the error equation holds, system (Σ_T) is $\mathcal{P}$-detectable if and only if the unobservable structure $A_2(t)$ is asymptotically stable.*

Our construction, also implies the result on the $\mathcal{OP}$-assignability.

Theorem 6. *Suppose, that the Wei-Norman's equation is generic and system structure is complete. Then, if the differential algebraic condition, corresponding to the error equation holds, system (Σ_T) is $\mathcal{OP}$-assignable if and only if that is observable.*

Example. (*Continuation*). If $a_2(a_1^2 + a_3^2) \neq 2(\dot{a}_3 a_1 - a_3 \dot{a}_1)$ holds, then our system is $\mathcal{OP}$-assignable.

We notice, that if the equation $a_2(a_1^2 + a_3^2) \neq 2(\dot{a}_3 a_1 - a_3 \dot{a}_1)$ holds, then the Wei-Norman's equation is not generic, however, our system is controllable and $\mathcal{P}$-assignable. However that is not observable hence it is not $\mathcal{OP}$-assignable, but $\mathcal{P}$-detectable if and only if, the non-observable part is asymptotically stable.

4 Conclusion.

The stabilizability and detectability plays important roles in the design of decoupling and fault isolation problems. Using the recent results, we can extend our earlier methods, [4], [5], to larger class of time varying linear systems.

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