Dual composition control of a binary distillation columns based on nonlinear state observer

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Abstract

Dual composition control of binary distillation columns is proposed and solved based on observer design and an elementary state elimination procedure. A particular observer allows us to recover the unmeasured feed composition, which is used in the controller design in order to maintain desired compositions. The method is an alternative against the disturbance decoupling.

1 Introduction

The distillation column is one of the most important example in process control theory and practice. Columns handle multicomponent feeds, however, many can be approximated by binary mixtures. A single feed stream is fed as saturated liquid at its bubblepoint onto the (m-th) feed tray. But it is very difficult to obtain accurate, continuous measurements of the composition of nearly pure streams. The large holdup of liquid in the column, reboiler and reflux drum tend to make distillation control systems sluggish. Besides, there is also an effective time delay of up several minutes before a change in the feed or reflux stream is noticed at the intermediate trays. Feed flow rate (mol/min) is a measured, and its composition is an unmeasured disturbance. Many efforts are made to control robustly the distillation process, against the unmeasured disturbance. Unfortunately, the decoupling matrix of the standard model of an idealized binary distillation column, see [5], is ill-posed. Its decoupling matrix is almost singular, becaused the compositions of successive trays are closed to each other. Dual composition control was proposed in [1]. Singular perturbation technics lead to aggregated models, see [2]. T. Takamatsu at al. in [7], have solved the disturbance attenuation problem, using differential geometric methods. The qualitative description of the stability of the binary distillation columns can be found in [3]. Our purpose is to design nonlinear observers, in order to recover the unmeasured disturbance. State observers are well known in fault detection and isolation, see [4], [6].

2 Binary Distillation Column

A binary system (two components) is assumed with constant relative volatility throughout the column, that is, the vapor leaving the tray is in equilibrium with the liquid on the tray. The relative volatility is a measure of the ease of separation. It is the ratio between the tendency to vaporize of two components. The greater value means easier separation (see Figure 1). A simple vapor-liquid equilibrium relationship can be used

$$k(x) = \frac{\alpha x}{1 + (\alpha - 1)x}.$$

By the other side, the efect of pressure on relative volatility is illustrated in Figure 2. The diagram shows that the relative volatility reduces as the pressure is raised. Besides, the relative volatility approaches unity as the pseudocritical point of the mixture is reached.

Any delay time will be neglected. The vapor rate $v(t)$ through all trays of the column is the same, dynamically as well as at steady state. The liquid flow rate leaving the trays is $l(t)$. The measured feed flow rate is

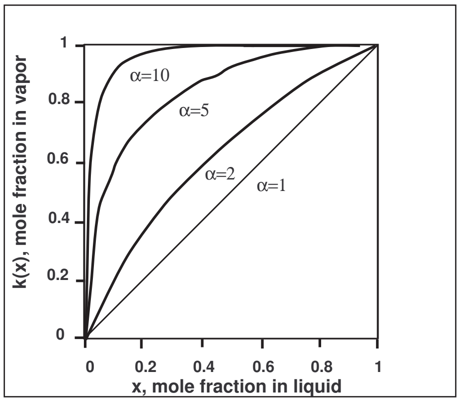

Figure 1: Relative Volatility.

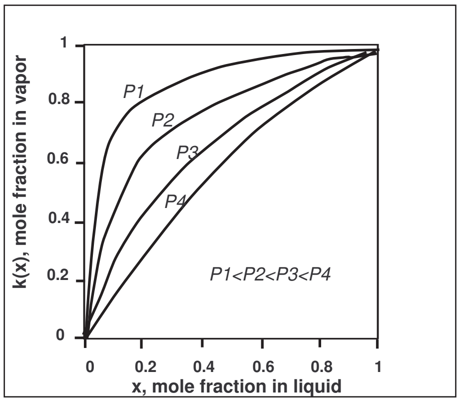

Figure 2: Relative Volatility. Effect of pressure.

$f(t)$, and the unmeasured composition is $z(t)$ at time t. The Figure 3 shows a schematic representation of the nomenclature used.

Thus, the nonlinear state equations are

$$
\begin{aligned}
H_1\dot{x}_1 &= -v(t)x_1 + v(t)k(x_2), \\
H_i\dot{x}_i &= l(t)\left(x_{i-1} - x_i\right) + \\
&\quad + v(t)\left(k(x_{i+1}) - k(x_i)\right), \ i = 2, \ldots, m-1, \\
H_m\dot{x}_m &= l(t)\left(x_{m-1} - x_m\right) - f(t)x_m + \\
&\quad + v(t)\left(k(x_{m+1}) - k(x_m)\right) + f(t)z, \\
H_i\dot{x}_i &= l(t)\left(x_{i-1} - x_i\right) + f(t)(x_{i-1} - x_i) + \\
&\quad + v(t)\left(k(x_{i+1}) - k(x_i)\right), \ i = m+1, \ldots, n-1, \\
H_n\dot{x}_n &= l(t)\left(x_{n-1} - x_n\right) + f(t)(x_{n-1} - x_n) + \\
&\quad + v(t)x_n - v(t)k(x_n),
\end{aligned}
$$

where $y_1 = x_1$, $z_1 = x_n$ are the top and the bottom components, respectively. Our purpose is to design a nonlinear observer to recover the unmeasured disturbance $z(t)$, in order to control more robustly, the desired output compositions y_1 and z_1.

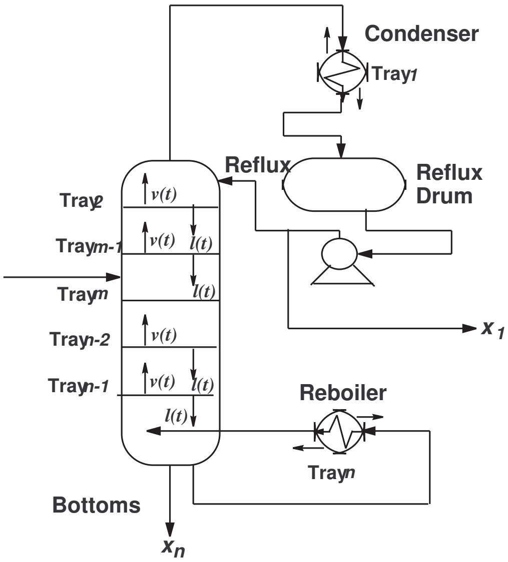

Figure 3: Distillation column.

3 Observer design

We notice that from the last equation

$$
\begin{aligned}
\left(l(t) + f(t)\right)x_{n-1} &= H_n\dot{z}_1 + \left(l(t) + f(t)\right)z_1 - \\
&\quad - v(t)z_1 + v(t)k(z_1),
\end{aligned} \tag{1}
$$

hence x_{n-1} can be expressed by z_1. Let us define new, computed outputs $z_2, z_3, \ldots, z_{n-m+2}$ and $y_2, y_3, \ldots, y_{m+1}$ in order to express the unmeasured states $x_2, x_3, \ldots, x_{n-1}$, similarly to expression (1):

$$
\begin{aligned}
z_I &= \sum_{i=n-I+2}^{n} H_i\dot{x}_i + \\
&\quad + (l + f - v)x_n, \ 2 < I \le n - m + 2, \\
y_2 &= -(H_1\dot{x}_1 + vx_1), \\
y_I &= l(x_1 - x_{I-1}) - vx_1 - \\
&\quad - \sum_{i=1}^{I-1} H_i\dot{x}_i, \ 2 < I \le m + 1.
\end{aligned}
$$

Then, recursively

$$
\begin{aligned}
x_I &= \frac{1}{l+f}\left(z_{n-I+1} + vk(x_{I+1})\right), \\
&\quad I = n-1, n-2, \ldots, m, \\
x_{m-1} &= \frac{1}{l}\left(z_{n-m+2} + vk(x_m) - fz\right).
\end{aligned}
$$

On the other hand,

$$
x_I = -\frac{y_I}{\alpha v + (\alpha - 1)y_I}, I = 2, \ldots, m.
$$

Let us consider the sum of all equations of the dynamics using the expressions from y_{m+1} and z_{n-m+1}:

$$
z_{n-m+1} - y_{m+1} - lx_m = fz \tag{2}
$$

Then, the observer is

$$
\begin{aligned}
H_1\dot{\hat{x}}_1 &= -(v+1)\hat{x}_1 + vk\left(\frac{-y_2}{\alpha v+(\alpha-1)y_2}\right) + y_1 \\
H_i\dot{\hat{x}}_i &= -(l+1)\hat{x}_i + v\left(k\left(\frac{-y_{i+1}}{\alpha v+(\alpha-1)y_{i+1}}\right) - \right. \\
&\quad \left. -k\left(\frac{-y_i}{\alpha v+(\alpha-1)y_i}\right)\right) - \frac{ly_{i-1}}{\alpha v+(\alpha-1)y_{i-1}} \\
&\quad -\frac{y_i}{\alpha v+(\alpha-1)y_i},\ i=2,\dots,m-1, \\
H_m\dot{\hat{x}}_m &= -(l+f+1)\hat{x}_m + v\Big(k(x_{m+1}) - \\
&\quad -k\left(\frac{-y_m}{\alpha v+(\alpha-1)y_m}\right)\Big) - \frac{ly_{m-1}}{\alpha v+(\alpha-1)y_{m-1}} \\
&\quad -\frac{(l-1)y_m}{\alpha v+(\alpha-1)y_m} + z \\
H_m\dot{\hat{x}}_m &= -(l+f+1)\hat{x}_m + v\Big(k(x_{m+1}) - \\
&\quad -k\left(\frac{-y_m}{\alpha v+(\alpha-1)y_m}\right)\Big) - \frac{ly_{m-1}}{\alpha v+(\alpha-1)y_{m-1}} \\
&\quad -\frac{(l-1)y_m}{\alpha v+(\alpha-1)y_m} + z_{n-m+1}.
\end{aligned}
$$

Here formula (2) is expressed in terms of the outputs, x_{m+1} is also computed recursively from $z_1,\dots,z_{n-m}$. Analogously, for $i=m+1,\dots,n-1$, the corresponding x_i, x_{i+1} can be computed from outputs $z_1, z_2, \dots,$:

$$
\begin{aligned}
H_i\dot{\hat{x}}_i &= -(l+f+1)\hat{x}_i + vk(x_{i+1}) + z_{n-i+2} + \\
&\quad +\frac{1}{l+f}\left(z_{n-i+1} + vk(x_{i+1})\right), \\
&\quad i=m+1,\dots,n-1, \\
H_n\dot{\hat{x}}_n &= (v-l-f)\hat{x}_n + z_2 + z_1 - \hat{x}_n.
\end{aligned}
$$

The observer must be stable, which means that the error $e(t)=x(t)-\hat{x}(t)$ must tend to zero at infinity. The error equation is

$$
\begin{aligned}
H_1\dot{e}_1 &= -(v+1)e_1, \\
H_i\dot{e}_i &= -(l+1)e_i,\ i=2,\dots,n, \\
H_i\dot{e}_i &= -(l+f+1)e_i,\ i=m,m+1,\dots,n.
\end{aligned}
$$

The vapor rate $v(t)$, liquid flow rate $l(t)$, feed flow rate $f(t)$, and the physical constants $H_i,\ i=1,\dots,n$ are positive, hence the error equation is asymptotically (exponentially) stable.

4 Controller design

The dual composition problem supposes the measurement (modelling) of the quality of the top or bottom product.

The quality is measured by analyzers, for example, by analyzer of final point. The final point temperature may depend on the states, and the disturbance terms. That dependence can be identified, and it is supposed to be functions of all variables

$$
\begin{aligned}
y_3(t) &= g_{\mathrm{Top}}(x(t), v(t), l(t), f(t), z(t)), \\
y_4(t) &= g_{\mathrm{Bottom}}(x(t), v(t), l(t), f(t), z(t)).
\end{aligned}
$$

The purpose is, to control the distillation column such that $y_3(t)=\mathcal{C}_1$, $y_4(t)=\mathcal{C}_2$, using the computed disturbance term. First, let us show the case $n=5$, as an example. Thus, the dynamics is written by

$$
\begin{aligned}
H_1\dot{x}_1 &= -vx_1 + vk(x_2), \\
H_2\dot{x}_2 &= l(x_1-x_2) + v(k(x_3)-k(x_2)), \\
H_3\dot{x}_3 &= l(x_2-x_3) + v(k(x_4)-k(x_3)) - fx_3 + fz, \\
H_4\dot{x}_4 &= (l+f)(x_3-x_4) + v(k(x_5)-k(x_4)), \\
H_5\dot{x}_5 &= (l+f)(x_4-x_5) + vx_5 - v(k(x_5)),
\end{aligned}
$$

$$y_1 = x_1 \quad z_1 = x_5.$$

Hence the computed outputs are

$$
\begin{aligned}
y_2 &= -(H_1\dot{x}_1 + vx_1) = -(H_1\dot{y}_1 + vy_1), \\
x_2 &= -\frac{y_2}{\alpha v+(\alpha-1)y_2}, \\
y_3 &= l(x_1-x_2) - vx_1 - H_1\dot{x}_1 - H_2\dot{x}_2, \\
y_4 &= l(x_1-x_3) - fx_3 - vx_1 - H_1\dot{x}_1 - H_2\dot{x}_2 - H_3\dot{x}_3, \\
z_2 &= H_5\dot{x}_5 + (l+f-v)x_5, \\
z_3 &= H_4\dot{x}_4 + H_5\dot{x}_5 + (l+f-v)x_5, \\
z_4 &= H_3\dot{x}_3 + H_4\dot{x}_4 + H_5\dot{x}_5 + (l+f-v)x_5.
\end{aligned}
$$

Hence,

$$fz = z_3 - y_4 - lx_3.$$

Now, let us suppose that the desired output compositions $y_1=y_c$, $z_1=z_c$ are constant. Then, $\dot{x}_1=0$, $\dot{x}_5=0$, and from $v\neq 0$ and $0=H_1 0=-vy_c+vk(x_2)$, $x_2=\frac{y_c}{\alpha v-(\alpha-1)y_c}$, obviously $y_c\neq\frac{\alpha}{\alpha-1}$ is supposed. x_2 is also a constant. Therefore the system dynamics is reduced to

$$0 = l\left(y_c - \frac{y_c}{\alpha-(\alpha-1)y_c}\right) + v(k(x_3)-y_c), \qquad (3)$$

$$0 = (l+f)(x_4-z_c) + vz_c - v(k(z_c)) \qquad (4)$$

By substituting the unmeasured term

$$fz = H_3\dot{x}_3 + H_4\dot{x}_4 + (l+f-v)z_c + (v-l)y_c$$

into the third equation

$$
\begin{aligned}
H_4\dot{x}_4 &= (l+f)x_3 - l\frac{y_c}{\alpha-(\alpha-1)y_c} + (l-v)y_c - \\
&\quad -v(k(x_4)-k(x_3)) + (v-l-f)z_c \qquad (5)
\end{aligned}
$$

is obtained.

Considering the coupled system (3), (4), (5), x_3, and x_4 can be expressed algebraically in terms of l, v, f. For example,

$$x_4 = \left(1 - \frac{v}{l+f}\right) z_c + \frac{v}{l+f} k(z_c). \qquad (6)$$

and

$$\dot{x}_4 = \frac{d}{dt}\left(\frac{v}{l+f}\right)(k(z_c) - z_c). \qquad (7)$$

Similarly for x_3, $k(x_3)$, and $k(x_4)$.

The substitution of all terms is elementary but rather tedious computation. The obtained equation is the implicit form of the control law with one free control term. For example choosing $l(t)$, $v(t)$ is computed.

For this example, let us suppose that $y_c = 0.98$, $z_c = 0.05$ thus, in the steady state, if $H_i = 100$, $F = 100$, $z = .04$, $\alpha = 5$, then, $v = 0.0501$ and $l = 0.55$. Let us choose $v = 0.0501$ and $l(t)$ is computed. The Figures 4, 5, and 6 show the temporal evolution:

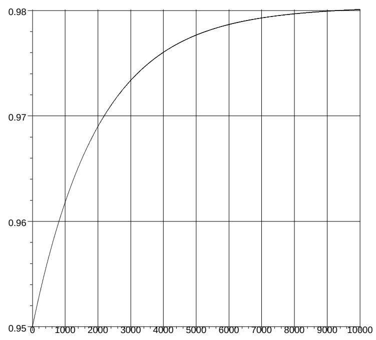

Figure 4: x_1 -vs- t.

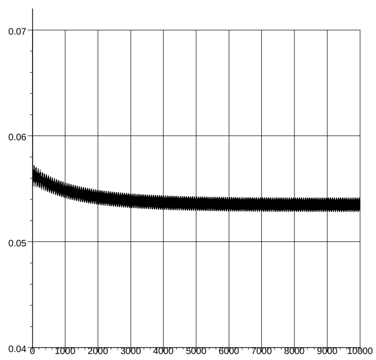

Figure 5: x_5 -vs- t.

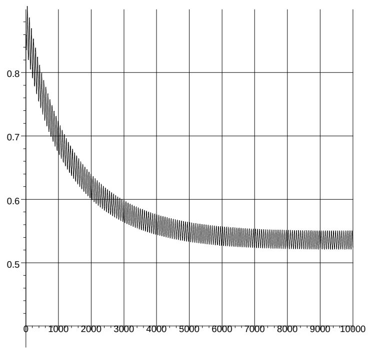

Figure 6: l -vs- t.

5 Conclusions

Dual composition control of a binary distillation columns based on the unmeasured feed composition estimation has been proposed. Unmeasured composition is estimated by using elementary state elimination procedure. This procedure simpler than the disturbance decoupling. The difficulty of the method due to the presence of the measured composition derives. However, low order dynamics allows us to use low order derivatives.

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