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FAULT DETECTION AND ISOLATION IN THE PRESENCE OF UNMEASURED DISTURBANCE: APPLICATION TO BINARY DISTILLATION COLUMNS

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Abstract: In this paper an exhaustive analysis is given in order to design a fault detection filter for the isolation of the pressure decay and the unmeasured feed composition in a binary distillation column. Our design scheme is based on nonlinear observers, state elimination, and the inversion of the error equation. The diagnostic model is developed from chemical principles.

Keywords: Binary distillation column, Diagnostic model, Nonlinear state observer, State elimination, System inversion.

1. INTRODUCTION

The distillation column is one of the most important example in process control theory and practice. Columns handle multicomponent feeds, however, many can be approximated by binary mixtures. A single feed stream is fed as saturated liquid at its bubblepoint onto the feed $(f-th)$ tray. Feed flow rate (mol/min) is a measured, and its composition is an unmeasured disturbance. Many efforts are made to control robustly the distillation process, in spite of the unmeasured disturbance. Unfortunately, the decoupling matrix of the standard model of an idealized binary distillation column, see (Luyben 1990), is ill posed. Its decoupling matrix is almost singular, because the compositions of successive trays are closed to each other. Dual composition control was proposed in (Creff *et al.* 1991). Singular perturbation technics lead to aggregated models, see (Levine and Rouchon 1991). T. Takamatsu *at al.* in (Takamatsu *et al.* 1979), have solved the disturbance attenuation problem, using differential geometric methods. The qualitative description of the stability of the binary distillation columns can be found in (Rosenbrock 1962).

Our purpose is to design nonlinear observers, in order to recover the unmeasured disturbance. State observers are well known in fault detection and isolation, see (Edelmayer *et al.* 1997), (Szigeti 1998), however those are used principally in the linear and bilinear case. Nonlinear observer based fault detection and isolation had success in the bilinear case. Nonlinear observer in itself is hard computationally. The design scheme which is followed in this paper can be outlined by a few steps. State elimination allows us to design a nonlinear observer such that the error equation is almost non linear and can be inverted the input (fault terms) and the output (output error). Hence, decoupling obtained by inversion, solve the problem of fault detection and isolation. However, it may occur that the inverted error equation is unstable. In that case a post filter is required. Our principal model is a binary distillation column. It is supposed that the feed composition is an unmeasured disturbance. Pressure decay will be isolated from the effect of the unmeasured feed composition. First a diagnostic model is developed in order to isolate the change of the pressure and the feed composition. Then, by elementary state elimination all state variables are expressed by the measured top and bottom composition. This allows us to design a nonlinear state observer with invertible and stable error equation. Thus the proposed fault detection and isolation problem is solved for th binary distillation column.

2. BINARY DISTILLATION COLUMN.

A binary system (two components) is assumed with constant relative volatility throughout the column, that is, the vapor leaving the tray is in equilibrium with the liquid on the tray. Thus a simple vapor-liquid equilibrium relationship can be used

$$k(x) = \frac{\alpha x}{1 + (\alpha - 1)x}.$$

Any delay time will be neglected. We notice that in the vapor-liquid equilibrium relationship the α is constant at given pressure and it varies by the pressure. This fact allows us to make diagnosis and detect anomalies at the pressure by monitoring the change of α. The vapor rate $v(t)$ through all trays of the column is the same, dynamically as well as at steady state. The liquid flow rate leaving the trays is $l(t)$. The measured feed flow rate is $f(t)$, and the unmeasured composition is $z(t)$ at time t. Then the nonlinear state equations are

$$\begin{aligned}
H_1\dot{x}_1 &= -v(t)x_1 + v(t)k(x_2),\\
H_i\dot{x}_i &= l(t)\left(x_{i-1} - x_i\right) + \\
&\quad + v(t)\left(k(x_{i+1}) - k(x_i)\right),\\
&\quad i = 2, \ldots, m-1,\\
H_m\dot{x}_m &= l(t)\left(x_{m-1} - x_m\right) - f(t)x_m + \\
&\quad + v(t)\left(k(x_{m+1}) - k(x_m)\right) + f(t)z,\\
H_i\dot{x}_i &= l(t)\left(x_{i-1} - x_i\right) + f(t)(x_{i-1} - x_i) + \\
&\quad + v(t)\left(k(x_{i+1}) - k(x_i)\right),\\
&\quad i = m+1, \ldots, n-1,\\
H_n\dot{x}_n &= l(t)\left(x_{n-1} - x_n\right) + f(t)(x_{n-1} - x_n) + \\
&\quad + v(t)x_n - v(t)k(x_n),
\end{aligned}$$

where $y_1 = x_1$, $y_2 = x_n$ are the top and the bottom components, respectively.

Our purpose is to design a nonlinear observer to detect and isolate the change of the pressure in the presence of the unmeasured disturbance $z(t)$.

3. DIAGNOSTIC MODEL.

Let us assume that the nominal value of the parameter α is α_0. The corresponding nominal function is

$$k_0(x) = \frac{\alpha_0 x}{1 + (\alpha_0 - 1)x}.$$

The nominal unmeasured feed composition can be time varying $z_0(t)$. Then, the corresponding diagnostic model can be delivered from the interpretation that the first fault function is $\phi_1(t) = \alpha(t) - \alpha_0(t)$, while the second one is $\phi_2(t) = z(t) - z_0(t)$.

$$\begin{aligned}
&H_1\dot{x}_1 + v(t)x_1 - v(t)k_0(x_2) = (v(t)k_0(x_2) - \\
&\left(v(t)x_1 + H_1\dot{x}_1\right)k_0(x_2))\,\frac{\phi_1(t)}{\alpha_0} \approx \\
&\approx v(t)\left(k_0(x_2)(1 - k_0(x_2))\right)\frac{\phi_1(t)}{\alpha_0},\\
&H_i\dot{x}_i - l(x_{i-1} - x_i) - v(k_0(x_{i+1}) - k_0(x_i)) = \\
&v\left(k_0(x_{i+1})(1 - k_0(x_{i+1})) - \right.\\
&\left.-k_0(x_i)(1 - k_0(x_i))\right)\frac{\phi_1(t)}{\alpha_0} + v\left(k_0(x_{i+1}) - k_0(x_i)\right)\\
&k_0(x_i)k_0(x_{i+1})\frac{(\phi_1)^2}{(\alpha_0)^2} \approx v\left(k_0(x_{i+1})(1 - k_0(x_{i+1}))\right.\\
&\left.-k_0(x_i)(1 - k_0(x_i))\right)\frac{\phi_1}{\alpha_0},\ i = 2, \ldots, m-1,\\
&H_m\dot{x}_m - l(x_{m-1} - x_m) + fx_m - \\
&-v\left(k_0(x_{m+1}) - k_0(x_m)\right) - fz_0 = \\
&= v\left(k_0(x_{m+1})(1 - k_0(x_{m+1})) - \right.\\
&-k_0(x_m)(1 - k_0(x_m)) - \\
&\left.-fz_0\left(k_0(x_m) + k_0(x_{m+1})\right)\right)\frac{\phi_1(t)}{\alpha_0} + \\
&+f\left(k_0(x_m) + k_0(x_{m+1})\right)\phi_1\phi_2,\\
&H_i\dot{x}_i - (l + f)(x_{i-1} - x_i) - v(k_0(x_{i+1}) - \\
&-k_0(x_i)) = v\left(k_0(x_{i+1})(1 - k_0(x_{i+1})) - \right.\\
&\left.-k_0(x_i)(1 - k_0(x_i))\right)\frac{\phi_1(t)}{\alpha_0} + v\left(k_0(x_{i+1}) - \right.\\
&\left.-k_0(x_i)\right)k_0(x_i)k_0(x_{i+1})\frac{(\phi_1)^2}{(\alpha_0)^2}\\
&\approx v\left(k_0(x_{i+1})(1 - k_0(x_{i+1})) - k_0(x_i)(1 - k_0(x_i))\right)\frac{\phi_1}{\alpha_0},\\
&i = m+1, \ldots, n-1,\\
&H_n\dot{x}_n - (l + f)\left(x_{n-1} - x_n\right) - vx_n + vk_0(x_n) = \\
&vk_0(x_n)(k_0(x_n) - 1)\frac{\phi_1}{\alpha_0}.
\end{aligned}$$

The second order terms are omitted in order to obtain linear expressions in the fault functions. It can be justified by our purpose, that is the detection and isolation of incipient faults. At the equation corresponding to the feed tray, the term corresponding to the second fault function can be reconsidered and redefined by $\phi_2 := f(k_0(x_m) + k_0(x_{m+1}))\,\phi_1\phi_2$. Then, the corresponding fault signature is $L_2 = e_m = (0,\ldots,0,1,0,\ldots,0)$. The first fault signature $L_1 = L_1(x,v,f)$ is a nonlinear function of the state x, the control $v(t)$ and the feed $f(t)$. Fortunately, the fault signatures $L_1(x(t),v(t),f(t))$, L_2 are linearly independent. Indeed,

$$L_1 = \begin{pmatrix} v\,(k_0(x_2)(1-k_0(x_2))) \\ \cdots \\ v\,(k_0(x_{i+1})(1-k_0(x_{i+1}))- \\ -k_0(x_i)(1-k_0(x_i))) \\ v\,(k_0(x_{m+1})(1-k_0(x_{m+1}))- \\ -\,k_0(x_m)(1-k_0(x_m))) + \\ +fz_0\,(k_0(x_{m+1})+k_0(x_m)) \\ v\,(k_0(x_{i+1})(1-k_0(x_{i+1}))- \\ -k_0(x_i)(1-k_0(x_i))) \\ \cdots \\ vk_0(x_n)(k_0(x_n)-1) \end{pmatrix}$$

and $L_2 = e_m$ are linearly independent.

4. OBSERVER DESIGN.

We notice that from the last equation x_{n-1} can be expressed by

$$\begin{aligned} &H_n\dot{y}_2 - (l+f)(x_{n-1}-y_2) - \\ &-vy_2 + vk_0(y_2)(k_0(y_2)-1)\frac{\phi_1}{\alpha_0} \end{aligned} \qquad (1)$$

and recursively, all x_{i-1} can be expressed in terms of $x_i, x_{i+1},\ldots,x_n$, hence, all x_{i-1} is a differential fraction of y_2, that is, the state elimination, in this case is purely (differential) algebraic. Then, the observer is

$$\begin{gathered} H_1\dot{\hat{x}}_1 = -v\hat{x}_1 + vk_0(x_2) + v\,(k_0(x_2)- \\ -\,(k_0(x_2))^2)\,(x_1-\hat{x}_1) + G_1(y_1-\hat{x}_1), \\ H_i\dot{\hat{x}}_i = l(\hat{x}_{i-1}-\hat{x}_i) + v\,(k_0(x_{i+1})- \\ -k_0(x_i)) + G_i\,(H_i\dot{x}_i + lx_i - v(k_0(x_{i+1})- \\ -\,k_0(x_{i-1})) - l\hat{x}_{i-1})\,; \\ H_{i+1}\dot{\hat{x}}_{i+1} + lx_{i+1} - v(k_0(x_{i+2})- \\ -k_0(x_{i+1})) - l\hat{x}_i, \end{gathered}$$

for $i = 2,3,\ldots,m-1$, and so on. The last equation, similarly to the first one, is

$$\begin{aligned} H_n\dot{\hat{x}}_n = {} & (l+f)\hat{x}_{n-1} + (l+f-v)\hat{x}_n - vk_0(x_n) - \\ & -v\left(k_0(x_n) - (k_0(x_n))^2\right)(x_n-\hat{x}_n) + \\ & +G_n\,(x_{n-1}-\hat{x}_{n-1}, y_2-x_n)\,. \end{aligned}$$

The observer must be stable, which means that the error $e(t) = x(t) - \hat{x}(t)$ must tend to zero at infinity. Then, the error equation is

$$\begin{aligned} H_1\dot{e}_1 = {} & -ve_1 - G_1(e_1) + \alpha_0^{-1}vk_0(x_2)(1-k_0(x_2))e_i \\ & +\alpha_0^{-1}vk_0(x_2)(1-k_0(x_2))\phi_1, \\ H_i\dot{e}_i = {} & l(e_{i-1}-e_i) - G_i(e_{i-1},e_i) + \\ & +\alpha_0^{-1}v\,(k_0(x_{i+1})(1-k_0(x_{i+1}))- \\ & -k_0(x_i)(1-k_0(x_i)))\,\phi_1, \quad i=2,\ldots,m-1; \\ H_m\dot{e}_m = {} & l(e_{m-1}-e_m) - fe_m - G_m(e_{m-1};e_m) + fz_0 + \\ & +\alpha_0^{-1}v\,(k_0(x_{m+1})(1-k_0(x_{m+1}))- \\ & -k_0(x_m)(1-k_0(x_m))) + \\ & +fz_0\,(k_0(x_{m+1}+k_0(x_m)))\,\phi_1\phi_2, \\ H_i\dot{e}_i = {} & (l+f)(e_{i-1}-e_i) - G_i(e_{i-1},e_i) + \\ & +\alpha_0^{-1}v\,(k_0(x_{i+1})(1-k_0(x_{i+1}))- \\ & -k_0(x_i)(1-k_0(x_i)))\,\phi_1, \quad i=m+1,\ldots,n-1; \\ H_n\dot{e}_n = {} & (l+f)(e_{n-1}-e_n) + ve_n - \alpha_0^{-1}G_n(e_{n-1},e_n) + \\ & +vk_0(y_2)(1-k_0(y_2))(e_i-\phi_1). \end{aligned}$$

with output error

$$\eta_1 = e_1 = x_1 - \hat{x}_1, \qquad \eta_2 = e_n = x_n - \hat{x}_n.$$

Then, we can choose the gain terms of the observer by

$$\begin{aligned} & G_1(y_1-\hat{x}_1) = (H_1-v)(y-1-\hat{x}_1), \\ & G_i(x_{i-1}-\hat{x}_{i-1}, x_i-\hat{x}_i) = l(x_{i-1}-\hat{x}_{i-1}) + \\ & +(H_i-l)(x_i-\hat{x}_i), \\ & G_m(x_{m-1}-\hat{x}_{m-1}, x_m-\hat{x}_m) = l(x_{m-1}-\hat{x}_{m-1}) + \\ & +(H_i-l-f)(x_m-\hat{x}_m), \\ & G_i(x_{i-1}-\hat{x}_{i-1}, x_i-\hat{x}_i) = (l+f)(x_{i-1}-\hat{x}_{i-1}) + \\ & +(H_i-l-f)(x_i-\hat{x}_i), \\ & G_n(x_{n-1}-\hat{x}_{n-1}, x_n-\hat{x}_n) = (l+f)(x_{n-1}-\hat{x}_{n-1}) + \\ & +(H_n-l-f+v)(x_i-\hat{x}_n), \end{aligned}$$

where the states $x_2, x_3, \ldots, x_{n-1}$ are computed by (1) and the related equations. Hence, the error equation is a linear system in the error state

$$\dot{e}(t) = \begin{pmatrix} -1 & 0 & \cdots & \mu_1 & 0 & \cdots & 0 \\ 0 & -1 & & & & & \vdots \\ \vdots & & \ddots & & & & \vdots \\ & & & -1 & & & \\ \vdots & & & & & \ddots & 0 \\ 0 & 0 & \cdots & \mu_2 & \cdots & 0 & -1 \end{pmatrix} e(t) + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} fz_0$$

$$+L_1\phi_1 + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \phi_2,$$

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix},$$

where

$$\begin{aligned}
&\mu_1 v\,(k_0(x_{m+1})(1-k_0(x_{m+1}))- \\
&-k_0(x_m)(1-k_0(x_m))) + \\
&+f z_0\,(k_0(x_{m+1}+k_0(x_m))) + \\
&+\frac{d}{dt}\,(v k_0(x_2)(1-k_0(x_2))- \\
&-v k_0(x_2)(1-k_0(x_2))) = 1, \\
&\mu_2 v\,(k_0(x_{m+1})(1-k_0(x_{m+1}))- \\
&-k_0(x_m)(1-k_0(x_m))) + \\
&+f z_0\,(k_0(x_{m+1}+k_0(x_m))) + \\
&+\frac{d}{dt}\,(v k_0(x_n)(1-k_0(x_n))- \\
&-v k_0(x_n)(1-k_0(x_n))) = 0.
\end{aligned}$$

Let us compute the disturbance term by successive derivation. From the first equation

$$e_2(t) = \dot{e}_1 + e_1(t) = \dot{\eta}_1(t) + \eta_1(t),$$

then, by induction

$$e_{i+1}(t) = \dot{e}_i(t) + e_i(t) = \left(\sum_{j=0}^{i-1} \binom{i-1}{j} \eta_1^{(j)} \right) + \\ + \sum_{j=0}^{i-1} \binom{i-1}{j} \eta_1^{(j)} = \sum_{j=0}^{i} \binom{i}{j} \eta_1^{(j)}.$$

Hence, the disturbance can be expressed by the output error η_1 and its derivatives

$$f(t) z_0(t) = \dot{e}_m(t) + e_m(t) = \sum_{j=0}^{m} \binom{m}{j} \eta_1^{(j)}. \quad (3)$$

Similarly, the disturbance can also be expressed by the output error η_2.

$$e_{n-1} = \dot{e}_n + e_n = \dot{\eta}_2 + \eta_2.$$

By induction

$$e_{n-i} = \sum_{j=0}^{i} \binom{i}{j} \eta_2^{(j)}, \ldots, e_m = \sum_{j=0}^{n-m} \binom{n-m}{j} \eta_2^{(j)},$$

hence

$$f(t) z_0(t) = \sum_{j=0}^{n-m+1} \binom{n-m+1}{j} \eta_2^{(j)} \quad (4)$$

is an another expression for the disturbance, in terms of the output error η_2 and its derivatives. The formula of lower order is used in the practice. Let us consider the second derivative of the output (2) of the error equation with the fault terms,

$$\begin{aligned}
\begin{pmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix} \Big((E^2 + \dot{E})e + E b_m f z_0 + \\
&\quad + (E L_1 + \dot{L}_1)\phi_1 + L_1 \dot{\phi}_1 + L_2 \phi_2 \Big), \\
&= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix} \Big((E^2 + \dot{E})e + E b_m f z_0 + \\
&\quad + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \phi_1 + \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \phi_2 + L_1 \dot{\phi}_1 \ .
\end{aligned}$$

The decoupling matrix $\begin{pmatrix} 1 & \mu_1 \\ 0 & \mu_2 \end{pmatrix}$ is invertible if and only if $\mu_2 \neq 0$, which is obvious. Substituting the expressed fault functions in the differential equations then inverse system is obtained, decoupling the fault functions ϕ_1, ϕ_2.

5. CONCLUDING REMARKS.

The appearing derivatives can be computed by spline approximation to beat down the unstabilizing effects of the derivatives. The robustness issues must be consider in different sense. Using the simple vapor-liquid equilibrium relationship mentioned above, is an ideal case. The method must be robust with respect to a more realistic, unmodelled situation. Mor general nonlinear plant can also be treated similarly. Obviously, in that case, instead of our simple elimination algorithm, the celebrated Diop's state elimination is used. Our method can be used to isolate 2 faults in the presence of the unmeasured feed composition, because the unmeasured disturbance can be expressed by the outputs and their derivatives. Substituting (3), or (4) in the error equation the appearing output error in the error equation does not modifies the invertibility of the error system.

6. REFERENCES

Creff, Y., J. Levine and P. Rouchon (1991). Qualitative behaviour of distillation columns and their control. In: *Proc. 1st European Control Conference*. Hermés Paris. pp. 1069–1075.

Edelmayer, A., J. Bokor, F. Szigeti and L. Keviczky (1997). Robust detection filter design in the presence of time varying systems perturbacions. *Automatica* pp. 471– 480.

Levine, J. and P. Rouchon (1991). Quality control of binary distillation columns based on nonlinear aggregated models. *Automatica* pp. 463–480.

Luyben, W. L. (1990). *Process Modeling, Simulation and Control for Chemical Engineers.* McGraw-Hill Publishing. New York.

Rosenbrock, H. H. (1962). A lyapunov function with applications to some nonlinear physical systems. *Automatica* pp. 31–53.

Szigeti, F. (1998). Detección y dignóstico de fallas: Métodos basados en diagonalización y planitud. In: *Proc. de las 4^{tas} Jornadas Científico-Tecnológicas de la Facultad de Ingeniería.* Mérida - Venezuela.

Takamatsu, T., I. Hashimoto and Y. Nakai (1979). A geometric approach to multivariable system design of a distillation column. *Automatica* pp. 387–402.